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Topology Optimization Of High Speed Flexible Robot Arms

التصميم التوبولوجى الأمثل لأذرع الانساليات السريعة المرنة

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ملخص البحث

تتمثل المتطلبات المتزايدة في مجال صناعة الانساليات في بناء هياكل خفيفة تسهم في زيادة سرعة حركة الإنسان الآلي ومن ثم زيادة الإنتاجية. وقد تم في الأبحاث السابقة الحصول على أقصر وقت ممكن لذراع الإنسان الآلي بتنفيذ التصميم البعدى والشكلي الأمثل بأخذ مرونة الذراع فى الاعتبار. أما فى البحث الحالي فقد تم استخدام التصميم التوبولوجى الأمثل باعتبار التصميم البنائي والتحكم معاً للحصول على سرعة الوصول المطلوبة. ولذلك فقد تم استخدام طريقة خطوط التقارب المتحركة كأسلوب للتصميم الأمثل حيث أن هذه الطريقة تتيح التعامل مع أي عدد من معادلات القيود كما أنها تعتبر من الطرق الشاملة والمرنة للتصميم الأمثل. وقد تم عمل مقارنة بين التصميم الناتج من هذه الطريقة والتصميم الاسمي (أو الأولى) باعتبار أبعاد مختلفة للذراع الآلي مع إهمال مقاومة الهواء فوجد أنه تم تخفيض زمن الوصول بنسبة 62% كما تم تخفيض وزن الذراع بنسبة 70% وهى نسب جديرة بالاعتبار. تم أيضا عمل مقارنة بين نتائج هذا البحث باستخدام التصميم التوبولوجى الأمثل مع نتائج الأبحاث السابقة باستخدام التصميم البعدى والشكلي الأمثل فوجد تميز واضح لطريقة التصميم التوبولوجى التي أعطت نتائج واعدة خصوصا عند أخذ مقاومة الهواء فى الاعتبار حيث أنها الحالة الأكثر واقعية حيث وصلت نسبة التخفيض فى زمن الوصول 44.8% والتي كانت فقط 23.5% فقط فى الأبحاث السابقة.

Abstract

There are growing demands in the robot industry for light structures in order to increase the speed of the robot motion and thus the productivity. Size and shape optimization are carried out previously to achieve minimum travelling time of the robot arm taking into consideration its flexibility. In this work, topology optimization is introduced for obtaining optimum robot arm to achieve minimum travelling time from both structure and control viewpoints. Method of Moving Asymptotes is used as optimization technique, because it can handle arbitrary number of constraints and considered to be a general and flexible optimization method. A comparison is accomplished between the optimal resulted design and its initial design for different dimensions of robot arm neglecting air damping. It is found that the reduction ratio of the travelling time reaches 62% and the reduction ratio of the weight reaches 70%. A comparison between this work using topology optimization technique and previous works that used size or shape optimization is carried out. It is found that the topology optimization of flexible robot arm outperforms the size and shape optimization when considering the real condition that takes into account the air damping. The optimum topological design in this case gives reduction ratio of travelling time equals 44.8%, while previous work gives 23.5%.

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1. Introduction

Novel robotic applications demand lighter robots that can be driven using small amounts of energy, for example robotic booms in the aerospace industry, where lightweight manipulators with high performance requirements such as, high operational speed, better accuracy and high payload/weight ratio are required [18]. Another example is the need for lightweight manipulators to be mounted on mobile robots, where power limitations imposed by battery autonomy have to be taken into account. Unfortunately, the flexibility of these lightweight robots leads to oscillatory behavior at the tip of the link, making precise pointing or tip positioning a daunting task that requires complex closed-loop control.

In the early 70's the necessity of building lighter manipulators able to perform mechanical tasks arises as a part of the space research. The abusive transportation costs of a gram of material into orbit and the reduced room and energy available inside an spacecraft cause the imperative need for reducing weight and size as far as possible in any device aboard. Unfortunately, as the manipulator reduces weight, it reduces also accuracy in its manoeuvres due to the appearance of structural flexibility and hence, vibrations of the device [5]. Many current approaches aimed to decrease the end-effectors residual vibration deal with improving the structural design of robot arm or adopting advanced control techniques. The approach which is presented here improves both the controller and the structural design simultaneously. The available methods for structural design modification can be classified as follows: (1) Optimizing the weight to stiffness ratio by varying the length and cross-section of each metallic robot arm of the system [17], (2) Implementing advanced composite material for the robot's structural designs [8], (3) Applying size/shape optimization for robot arm structural design integrated with time optimal control theory as presented in [2]. Topological optimization approach is introduced in this work for structural design of robot arms integrated with time optimal control theory. This approach leads to optimum robotic arm from both structural design and control viewpoints.

2. Time Optimal Control Of Flexible Robot Arm

2. 1. Mathematical Model Without Air Damping

The application of time optimal control theory, to get the minimum traveling time of a flexible robot arm, results in a multi switch bang-bang control. The problem can be formulated as follows: given a single flexible robot arm which moves in a horizontal plane and an actuator with a maximum torque $\mathbf{T_0}$. How should the actuating torque vary with time to make the arm rotates a given angle $\boldsymbol{\theta_0}$, from initial rest to a final rest state in a minimum time (see Fig. 1). The arm is represented in this model by one rigid mode and one flexible mode without damping, see [2]. The state equations are:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}T(t), \quad (1)$$

Where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ b_0 \\ 0 \\ b_1 \end{bmatrix}$$

$\mathbf{x}(t)$ is the modal coordinate vector, $T(t)$ is the actuating torque, ω is the natural frequency of the flexible mode, b_0 is the component of weighted rigid mode shape at a coordinate where the torque is applied, $((1/b_0)^2 = J$ is the moment of inertia the arm about the rotating axis) and b_1 is the same as b_0 but for flexible mode, $(b_1^2 = q$ is the flexible mode participation coefficient [15]). Both b_0 and b_1 can be obtained using finite element method (FEM). See also [8, 12, 15, 16].

Through applying the Pontryagin maximum principle, the optimal solution is found to be a multi-switch bang-bang control, (see [16]). The switches are symmetrical about the middle switch so one needs to calculate only two time intervals ($t_f/2$-t_a), $t_f/2$, and ($t_f/2$+t_a).

$$(t_f/2)^2 - 2t_a^2 = \frac{\theta_o J}{T_o} \qquad (2)$$

$$\cos(\omega\, t_f/2) - 2\cos(\omega\, t_a) + 1 = 0 \qquad (3)$$

Figure 2, shows the three switches, where t_f is the optimal travelling time. It is seen from equations (2, 3) that, the optimal traveling time, t_f, depends on the inertia J as well as the natural frequency ω of the arm. Both quantities depend on the construction of the robot arm. So, through optimizing the arm topology, one can further decrease the traveling time. So, a minimum traveling time is obtained from control as well as structure view points.

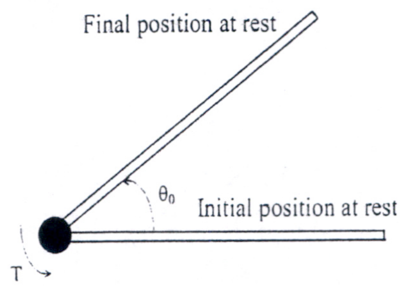

Fig. 1. Flexible robot arm.

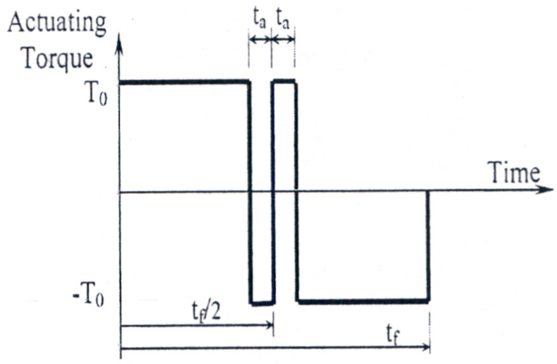

Fig. 2. Bang-bang control of flexible robot arm.

2. 2. Mathematical Model With Air Damping

The air damping is modeled by an equivalent viscous damper. The angular velocity of the arm is measured at different voltages and the relation between the arm angular velocity and the damping torque is linearized about an average operating arm velocity. Another way is averaging directly the viscous damping coefficients measured at different operating velocities. Both methods give close results. The second method is considered further because of its simplicity.

It is clear that, the air damping is negligible for the flexible mode, because the deflection of the flexible mode is small. The state equations of the flexible arm is the same as equation (1) except matrix A which becomes:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega^2 & 0 \end{bmatrix} \quad (4)$$

where α equals the damping coefficient, C, divided by moment of inertia of the arm, J. The solution of the Pontryagin maximum principle is a multi-switch bang-bang control but not symmetrical about the middle switch as in the previous case without damping. Through applying the final state conditions, which dictate that the angular velocity must be zero and the angular displacement must equal $\boldsymbol{\theta}_o$, the following equations (in dimensionless form) are derived:

$$1 - 2e^{-y_4/x} + 2e^{-(y_3+y_4)/x} - 2e^{-(y_2+y_3+y_4)/x} + 2e^{-(y_1+y_2+y_3+y_4)/x} = 0 \quad (5)$$

$$y_1 - y_2 + y_3 - y_4 - z = 0 \quad (6)$$

$$1 - 2\cos y_4 + 2\cos(y_3 + y_4) - 2\cos(y_2 + y_3 + y_4) + \cos(y_1 + y_2 + y_3 + y_4) = 0 \quad (7)$$

$$2\sin y_4 - 2\sin(y_3 + y_4) + 2\sin(y_2 + y_3 + y_4) - \sin(y_1 + y_2 + y_3 + y_4) = 0 \quad (8)$$

where the dimensionless quantities are defined as:

$$x = \frac{J\omega}{C}, \quad z = \frac{\theta_0 \omega C}{T_0}, \quad y_i = \omega t_i\,, \quad i = 1, \dots 4 \quad (9)$$

where t_1, t_2, t_3 and t_4 are the time intervals between each two consecutive switchings. Note that:

$$y_1 + y_2 + y_3 + y_4 = y = \omega t_f \quad (10)$$

The solution of these equations for minimum t_f is now obtained and is found to have the following characteristics, see Fig. 3. The first and last intervals (t_1, t_4) are much larger than the inner intervals, which is similar to the previous case without damping. The time of positive torque is larger than that of negative torque (unlike the previous case, where they are equal). Then y as function of x and z are obtained by means of multi-regression technique [2]:

$$y = \sqrt{0.4z^{2.23264} + 3.93xz} \quad (11)$$

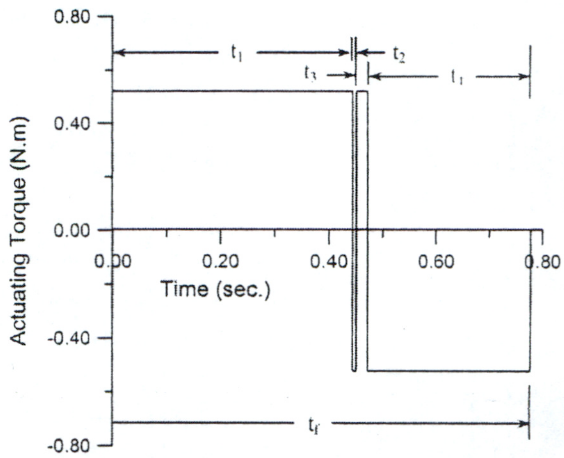

Fig. 3 Torque-Time diagram for the damped model.

3. Topology Optimization

3. 1. Problem Definition

Topology optimization is a relatively new but extremely rapidly expanding research field, which has interesting theoretical implications in mathematics, mechanics, multi-physics and computer science, but also important practical applications by the manufacturing industries such as car and aerospace industries, and is likely to have a significant role in micro- and nano-technologies [13]. Topology optimization strives to achieve the optimal distribution of material within finite volume design domain, which maximizes a certain mechanical performance under specified constraints. Its algorithms selectively removes and relocates the elements to achieve the optimum performance [14]. It can provide a good configuration concept for the structure as a minimum compliance or maximum stiffness design ...etc.

The presently most popular numerical FE-based topology optimization method is the Solid Isotropic Material with Penalization method (SIMP), which was developed in the late eighties. The basic idea of this approach was proposed by Bendsøe in [19], the material properties are assumed constant within each element that is used to discrete the design domain, normally, a continuous relative density is used as a design variable [1]. The elastic modulus of each element, E_i , is modeled as a function of the relative density, x_i, using a power law. That is used for updating the elastic modulus which is entering to the structural model at each iteration as in the following equation.

$$\rho_i(x_i) = \rho_0 \, x_i$$

$$E_i(x_i) = E_0 \, x_i^p \quad ,(0 \le x_i \le 1) \qquad (12)$$

Where ρ_i is element density, ρ_0 is the initial density, E_0 is the elastic modulus of the base material and p is a penalization power. This power penalizes intermediate densities and drives the design to a black (material) and white (holes) structure. The proper value of p, depends on Poisson's ratio v, and can be calculated from equations (13, 14) for 2D and 3D model respectively, see [3].

$$\text{P} \ge \max\left\{\frac{2}{1-\text{v}} \, , \, \frac{4}{1-\text{v}}\right\} \qquad (2D) \quad (13)$$

$$\text{P} \ge \max\left\{(15)\frac{1-\text{v}}{7-5\text{v}} \, , \, \left(\frac{3}{2}\right)\frac{1-\text{v}}{1-2\text{v}}\right\} \quad (3D) \, (14)$$

3. 2. Numerical Instability In Topology Optimization

There are some problems in the convergence of topology optimization process. Some of these problems are checkerboard and mesh-dependency. Using mesh dependent filtering that proposed by Sigmund and Peterson in [9] which is also an extension of the checkerboard filter will enhance the convergence. This filter modifies the design sensitivity of a specific element based on a weighted average of the element sensitivities in a fixed neighborhood.

The mesh dependence scheme works by modifying the element sensitivities as follows:

$$\frac{\widehat{\partial f}}{\partial x_k} = (x_k)^{-1} \frac{1}{\sum_{i=1}^{N} \hat{H}_i} \sum_{i=1}^{N} \hat{H}_i \, x_i \frac{\partial f}{\partial x_i} \qquad (15)$$

The convolution operator (weight factor) $\hat{H}_i$ is written as

$$\hat{H}_i = r_{min} - dist(k, i),$$
$$\{i \in N | \, dist(k, i) \le r_{min} \}, k = 1, \dots, N$$

where f is the objective function, $\frac{\widehat{\partial f}}{\partial x_k}$ is the sensitivity of objective function, the operator $dist(k, i)$ is defined as the distance between the center of element k and the center of element i. The convolution operator $\hat{H}_i$ is zero outside the filter area. The convolution operator for element i is seen to decay linearly with the distance from element k.

4. Method of Moving Asymptotes

Method of Moving Asymptotes is a method that turned out to be very efficient for topology optimization problems in an academicals and industrial environments, as its mother method CONLIN see (Fleury and Braibant, 1986). The MMA works with a sequence of simpler approximating subproblems (similar to Sequential Linear Programming SLP, and SQP), but their approximation is based on terms of direct and reciprocal design variables. A major advantage of the MMA is that these local models are convex and separable and only require one function and gradient evaluation at the iteration point. Separability here means that the necessary optimality conditions of the subproblem do not couple the design variables. This yields that instead of one n-dimensional problem we have to solve n one-dimensional problems. Convexity means that dual or primal-dual methods can be used to attack the subproblems. These valuable properties allow reducing computational costs for solving the subproblems significantly [7]. A solution of a subproblem is then used as the next iteration point. Consider a structural optimization problem of the following form:

$$\text{Min: } f_0(x) \quad (x \in R^n) \tag{16}$$

$$\text{S. t.: } f_i(x) \le \hat{f}_i, \text{ for } i = 1, \dots, m \tag{17}$$

$$\underline{x}_j \le x_j \le \bar{x}_j, \text{ for } j = 1, \dots, n \tag{18}$$

Where $x = (x_1, \dots\dots, x_n)^T$ is the vector of design variables, $f_0(x)$ is the objective function, $f_i(x) \le \hat{f}_i$, is the behavior constraints, $\bar{x}_j$ and $\underline{x}_j$ are given upper and lower bounds.

The method is interpreted in brief that each $f_i^{(k)}$ where k is the current iteration, is obtained by a linearization of f_i in variables of the type $1/(x_j - L_j)$ or $1/(U_j - x_j)$ dependent on the signs of derivatives of f_i at $x^{(k)}$. The values of the parameters L_j and U_j are normally changed between the iterations, and we will always refer to L_j and U_j as "Lower and Upper Moving Asymptotes".

At each iteration, the current iteration point $x^{(k)}$ is given. Then an approximating explicit subproblem is generated. In this subproblem, the functions $f_i(x)$ are replaced by approximating convex functions $\tilde{f}_i^{(k)}(x)$ which are chosen as:

$$\tilde{f}_i^{(k)}(x) = \sum_{j=1}^{n} \left(\frac{p_{ij}^{(k)}}{U_j - x_j} + \frac{q_{ij}^{(k)}}{x_j - L_j} \right) + r_i^{(k)}, \; i = 0,1,\dots,m \tag{19}$$

$$L_j^{(k)} < x_j^{(k)} < U_j^{(k)} \tag{20}$$

Where,

$$p_{ij}^{(k)} = \begin{cases} (U_j^{(k)} - x_j^{(k)})^2 \, \partial f_i/\partial x_j, & \text{if } \partial f_i/\partial x_j > 0 \\ 0, & \text{if } \partial f_i/\partial x_j \le 0 \end{cases} \tag{21}$$

$$q_{ij}^{(k)} = \begin{cases} 0, & \text{if } \partial f_i/\partial x_j \ge 0 \\ -(x_j^{(k)} - L_j^{(k)})^2 \, \partial f_i/\partial x_j, & \text{if } \partial f_i/\partial x_j < 0 \end{cases} \tag{22}$$

$$r_i^{(k)} = f_i(x^{(k)}) - \sum_{j=1}^{n} \left(\frac{p_{ij}^{(k)}}{U_j - x_j} + \frac{q_{ij}^{(k)}}{x_j - L_j} \right) \tag{23}$$

Further, the second derivatives of $\tilde{f}_i^{(k)}$, at any point x are given by

$$\frac{\partial^2 \tilde{f}_i^{(k)}}{\partial x_j^2} = \frac{2p_{ij}^{(k)}}{\left(U_j^{(k)} - x_j\right)^3} + \frac{2q_{ij}^{(k)}}{\left(x_j - L_j^{(k)}\right)^3}$$

$$\frac{\partial^2 \tilde{f}_i^{(k)}}{\partial x_j \, \partial x_i} = 0 \quad \text{if } j \ne i \tag{24}$$

Thus the closer $L_j^{(k)}$ and $U_j^{(k)}$ are chosen to $x_j^{(k)}$, the larger become the second derivatives, the more curvature is given to the approximating function $\tilde{f}_i^{(k)}$ and more conservative becomes the approximation of the original problem. Correspondingly, if $L_j^{(k)}$ and $U_j^{(k)}$ are chosen far away from $x^{(k)}$, then $f_i^{(k)}$ becomes close to linear. In the extreme case that $L_j^{(k)} = -\infty$ and $U_j^{(k)} = +\infty$ for all j, then the $f_i^{(k)}$ become identical to linear functions below,

$$f_i^{(k)}(x) = f_i(x^{(k)}) + \Sigma_j(\partial f_i/\partial x_j)(x_j - x_j^{(k)}) \quad (25)$$

For more details see [4, 11].

4. 1. Effect Of Asymptotes L_j , U_j On Optimization Process Convergence.

Since the method of moving asymptotes is a general method, so the asymptotes can be adopted to be suitable for seeking the demanded convergence of specific problems.

A general (although heuristic) rule for how to change the values of $L_j^{(k)}$ and $U_j^{(k)}$ is the following:

a) If the process tends to oscillate, then it needs to be stabilized. This stabilization may be accomplished by moving the asymptotes closer to the current iteration point.

b) If, instead, the process is monotone and slow, it needs to be relaxed. This may be accomplished by moving the asymptotes away from the current iteration point. See [4].

The default rules for updating the lower asymptotes $L_j^{(k)}$ and the upper asymptotes $U_j^{(k)}$ are as follows.

The first two iterations, when k =1 and k =2.

$$L_j^{(k)} = x_j^{(k)} - asyint\ (x_j^{max} - x_j^{min})$$

$$U_j^{(k)} = x_j^{(k)} + asyint\ (x_j^{max} - x_j^{min})$$

In later iterations, when $k \geq 3$

$$L_j^{(k)} = x_j^{(k)} - \gamma_j^{(k)}\left(x_j^{(k-1)} - L_j^{(k-1)}\right)$$
$$U_j^{(k)} = x_j^{(k)} + \gamma_j^{(k)}\left(U_j^{(k-1)} - x_j^{(k-1)}\right)$$

Where,

$$\gamma_j^{(k)} = \begin{cases} asydecr, & if(x_j^{(k)} - x_j^{(k-1)})(x_j^{(k-1)} - x_j^{(k-2)}) < 0 \\ asyincr, & if(x_j^{(k)} - x_j^{(k-1)})(x_j^{(k-1)} - x_j^{(k-2)}) > 0 \\ 1, & if(x_j^{(k)} - x_j^{(k-1)})(x_j^{(k-1)} - x_j^{(k-2)}) = 0 \end{cases}$$

Where the default value of $\boldsymbol{asyint}$ equals 0.5, $\boldsymbol{asydecr}$ equals 0.7 and $\boldsymbol{asyincr}$ equals 1.2, see [11].

It is also found that there are some rules that can be used in the sub-problem file in MMA code. That can be added to the default one, see [6]. These rules are:

$$L_{j\,min}^{(k)} = x_j^{(k)} - S_{max}\ (x_j^{max} - x_j^{min})$$

$$L_{j\,max}^{(k)} = x_j^{(k)} - S_{min}\ (x_j^{max} - x_j^{min})$$

$$U_{j\,min}^{(k)} = x_j^{(k)} + S_{min}\ (x_j^{max} - x_j^{min})$$

$$U_{j\,max}^{(k)} = x_j^{(k)} + S_{max}\ (x_j^{max} - x_j^{min})$$

$$L_j^{(k)} = \max\ (L_j^{(k)}, L_{j\,min}^{(k)})$$

$$L_j^{(k)} = \min\ (L_j^{(k)}, L_{j\,max}^{(k)})$$

$$U_j^{(k)} = \min\ (U_j^{(k)}, U_{j\,max}^{(k)})$$

$$U_j^{(k)} = \max\ (U_j^{(k)}, U_{j\,min}^{(k)})$$

Where the default values of S_{max} and S_{min} is 10 and 0.01. These values can be changed to suit any optimization problem.

5. Optimal Integrated Structural/ Control Design

5. 1. Problem Formulation For Topology Optimization Of Flexible Robot Arm Without Air Damping

Using the design domain as a slender arm made of Aluminum with rectangular section which is attached to the motor hub in such a way that it rotates only in the horizontal plane, so that the effect of gravity can be ignored, and subjected to a force **(F)** at the tip of the arm see Fig. 4. The arm has dimensions of 300x20x3 mm. All specifications for the model are shown in Table.1.

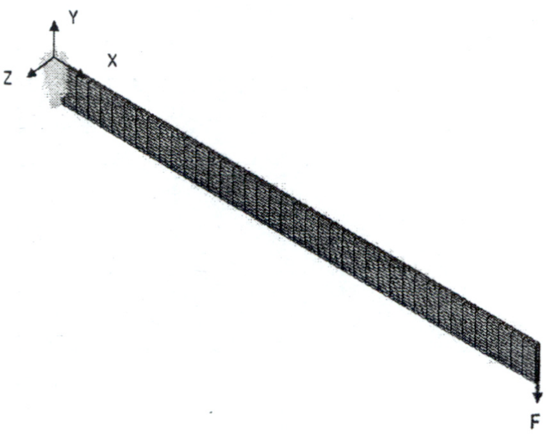

Fig. 4. The initial design of high speed flexible robot arm.

From equations (2) and (3) we can deduce an expression for the travelling time t_f , to use it as an objective function in MMA algorithm, where t_f depends on the inertia as well as the natural frequency of the arm, as shown in the following implicit equation:

$$\cos(\omega t_f/2) - 2\cos\left(\omega\sqrt{\left((t_f/2)^2 - \theta_o J/T_o\right)/2}\right) + 1 = 0 \quad (27)$$

Then the general form of topology optimization problem for flexible robot arm without air damping is:

$$min: \qquad t_f$$

$$\text{s. t. :} \qquad defl - \delta_{max} \leq 0$$

Where the angle $\boldsymbol{\theta_o}$ and torque $\boldsymbol{T_o}$ are constant through the optimization process while the natural frequency $\boldsymbol{\omega}$ and mass moment of inertia $\boldsymbol{J}$ varies with the relative density of each element $\boldsymbol{x_i}$ that considered as the design variable. The maximum deflection of the robot arm, $defl$, is constrained to be lower the prespecified value, δ_{max}.

Table .1 Aluminum arm 300x20x3 mm

Properties	**Values**
Young's modulus, E (Gpa)	70
Density, ρ (kg/m^{3})	2700
Poisson's ratio, ν	0.3
Force, F (N)	1
SIMP factor, P	3
No. of elements along x, y and z direction	40*12*2
Initial design variable *x* for all elements.	1

The maximum torque of the DC motor $\mathbf{T_o}$ is assumed 0.20 N.m. and. The angular displacement $\boldsymbol{\theta_o}$ is 180°. For the initial design of the arm, the natural frequency, ω, is 2.01230 HZ, and the mass moment of inertia of the arm and the hub, J, is 0.00191007 kg.m^2. Where the ratio of the inertia of hub to the arm equal to 0.31. Both ω and J are calculated using Finite element method.

The Moving Asymptotes, MMA, code that presented by K. Svanberg [6] written with MATLAB program is used. The default values of S_{max} and S_{min} for the asymptotes is changed to suit this topology optimization problem to be 400 and 0.01.

The finite element analysis using ANSYS is implemented. The arm is modeled using 3D solid elements, SOLID45. The inertia of the hub is modeled by concentrated mass element, MASS21, located at certain distance from the vertical rotating axis of the arm. The static analysis is used for determining the maximum deflection of the arm. The modal analysis is carried out for determining the natural frequency of the flexible robot arm. δ_{max} for the deflection constraint is chosen equal to 1 mm.

5. 2. Problem Formulation For Topology Optimization Of Flexible Robot Arm With Air Damping

The problem of topology optimization for a flexible robot arm with air damping will be formulated using the same constraints as previous problem. But the difference here is in the deduction of the travelling time, t_f, equation that will be used in the objective function. From equations (9) to (11), travelling time equation can be obtained as follow:

$$t_f = \sqrt{0.4\left(\frac{\theta_0 \omega C}{T_0}\right)^{2.23264} + 3.93\left(\frac{J\theta_0\omega^2}{T_0}\right)} \Big/ \omega \quad (30)$$

Where, C is the damping coefficient and it is a function of the arm dimension, as follows:

$$C = D\int_0^l r^3 w(r)dr \qquad \text{then,}$$

$$C = D\sum_{i=1}^{n} \rho_i b\left[\frac{(k_i + a)^4}{4} - \frac{k_i{}^4}{4}\right] \quad (31)$$

Where, D equal to 15.16645 in [2]. a and b are the element size in x and y direction. k_i is the horizontal distance between axis of rotation and element i.

Two robot arms are used for optimization. The first one has initial dimensions of 300x20x3 mm and the second one has initial dimensions of 760x20x3 mm. Both arms are made of Aluminum with properties shown in Table. 1.

6. Results And Discussions

6. 1. Results Of Topology Optimization Of Flexible Robot Arm Without Air Damping

By running the MMA algorithm [6], with the interface between MATLAB and ANSYS program, we can see that the travelling time at the beginning of optimization process is 0.540 sec. and finally, after 49 iterations becomes 0.201 sec., see Fig. 5. Consequently, the reduction ratio of travelling time is 62.7%. The deflection of the initial design is 0.06412 mm and becomes 0.471 mm at final design. Also the final volume has become 30.3% of the initial one. All results are shown in Table.2.

Table. 2. The results of topology optimization of Aluminum robot arm with 300x20x3 mm.

Prop.	t_f (sec)	Mass (gm)	Defl. (mm)	J_{total} (kg.m^2)	Freq (HZ)
Initial design	0.540	48.6	0.064	1.910e-3	49.14
Final design	0.201	14.7	0.471	6.372e-4	34.86

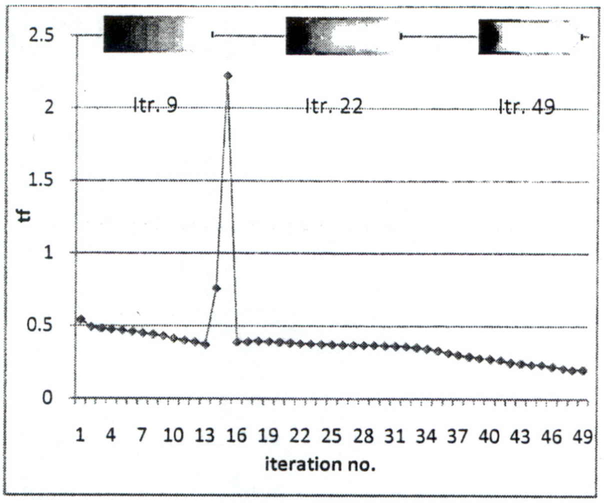

Fig. 5. Travelling time for a robot arm with dimensions of 300x20x 3 mm versus iteration no.

The final optimum topological design is then obtained and we implement finite element analysis to check if the final design is safe according to the strength and rigidity viewpoints or not? Using at the beginning Solid Edge program, see Fig. 6, for modeling the final design of the arm according to the values of the relative densities for all elements at the last iteration. The relative density for each element is interpreted as the thickness of the element as mentioned in [9]. Then the solid model file is exported to ANSYS program, see Fig. 7, for finite element analysis to determine the maximum stress and the maximum deflection to make sure that this final topological optimum design is safe and the deflection will be within the predetermined value.

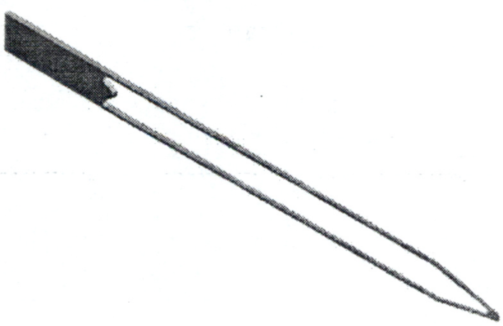

Fig. 6. The final shape of the flexible robot arm drawn by Solid Edge.

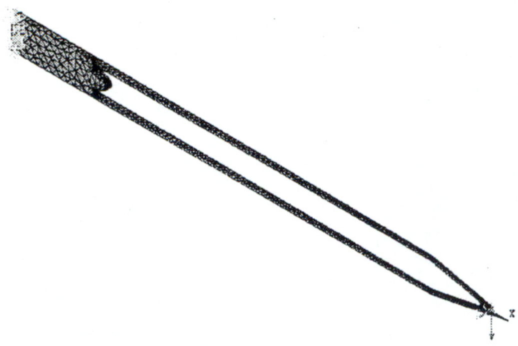

Fig. 7. Meshing the final topological optimum design using ANSYS.

The maximum equivalent stress of the arm is found to be 26.44 Mpa. Aluminum has a tensile strength of about 90 MPa, so the final topological design is safe, see Fig. 8.

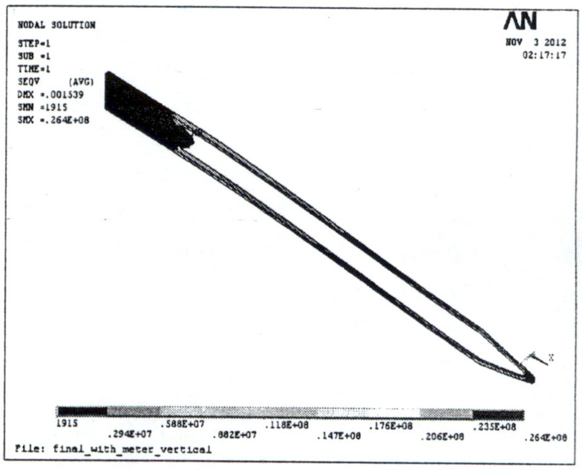

Fig. 8. Stress analysis for the optimum topological design using ANSYS.

Another model that has an initial design consist of an Aluminum beam with a dimensions of 710x38x20 mm and a hub of rectangular block of 25x25x100 mm. This Aluminum arm has a density and a modulus of elasticity as shown in Table.1. Using a torque equal to 20 N.m, angular displacement of 20° and a payload of 0.125 kg. and using δ_{max}= 0.2 mm. The topology optimization is performed and the optimal topological design is obtained and modeled with Solid Edge pro. for 3D modeling as shown in Fig. 11. The results are shown in Table. 3.

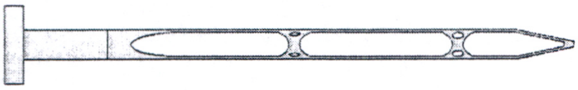

Fig. 11. Optimal topological design of robot arm (710x38x20 mm) modeled with Solid Edge Pro.

Table. 3. Results of topology optimization of Aluminum robot arm 710x38x20 mm

Prop.	t_f (sec)	Mass (kg)	Defl. (mm)	J_{total} ($kg.m^2$)	Freq. (HZ)
Int. Design	0.1468	1.456	0.1222	0.3079	51.510
Opt. Topol.	0.0836 (43.05%	0.439	0.1985	0.0952	31.578

6. 2. Results Of Topology Optimization Of Flexible Robot Arm With Air Damping

For the first short robot arm, with δ_{max} =1 mm. It is found that the travelling time t_f of the initial design at the beginning of the optimization process is 0.3435 sec., while at final optimal topological design becomes 0.1923 sec., see Fig. 11 with reduction ratio of 44.01%. The final total mass of arm is found to be 24.4% of the initial arm. The results of the topology optimization process is shown in Table.4

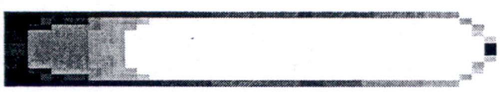

Fig. 11 Optimal topological design of Aluminum robot arm 300x20x3 mm with consideration of the air damping.

Table. 4. The results of topology optimization of Aluminum robot arm with dimensions 300x20x3 mm with air damping model.

Prop.	t_f (sec)	Mass (gm)	Defl. (mm)	J_{arm} ($kg.m^2$)	Freq (HZ)
Initial design	0.343	48.6	0.064	1.910e-3	49.867
Final design	0.192	11.86	0.846	5.98e-4	24.588

6. 3. Comparison Between Topology And Size Optimization For Air Damped Model

The second long robot arm is the same as that of [2]. The model has 50, 12, 2 elements along x, y, z directions. The torque T_o is 0.52125 N.m and the angular displacement θ_o is 180^o. It is found that the travelling time for the initial design is 0.772 sec. The size optimization result in [2] shows that the optimal travelling time is 0.593 sec. with reduction ratio of 23% see Fig. 12. Whereas, using the proposed topology optimization process and after 64 iterations, the travelling time became 0.426 sec. with a travelling time reduction ratio of 44.8%. The final mass is 55.1% of the initial one, see Fig. 13. All results of topology and size optimization for this robot arm are shown in Table. 5.

The bang- bang control is not very robust and accurate. So, a PD control is made to follow the bang-bang control. In order to keep the settling time of the flexible robot arm small, the first open loop zero and first open loop pole must be kept high. Additional to these requirements, another practical requirement is found in [2]. That is: the open loop gain (K) must be kept high also.

The high value of the open loop gain increases the upper limit of the possible obtained optimal gain. The transfer function from the actuating torque to the robot arm angle, see [2], is given by:

$$\frac{\theta(s)}{T(s)} = \frac{K(s^2 + Z^2)}{s^2(s^2 + P^2)}$$

where the open loop gain (K), first zero (Z) and first pool (P) are given by:

$$K = \frac{1 + qJ}{J}, Z = \omega\sqrt{1/(1 + qJ)}, P = \omega$$

The value of the open loop gain, first zero, first pool and damping coefficient at the initial design and optimal topological design is shown in Table. 6.

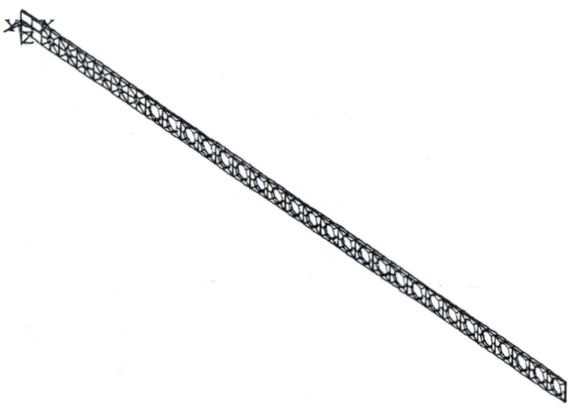

Fig. 12. Optimal size design of robot arm considering the air damping with dimensions of 760x20x3 mm.

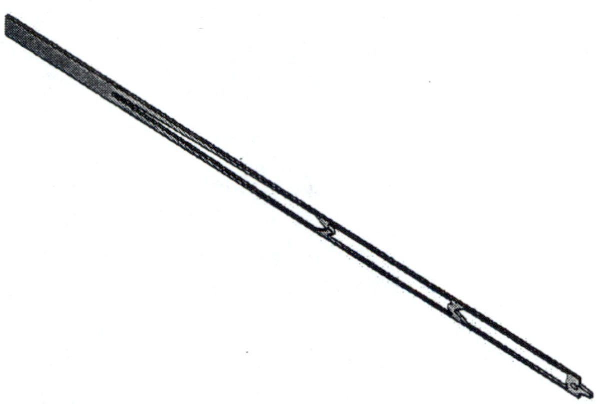

Fig. 13. Optimal topology design of robot arm considering the air damping with dimensions of 760x20x3 mm.

Table. 5. The results of size and topology optimization of Aluminum robot arm with air damping 760x20x3 mm

Prop.	t_f (sec)	Mass (gm)	Defl. (mm)	J_{total} (kg.m^2)	Freq. (HZ)
Int. Design	0.772	123.12	3.073	0.02441	16.046
Opt. Sizing	0.593 (23.5 %)	82.36	3.200	0.01400	16.300
Opt. Topol.	0.4261 (44.8%)	67.83	3.1995	0.00758	17.272

Table. 6. The damping coefficient, gain, zero and pole for the Aluminum robot arm with air damping 760x20x3 mm.

Prop.	Damp. (C) N.m.s / rad	Gain (K) (Kg. m^2)$^{-1}$	Pool (P) HZ	Zero (Z) HZ
Int. Design	0.02529	448.914	16.046	4.846
Opt. Sizing	0.0150	565	16.30	5.70
Opt. Topol.	0.00904	505.308	17.272	8.820

The optimal topology robot arm is also safe where the maximum stress is 49.9 Mpa.

7. Conclusion

In this paper, a methodology for the optimum topology design of high-speed robotic manipulator arm is proposed. The purpose of this work is utilizing the advantage of modern structural design and control techniques simultaneously to obtain robot design of minimum travelling time from both structure and control viewpoints.

By applying the topology optimization for different robot arms, one can see obviously, that at all the optimized models, the mass decrease from the fixed end to the free end (tip) of the arm.

The results of robot arms optimization without air damping show reduction ratio of the travelling time (25.7% to 44.23%) with respect to initial design.

The proposed optimal topology of the robot arm outperforms the previously published size optimization of the same robot arm with consideration of air damping. The reduction ratio of the topology design optimization is nearly twice that of the corresponding size design optimization.

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References

[1] Bendsøe, M. P. and Sigmund, O., Topology Optimization - Theory, Methods and Applications, Springer Verlag, Berlin Heidelberg, 2004.

[2] Fanni, M., and El-Keran, A. : "Theoretical And Experimental Investigation Of Integrated Structure /Control Design Of High Speed Flexible Robot Arm", Current Advances in Mechanical Design and Production,MDP-7

[3] Bendsøe M. P., Sigmund O., "Material interpolation schemes in topology optimization", Archive of Applied Mechanics **69** (635-654) Springer-Verlag, 1999.

[4] Svanberg, K., The Method of Moving Asymptotes - A new method for structural optimization, International Journal for Numerical Methods in Engineering **24**, (359-373), 1987.

[5] Pereira, E.; Becedas, J.; Payo, I.; Ramos, F. and Feliu, V. :Control of Flexible Manipulators. Theory and Practice, Chapter 12 , Robot Manipulators Trends and Development, Handbook, ISBN 978-953-307-073-5, 2010.

[6] MATLAB MMA-code written by Krister Svanberg.

[7] Roman Stainko, Advanced Multilevel Techniques to Topology Optimization, PhD thesis, Johannes Kepler University, Linz, Austria, 2006.

[8] Ghazavit A. and Gordaninejad F.,: A comparison of the control of a flexible robot arm constructed from Graphite/Epoxy versus Aluminum, computer & structure Vol. **54** No. 4, pp. 621-632, 1995.

[9] Sigmund, O. and Petersson, J.: "Numerical instab-ilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima", Struct. Optim. **16** (68-75), 1998.

[10] Jasbir S. Arora, Handbook, 'Introduction to Optimum Design', Second Edition, Elsevier Inc. , 2004.

[11] Svanberg K., 'Some modelling aspects for the Matlab implementation of MMA ' September 2004.

[12] Dwivedy S. K., Eberhard P.,; Dynamic analysis of flexible manipulators, a literature review, Mechanism and Machine Theory **41** (749-777), 2006.

[13] Rozvany G. I. N., A critical review of established methods of structural topology optimization, Struct Multidisc Optim **37**, (217-237), 2009.

[14] A. Tovar et al, 'Topology Optimization Using a Hybrid Cellular Automaton Method With Local Control Rules', Journal of Mechanical Design, Vol. **128**, 2006.

[15] Park, J. H. and Asada, H., "Concurrent Design Optimization of Mechanical Structure and Control for High Speed Robots", ASME Journal of Dynamic Systems, Measurement, and Control, Vol. **116**, pp. 344-456, 1994.

[16] Fanni, M., "Design of Non-Traditional High Speed Flexible Robot Arm Based on Time Optimal Control", 6th Int. Conf. on Artificial Intelligence Applications, Cairo, 1998.

[17] Imam I. and Sandor G. N., "High speed mechanism design: a general analytical approach", ASME J. Engng Industry 97, 6099628, 1975.

[18] Wang, F. and Gao Y., "Advanced studies of flexible robotic manipulators, modeling, design, control and applications", World Scientific, ISBN: 978-981-279- 672-1, New Jersey, 2003.

[19] Bendsøe, M. P., "Optimal Shape Design as a Material Distribution Problem," Struct. Optim., **1**, pp. 193–200, 1989.