Investigation of Natural Convection in Enclosed Concentric Cylindrical Annuli

دراسة تجريبية للحمل الحر في حلقات اسطوا نية مركزية مغلقة

E. A. M. Elshafei
Mechanical Power Engineering Department
Mansoura University, Mansoura, Egypt
Eelshafei@mans.edu.eg

ملخص البحث

يهدف البحث إلي إجراء دراسة تجريبية لبيان تاثير كل من نسبة قطري فراغ حلقي مغلق مملوء بالهواء بين اسطوانتين مركزيتين وكذلك تاثير زاوية ميل المحور على الأفقي على معدل الانتقال الحراري بالحمل الحر خلاله وكذلك من السطح الخارجي إلى الهواء المحيط. أثناء التجارب يتم تسخين السطح الداخلي للحلقة بفيض حراري منتظم وتبرد الاسطوانة الخارجية بواسطة الهواء المحيط. نسبة طول الحلقة إلى سمك الثغرة أكبر بكثير من الواحد الصحيح $(L/\delta>>1)$. وقد أجريت التجارب على الحلقة في الوضع الأفقي لمدى تغير رقم رايلى المعدل (Ra^{*}_{δ}) من 5 الى 10^{5} ونسبة قطريها $1.15 \le D_o/D_i \le 2.3$. اجريت ايضا التجارب على حلقي له نسبة اقطار $D_o/D_i = 2.88$ عند زوايا ميل مختلفة لدراسة تاثير ميل محوره على ادائه الحراري. أظهرت النتائج للفراغ الحلقي في الوضع الأفقي أن معامل انتقال الحرارة المتوسط يعتمد بشدة على طبيعة السريان، فعلى مدى منخفض لمعامل رايلي $Ra^{*}_{\delta} < 1.3\times10^{3}$ لم يظهر تاثير لكل من رقم رايلي ونسبة قطري الفراغ الحلقي عليه، بينما أظهرت التاثير الواضح لكليهما على معامل انتقال الحرارة المتوسط في المدى $1.3\times10^{3} \le Ra^{*}_{\delta} \le 10^{5}$. و قد أظهرت النتائج ايضا أن معدل الانتقال الحراري بالحمل الحر عبر الفراغ الحلقي $(D_o/D_i = 2.88)$ وكذلك من سطح اسطوانته الخارجي للهواء المحيط يزداد بزيادة زاوية ميل محوره على الأفقي، بينما يقل تاثير كلا من رقم رايلي ونسبة قطريه لقيم $\theta>60^{\circ}$. وقد استنتجت علاقات معملية لا بعدية لمعامل انتقال الحرارة المتوسط في مدى التغير لرقم رايلي ونسبة قطري الفراغات الحلقية المختبرة وقورنت النتائج في حالتي الوضع الأفقي والمائل لها بالنتائج المتاحة في الأبحاث السابقة.

Abstract

An experimental study of natural convection in a cylindrical enclosed annulus filled with air has been carried out to investigate the effect of diameter ratio and the angle of inclination on the rate of heat exchanged across it. In different case studies the inner cylinder surface is heated up at a uniform heat flux, surrounding atmospheric air convectively cools the outer cylinder. The annulus is long with respect to the gap width ($L/\delta>>1$). The experiments are performed in the horizontal case for modified Rayleigh number (Ra^{*}_{δ}) ranging from 5 to 10^{5} and diameter ratio of $1.15 \le D_o/D_i \le 2.3$. Heat transfer results are also given for the annulus of $D_o/D_i=2.88$ at different inclination angles. For the horizontal case, the average heat transfer coefficient is found to be strongly dependent on flow regime. Over the lower range $Ra^{*}_{\delta}<1.3\times10^{3}$, the effect of both Rayleigh number and the diameter ratio is diminished. For the higher range $1.3\times10^{3} \le Ra^{*}_{\delta} \le 10^{5}$, the average heat transfer coefficient is strongly affected by Rayleigh number as well as the diameter ratio. The results also show that the increase of inclination angle of the annulus axis causes an increase in the rate of convective heat transfer, while the effect of Rayleigh number diminishes for $\theta>60^{\circ}$. Empirical correlations are given for the average Nusselt number as a function of the modified Rayleigh number and comparisons are made with the available literature.

Keywords: Natural convection; Horizontal annulus; Inclined annulus; Uniform heat flux; Air-gap insulation.

Accepted March 8, 2005.

Nomenclature

A surface area, m^2
CFD Computational Fluid Dynamics
D outside diameter of outer cylinder, m
D_i , D_o inner and outer annulus diameters, m
Gr^*_δ modified Grashof number based on gap width, $(g\ \beta\ q\delta^4 / k\ \upsilon^2)$
Gr^*_D modified Grashof number based on outer diameter, $(g\ \beta qD^4/ k\ \upsilon^2)$
h heat transfer coefficient, $W/m^2.K$
k thermal conductivity (W/m.K)
L length of annulus, m
L_c characteristic length, m
Nu_D Nusselt number based on D, $(h_{av} D /k)$
Nu_{Lc} Nusselt number based on L_c, $(h_{av} L_c /k)$
Nu_δ Nusselt number based on gap width, δ $(h_{av}\ \delta/k)$
Pr Prandtl number, υ/α
Q rate of heat transfer, W
q heat flux, W/m^2.
R annulus diameter ratio (R= Do/Di)
Ra^* modified Rayleigh number, $(Gr^*.Pr)$.
Ra^*_D modified Rayleigh number based on diameter; D $(g\beta D^4 q/k\upsilon\alpha)$
Ra^*_δ modified Rayleigh number based on gap width, $(g\beta\delta^4 q/k\upsilon\alpha)$
Ra^*_{Lc} modified Rayleigh number based on characteristic length, $(g\beta\ L_c^4 q\ /k\upsilon\alpha)$
T temperature, K
t thickness of the outer tube, mm

Greek symbles

α thermal diffusivity, m^2/s
β coefficient of thermal expansion, 1/K
δ gap width (δ= $(D_o$-$D_i)/2$), m
ε emissivity of annulus material.
θ angle of inclination.
σ Stefan-Boltzman constant, $W/m^2\ K^4$.
υ kinematic viscosity, m^2/s

Subscripts

av average
b bulk
c convection
i inner
in input
L loss
o outer
r radiation

1-Introduction

The natural convection between two horizontal cylinders has been widely studied by many authors. Investigation of such problem is usually encountered in many fields especially in solar concentrators, thermal storage plants, horizontal pressurized water reactors and electrical gas insulated transmission systems.

Many experimental and numerical studies are carried out for laminar case or turbulent flow with low Rayleigh number (up to 10^7) and for isothermal cylinders as boundary conditions. Only few researchers studied the convection with non-isothermal boundary conditions and for a Rayleigh over than 10^7. The results of these experimental and numerical studies show the dependence of the flow patterns on the Rayleigh number, the Prandtl number and radius ratio. Kuehn and Goldstein [1-2] studied the problem experimentally and numerically. They determined the temperature distribution and the local heat transfer coefficients in air and in water.

Powe et al. [3] presented a numerical solution of the steady laminar flow for a Prandtl number of 0.7 and for inverse relative gap widths (2.8–12.5). They presented photographs showing the dependence of flow patterns on Rayleigh number and diameter ratio.
They compared their results to the one of Crawford and Lemlich [4], Abbott [5] and Bischop [6], and reported good agreement for flow patterns and the temperature profiles.

A numerical investigation for natural convection in horizontal annulus was also carried out and reported in [7 and 8]. The temperature was maintained constant at one cylinder while there was a uniform heat flux at the other. Hessami et al [9] studied experimentally and numerically the free convection heat transfer in a horizontal annulus with large radii ratio (Do/D_i =11.4) for $0.023 < Pr < 10^4$, and $0.03 < Gr < 3\times10^6$.

It is reported that the computed heat transfer coefficients did not affected by the variation of fluid properties whereas for glycerin, a significant discrepancy in local heat transfer distribution and flow patterns were observed.

A numerical prediction of the problem was also presented by Date [10]. Since the presence of natural convection augments the heat transfer rate compared with that which would be obtained by conduction alone, the author obtained a correlation for the equivalent conductivity factor as a function of Rayleigh number. Comparing his results with that reported by Kuhen and Goldstein [1], a good agreement was obtained. However, for low gap width relative to the inner cylinder diameter (δ/D_i) as in PWRS, Date [10] claimed that all the previously correlations did not match each other and over predicted his computed heat transfer rates.

Awad [11] presented an analytical procedure for optimizing the performance of solar energy receiver. It is concluded that the rate of convetive heat transfer is decreased with the increase of radii ratio until a certain value of $R_o/R_i = 1.348$ at which the minimum heat loss occurred. The rate of heat transfer was reported to increase again with $R_o/R_i > 1.348$.

Roe et al [12] investigated theoretically and experimentally the natural convection flow patterns and temperature distribution in horizontal annuli. Comparing the experimental results for temperature and flow patterns with the predicted ones, they determined the dominant flow pattern at a given modified Rayleigh number and diameter ratio.

Glakpe et al [13] studied theoretically the natural convection in gap between concentric and vertically eccentric horizontal circular cylinders. The calculations were performed for a uniform heat flux at the boundaries with $Pr = 0.7$, $D_o/D_i = 1.8$ and $0 < Ra_\delta < 10^6$. The average Nu, flow patterns, and temperature distributions on both inner and outer annulus surfaces were presented as a function of Ra_δ. It was reported that the flow patterns look similar to those obtained by using isothermal boundary conditions. However, the isotherm contours showed different heat flow patterns.

A visualization study was carried out on natural convection in a horizontal annulus by Kumar and Keyhani [14]. The predicted values of Nu for the range of $1.8 \leq D_o/D_i \leq 15$ and Pr of 0.7, 5 and 100, were compared with the experimental data for $D_o/D_i = 11$ and $74 \leq Pr \leq 173$. It was reported that the critical Ra_δ at which the flow regime changed from conduction to convection was nearly constant at about 740 ± 45. For $D_o/D_i \leq 1.8$, the critical Ra_δ was shown to be calculated from the following formula: $Ra_\delta = 6735(D_o/D_i)^{-2.89}$.

Hamad [15] investigated the natural convection heat transfer in an inclined annulus of D_o/D_i equals to 1.63, $Pr = 0.7$ and $4\times10^4 \leq Ra_{Di} \leq 2.5\times10^5$. It is concluded that both the Ra_{Di} and the angle of inclination have a very small effect on the average heat transfer coefficient through the annulus.

Hamad and Khan [16] investigated the effect of Rayleigh number, diameter ratio, and angle of inclination on natural heat transfer from an annulus. The experimental heat transfer results for horizontal annulus of diameter ratios 1.63 and 2.57 for $1.5\times10^3 \leq Ra_\delta \leq 4\times10^4$ and that of previous experimental data for an annulus with diameter ratio of 2.6 as well as with their predictions using CFD backage were compared. It was concluded that the effect of both Rayleigh number and diameter ratio on the average heat transfer coefficient is more significant than the effect of inclination angle.

Numerical investigation of natural convection of gases (mixture of SF_6 and N_2) in a horizontal concentric annulus was carried out by Chakir et al [17]. The CFD-code based on a finite element formulation was used in their analysis. The two-dimensional analysis of the heat transfer and fluid motion was performed for Rayleigh number ranging from 10^5 to 10^{10}, and radius ratio of 2.5. Comparisons were made with experimental measurements. It was reported that there were no effects noticeable for a low turbulent flow. The differences to the achieved Nusselt results of Kuehn and Goldstein [1] were small and the maximum temperature deviation was about *15%*.

It is of engineering importance that how much natural convection in the air gap of concentric annulus will reduce the rate of heat exchanged across it. This is actually dependent on geometrical parameters of this air gap and the boundary conditions. Although many experimental studies have been carried out in this field, the correlations published have limited validity for certain geometry, flow regime and boundary conditions. More research is needed to study the influence of geometrical parameters of the annulus as well as the angle of inclination on the rate of heat exchanged through it and to detect the optimum gap width at which the minimum rate of heat transfer could be achieved.

The aim of the present study is to investigate the effect of diameter ratio as well as the angle of inclination on the rate of heat exchanged between an enclosed concentric annulus surfaces. The inner cylinder is heated up at a uniform heat flux while the outer one is convectively cooled by surrounding air. The experimental data for horizontal case is correlated in terms of Nu_δ and modified Rayleigh number, Ra_δ for $1.35 \le R \le 2.3$, $Pr= 0.7$ and $5 \le Ra_\delta \le 10^5$. The detection of the proper gap width might be helpful in the design of gas-insulated lines, solar collectors and other similar applications. The experimental data of the annulus of diameter ratio equals to *2.88* with variable inclination angle ($0^\circ \le \theta \le 90^\circ$) are also presented and compared with the available literature.

2- Experimental Apparatus

2-1. Horizontal Annulus

The main parts of the experimental apparatus used in carrying out the experiments are the outer cylinder, inner coaxial cylinder that is heated for the inducement of natural convection, end plates, heater and power supply with associated controls and instrumentations. A schematic diagram of the apparatus is shown in Fig.1. The outer cylinder (2) of 2 mm thickness and has a fixed inner diameter of 57.5 mm. Meanwhile, The inner cylinder (4) of maximum outer diameter of 50 mm, which gave the smallest gap width. To obtain the varying annulus diameter ratio, this cylinder was machined for every case study to reduce its outer diameter. Both cylinders are made of brass and of 520 mm effective length. The inner cylinder was internally heated by an electrical current passing through a nickel chrome wire (6) of 0.8 mm diameter. This wire was exerted inside a central Pyrex glass tube (5) of 10-mm in diameter with an effective length of 520 mm, which is fitted in a machined central hole among the inner cylinder.

The power input necessary to heat up the inner cylinder with a uniform heat flux was measured by an ammeter (A) and voltmeter (V), and controlled by a variac unit (AT).

The axial temperature of the inner and outer annulus surfaces was measured by using 22 insulated copper-constantan thermocouples of 0.25 mm size distributed as shown in Fig. 1. Eleven of these thermocouples are inserted, glued and polished to assure good contact with the surface along the axis inside 11 rectangular grooves of 1-mm depth on the outer surface of the inner cylinder and the other 11 thermocouples were pressed into 11 holes drilled along the axial length of the outer cylinder outside surface. On both annulus surfaces, 5 thermocouples were placed 45° apart at the middle. Two of the remaining six thermocouples were fixed at about 10 mm from both ends of the annulus and the rest are fixed 90° apart at equal distance along the axis of the annulus. All thermocouples were connected to a 6-channel digital temperature recorder (TR) via a multi-point switch (MPS).

To minimize the axial end heat losses, two Teflon carriers (1) are pressed against both ends of the inner cylinder. The effective length and the outer diameter of the annulus remained fixed at 520 mm and 57.5 mm, respectively. The annulus geometrical parameters are listed in Table.1.

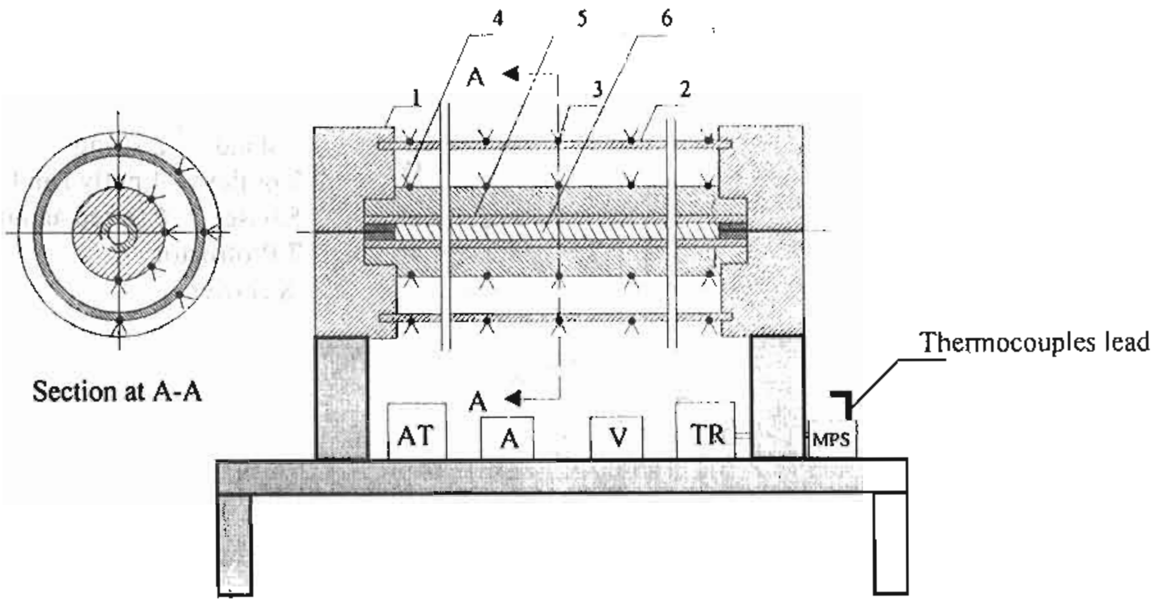

Fig.1 Experimental apparatus for the investigated horizontal annulus.

1- Teflon carrier, 2- outer cylinder, 3- copper-constantan thermocouples, 4- inner cylinder, 5- glass tube, 6- electric heater, A- ammeter, AT- Auto-transformer, TR- temperature recorder, V- voltammeter MPS-multi-point switch.

Table 1: Geometrical parameters of the tested horizontal annulus.

Parameter	Range							
Inner diameter; D_i (mm)	50	46	42.6	39.7	36	30.3	28.8	25
Gap width; δ (mm)	7.5	11.5	14.9	17.8	21.5	27.2	28.7	32.5
Aspect ratio; δ / D_i	0.15	0.25	0.35	0.45	0.6	0.9	1.0	1.3
Aspect ratio; L/δ	69.3	45.2	34.9	29.2	24.2	19.1	18.1	16
Diameter ratio; $R = D_o / D_i$	1.15	1.25	1.35	1.45	1.6	1.9	2.0	2.3

2.2. Inclined Annulus

To investigate the effect of annulus inclination on the rate of heat transfer by natural convection, a special test loop for an annulus of a diameter ratio (D_o/D_i) equals to 2.88 is built as illustrated in Fig. 2. The annulus was tilted at the required angle using the string 2, the pulley 3 that is fixed at the upper end of the stand 1 and another pulley provided with hand 4 at the other end of that stand. The inclination angle was detected by the protractor 7, which is pivoted into a carrier 8 at the lower end of the annulus.

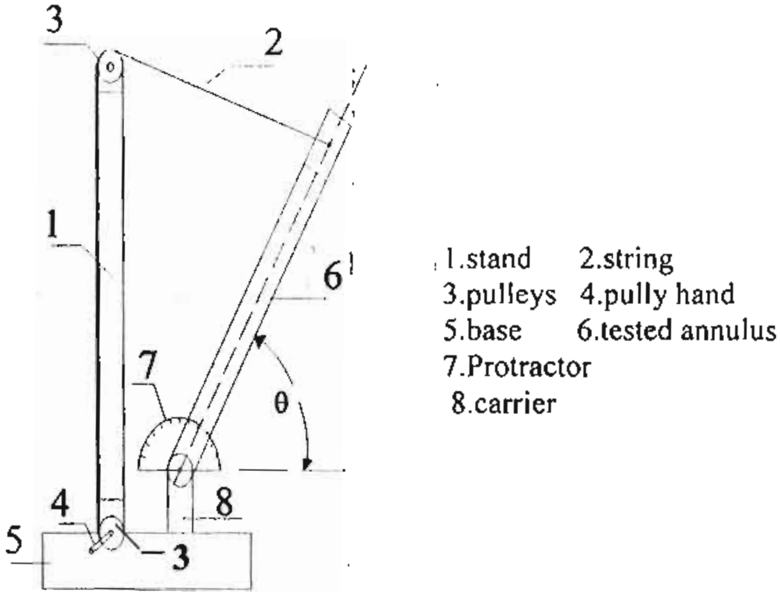

Fig. 2. Test apparatus for inclined annulus

The annulus 6 has a fixed inner diameter at 57.5 mm. The inner one is of 20 mm outside diameter and internally heated by an electrical heater exerted in the centre of the inner tube. The space between the heater and the inner surface of the inner tube is filled partially with asbestos tape wrapped around the heater and sand to avoid convection currents. Both tubes are made of brass with 2 mm thickness and of 922 mm effective length. Nine copper-constantan thermocouples are fixed on each of the inner and outer surfaces of the annulus, 5 at the midpoint and the other 4 are distributed at equal distance along the axial length of the annulus as described in Fig.1. Instrumentation and power input control for the inclined annulus is similar to those belongs to the horizontal annuli, which are detailed in the previous section. The annulus geometrical parameters are listed in Table 2.

Table 2. Geometrical parameters of the tested inclined annulus.

Parameter	$D_o(mm)$	$Di(mm)$	Do/Di	δ (mm)	δ/Di	$L(mm)$	L/δ
Range	57.5	20	2.88	18.75	0.94	922	49

In each experimental case study, the system was run for an average of three hours to reach the steady state. This was recognized by the repeatable-recorded values of annulus surfaces temperatures.

3. Data Reduction

The data collected from experiments have been used for heat transfer calculations through the annulus. The rate of heat exchanged across the annular gap by convection; Q_c is given by

$$Q_c = Q_{in} - Q_r - Q_L \qquad (1)$$

Where Q_{in} is the rate of power input, Q_r is the rate of heat exchanged by radiation and Q_L is the rate of axial heat loss by conduction. The recorded values of the temperatures along the axial length of the annulus showed very little temperature gradients near the ends. Therefore, the rate of heat loss can be ignored.

For the annular gap, the radiated heat exchanged; Q_r can be expressed as reported by [18] in the form:

$$Q_r = \frac{\sigma A_i \left(T_i^4 - T_o^4\right)}{\frac{1}{\varepsilon_i} + \left(\frac{1-\varepsilon_o}{\varepsilon_o}\right)\left(\frac{A_i}{A_o}\right)} \qquad (2)$$

Where ε_i , ε_o are the emissivity of inner and outer annulus surfaces. For brass, those coefficients are taken as 0.2 [18]. The average heat transfer coefficient can then be calculated as

$$h_{av} = \frac{Q_c}{A_i \left(T_{i\,av} - T_{o\,av}\right)} \qquad (3)$$

Where A_i is the inner surface area of the annulus ($A_i = \pi$ Di L) and T_{iav}, T_{oav} are the inner and outer surface mean temperatures of the annulus, respectively.

The heat transfer results are presented in a dimensionless form. The average heat transfer coefficient; h_{av} is expressed as an average Nu_{Lc}, which is given as follows:

$$Nu_{L_c} = \frac{h_{av}\, L_c}{k} \qquad (4)$$

Where L_c is the characteristic length, expressed as

$$L_c = L\sin\theta + \delta\cos\theta \qquad (5)$$

To find useful correlations for heat exchanged across the air gap and for the purpose of comparison with others, analysis using dimensionless parameters is presented. The average Nueeslt number is presented as a function of the modified Rayleigh number and the annulus geometry (Nu = f (Ra*, geometry), as follows:

$$Nu_{L_c} = C_1 \left(Ra_{L_c}{}^{*}\right)^n \qquad (6)$$

The values of Nu and Ra* are all based on the characteristic length (L_c). For the horizontal annulus, the characteristic length L_c is reduced to the gap width (δ) and the above equation becomes:

$$Nu_\delta = C\left(Ra_\delta{}^{*}\right)^n \qquad (7)$$

Where C_1, C and n are constants and their values depend on the annulus geometry.

The thermo-physical properties appearing in the above equations are evaluated at the average temperature of inner and outer annulus surfaces, $\left(T_{av} = \frac{T_{o_{av}} + T_{i\,av}}{2}\right)$.

4. Experimental Results

The experimental results for the present investigation covers two cases for the annulus; the horizontal annulus and the inclined annulus.

In the horizontal annulus case, the rate of heat transfer by natural convection from the outside surface of the outer cylinder to the ambient and that exchanged across the annular air gap for $5 \le Ra^*_\delta \le 10^5$ and with variable diameter ratio of $1.15 \le D_o / D_i \le 2.3$, which corresponds to $0.15 \le \delta/D_i \le 1.3$.

In the Inclined annulus, the experiments are performed on an annulus of D_o/D_i equals to 2.88 with gap width of 18.75 mm over a range of $10^5 < Ra^*_{Lc} < 10^9$ and $0^\circ \le \theta \le 90^\circ$. The experiments covered the results of heat exchanged by natural convection across the annular gap as well as that from the outside surface of the outer cylinder to the surroundings.

In both cases, the radiative heat exchanged between the annulus surfaces is determined from equation (2) and is taken into account during the heat transfer mode analysis. The maximum rate of heat loss by radiation inside the air gap was accompanied with the maximum power input and was found to be less than 10%. However, its percentage value was ignored from the outside surface of the outer cylinder to the surroundings where there were very little temperature differences.

4.1. **Horizontal case**

Table.2 shows a summary of ranges for the considered parameters studied in the present work for horizontal annulus. The Prandtl number for air is nearly constant at 0.71.

Table.2 Investigated parameter and ranges

Parameter / Range	T_i (°C)	T_o (°C)	T_a (°C)	q (W/m²)	Ra^*_D	Nu_D	Ra^*_δ	Nu_δ
Min.	28.4	25.4	23.0	6.0	2.3×10^5	7.0	5.0	0.9
Max.	132.7	75.7	29.0	750	1.65×10^8	17.8	1.0×10^5	5.17

It should be emphasized that the investigated horizontal annulus had a characteristic that the outer cylinder outside surface was not isothermal but convectively cooled, causing the upper part to be hotter than the bottom. The average surface temperature of the outer cylinder was used for calculation in each case study data. The rate of heat loss by natural convection from the outside surface of the outer cylinder to its surroundings for $2\times10^5 < Ra^*_D < 1.8\times10^7$ is calculated considering the whole annulus as a single cylinder ($L_c = D$) and presented as shown in Fig. 3. The values of constants C_l and n in Eq. [6] are calculated and the resulting empirical correlation (± 15%) is given as;

$$Nu_D = 0.56\left(Ra^*_D\right)^{0.25} \qquad (8)$$

In order to check the validity of the experimental procedures and instrumentations, the present results have been compared with the correlation for isothermal horizontal cylinder reported by McAdams [19] ($Nu_D = 0.53(Ra^*_D)^{0.25}$ for $10^4 < Ra^*_D < 10^9$), Fishenden [25] and Morgan [26] for convectively cooled horizontal isothermal cylinders, and this shows a reasonable agreement (about 12% higher than that of MacAdams) as can be seen in Fig.3.

Based on the heat transfer analysis for natural convection through enclosed spaces [3,20,21and22], there exist three heat transfer regimes for atmospheric-pressure air inside a concentric gap identified as: the conduction regime, the transition regime and the fully developed convection regime. The existence of any of these regimes is dependent not only on the range of Ra^*_δ and D_o/D_i, but also on the radius of curvature of the annulus .

The experimental results for heat transfer through the horizontal annulus for Nu_δ versus Ra^*_δ at $1.15 < D_o/D_i < 2.3$ over the tested range of Rayleigh number; $5 < Ra^*_\delta < 10^5$ are shown in Fig. 4. It can be assured that the data are significantly affected by the flow regime. The data can be analyzed by considering only two regimes over the tested range of modified Rayleigh number and diameter ratio as:

- Conduction regime,
  $5 \le Ra^*_\delta \le 1.3\times10^3$,
  $1.15 \le R \le 1.45$
- Transition regime.
  $1.3\times10^3 \le Ra^*_\delta \le 10^5$,
  $1.35 \le R \le 2.3$

In the first region, the heat transfer mechanism seems to have no response to the change of both Ra^*_δ and the diameter ratio R. The heat is expected to transfer by conduction alone and the value of Nusselt number is independent of both Ra^*_δ and R. The mean value of Nu_δ is determined using the least square method, and approximately equal to 1.1 ± 10%, which agrees with the results reported by several investigators [17, 18 and 21] within about 10%.

The difference between the present results and others is the critical value of Rayleigh number at which the flow changes from conduction mode to transient region. Grigull and Hauf [20] reported that the conductive regime exists for $Ra^*_\delta < 1.7\times10^3$ while Kuehn and Goldstein [21] concluded that the Nusselt numbers approach unity when

$Ra^{*}_{\delta} < 2.3\times10^{3}$. This is attributed to the change of radius of curvature of the tested annuli whether the range of diameter ratios are similar or not. A study of Powe et al [3] concluded that the radius of curvature, though not affecting the type of flow pattern, did affect the specific values of the Grashof number at which transition from steady to unsteady flow occurred. They also pointed out that the decrease in radius of curvature resulted in an increase in the transition Gr and suggested that further studies be carried out to substantiate this effect.

As the Rayleigh number increases, the heat transfer mode changes into the transition mode. The critical value of Ra^{*}_{δ} to move forward into the fully developed convection mode is also dependent on the geometrical parameters of the annulus, specially on its radius of curvature as reported by Powe et al [3] and Yunus [22] who recommended that could be happened for $Ra^{*}_{\delta} \geq 5\times10^{5}$, while its value reduced to $Ra^{*}_{\delta} \geq 2.1\times10^{4}$ for air as reported by Grigul and Hauf [20]. Therefore, the second part of the present data belongs to what has been considered as a transition regime.

For that regime, where $1.3\times10^{3} \leq Ra^{*}_{\delta} \leq 10^{5}$ and $1.35 \leq R \leq 2.3$; however, it is noticed from Fig. 4 that Nu_{δ} is greatly affected by both Ra^{*}_{δ} and diameter ratio; R. For all tested annuli, the heat transfer coefficient increases with the increase of Ra^{*}_{δ} as well as with the increase of diameter ratio; R at constant Ra^{*}_{δ}. The highest values of Nu_{δ} is accompanied with the annulus of the highest diameter ratio; $R= 2.3$, while the lowest values of Nu_{δ} resulted for the annulus of diameter ratio equals to 1.35. This fairly agreed with the analytical results reported by Awad [11].

The experimental results for annuli of $1.35 \leq D_o/D_i \leq 2.3$ in the transition region were fitted, using the least square analysis, to determine the values of constant; C in Eq. (7) The numerical value of the constant; C including the effect of the annulus diameter ratio can be written as:

$$C=0.219\left(\frac{D_o}{D_i}\right)^{0.9145} \qquad (9),$$

and the exponent; n is equal to 0.2

The empirical correlation for the horizontal annulus with variable diameter ratios for $1.35 \leq D_o/D_i \leq 2.3$ and $1.3\times10^{3} \leq Ra^{*}_{\delta} \leq 10^{5}$ may be described as:

$$Nu_{\delta}=0.2191\left(Ra_{\delta}^{*}\right)^{0.2}\left(\frac{D_o}{D_i}\right)^{0.9145} \qquad (10)$$

The experimental results for the tested annulus of $D_o/D_i = 1.6$ are compared with numerical and experimental results of Hamad and Khan [16] for similar annulus geometry of $D_o/D_i = 1.63$, and that reported by VDI [24] for $1.3 \leq D_o/D_i \leq 6.3$ as shown in Fig. 5. It can be seen that the present results are fairly agreed with those of Hamad and Khan [16], and a little bit higher than that for VDI correlation, which belongs to annuli of lower radius of curvature. So, it can be expected that there the existence of a relatively higher transition Rayleigh number at which heat transfer mode changes from regime to another [3 and 22].

4.2. Inclined case

The experimental apparatus shown in Fig.2 has been used to carry out experiments to study the effect of inclination angle on heat transfer coefficient for the annulus of D_o/D_i equals to 2.88 over the range $10^{5} \leq Ra^{*}_{Lc} \leq 10^{9}$ and $0^{\circ} \leq \theta \leq 90^{\circ}$. Both Nu_{Lc} and Ra^{*}_{Lc} are all based on the characteristic length; L_c.

The relation between Nu_{Lc} and Ra^{*}_{Lc} for various angle of inclination for the outer cylinder of the annulus is shown in Fig. 6. It is seen that the increase of θ accompanied with an increase in the values of Nu_{Lc}. It is also noticed, as previously found in the horizontal case, the values of Nusselt number increases with the increase of Rayleigh number for $\theta \leq 60^{\circ}$ and becomes flattened as θ is increased where the effect of Rayleigh number becomes small.

The variation of Nu_{Lc} with Ra^{*}_{Lc} for the annulus at different inclination angle is represented as shown in Fig. 7. It is also seen that the values of Nu_{Lc} increased with increasing the value of θ, and the effect of

Rayleigh number seemed to be diminished also for $\theta > 60^\circ$.

The increase in the rate of heat exchanged across the annulus with the increase of inclination angle may be attributed to flow distortion resulted from the change in the magnitude of gravitational force components with increasing θ up to 60° as well as the change of the flow to a three dimensional one. As the angle $\theta > 60^\circ$, the effect of both Rayleigh number as well as the inclination angle became smaller.

The heat transfer results for the tested inclined annulus ($D_o/D_i = 2.88$) are compared with the reported data by Hamad and Khan [16] for an annulus of $D_o/D_i = 2.57$ and shown in Fig. 8. It can be observed that the Nu-Ra* relation of the present work have the same features as those of [16]. The discrepancies may be attributed to the differences in geometry for their annulus and the present tested one. As previously discussed, the heat transfer coefficient increases with the increase of D_o/D_i.

5. Conclusions

An investigation of natural convection in a horizontal concentric enclosed annulus has been carried out for $5 \le Ra^*_\delta \le 10^5$, with variable diameter ratio; $1.35 \le R \le 2.3$ as well as for an inclined annulus of diameter ratio of 2.88 with varying the angle of inclination.

In case of horizontal annulus, the data was divided into two sections; for conduction regime and for transition regime. In the inclined case, the inclination angle is changed over the range $0^\circ \le \theta \le 90^\circ$.

The heat transfer results for both cases are presented as a function of the modified Rayleigh number, correlated and compared with the available literature. The main conclusions can be summarized as:

1. The Nu_δ is nearly constant at $1.1 \pm 10\%$ and has no sense to the variation of both the Ra^*_δ and the diameter ratio for $Ra^*_\delta < 1.3\times10^3$ and $D_o/D_i \le 1.45$, where the conduction was the only existed mode. The disagreement with the literature concerning the critical value of Ra^*_δ is attributed to geometrical parameters of the annulus, specially its radius of curvature.
2. The Nu_δ increased with increasing both the Ra^*_δ and diameter ratio, R in the transition region; $1.3\times10^3 \le Ra^*_\delta \le 10^5$ and $1.35 \le D_o/D_i \le 2.3$.
3. The minimum heat exchanged occurred through the annulus air gap of diameter ratio equals to 1.35 over the tested range of modified Rayleigh number.
4. Correlations for Nu_δ as a function of Ra^*_δ are given for horizontal annulus of different diameter ratios and compared with previous work.
5. The increase of inclination angle resulted in an increase in the values of Nu_{Lc} and reduced the effect of Ra^*_{Lc}.
6. The results for inclined annulus are compared with the available literature and discrepancies between them are discussed.

References:

1. Kuehn, T.H., Goldstein, R.J., "An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders", *J. Fluid Mech.*, 74, pp.695–719, 1976.
2. Kuehn, T.H., Goldstein, R.J., "An experimental study of natural convection heat transfer in concentric and eccentric horizontal cylindrical annuli", *ASME J. Heat Transfer*, 100, pp.635–640, 1978.
3. Powe, R.E., Carley, C.T, Carruth, S.L, "A numerical solution of natural convection in cylindrical annuli", *ASME J. Heat Transfer*, 92, pp. 210–220, 1971
4. Crawford, L., Lemlich, R., "Natural convection in horizontal concentric cylindrical annuli", IEC Find 1 (1962) 260–264.

5. Abbott, M.R, "Numerical method for solving the equations of natural convection in a narrow concentric cylindrical annulus with a horizontal axis", *J. Mech. Appl. Math.* XVII (Part 4) (1964) 471–481.
6. E.H. Bishop, "Discussion of Paper no. 60, Third International Heat Transfer Conference", Proceeding of the Third International Heat Transfer Conference, Vol. 6, 1967, pp. 155–156.
7. Cho, C. H. , Chang, K. S., and Park, K. H., " Numerical Solution of Natural Convection in Concentric and Eccentric Cylindrical Annuli", *ASME J. of Heat Transfer*, Vol. 104, pp.624-630, 1982.
8. Van Sande, E., and Hamer, B. J. G., "Steady and Transient Natural Convection in Enclosures between Horizontal Circular Cylinders (Constant Heat Flux)", *Int. J. Heat Mass Transfer*, Vol. 22, pp. 361-370, 1979.
9. Hessami, M. A., Pollard, A., and Rowe, R. D., and Ruth, D. ,W., "A Study of Free convective Heat Transfer in a Horizontal Annulus with a Large Radii Ratio", *ASME J. of Heat Transfer*, Vol. 107, pp.603-610, 1985.
10. Date, A. W., "Numerical Prediction of Natural Convection Heat Transfer in Horizontal Annulus", *Int. J. Heat Mass Transfer*, Vol. 29, No. 10, pp.1457-1464, 1986.
11. Awad, M. M, "Optimum Radius of Cylinderical Envelop of Solar Energy Receiver", Mansoura Faculty of Engineering Bullletin, Mansoura University, Vol.8, No.2, 1983.
12. Rao, Y. F., Miki, Y., Fukuda, K. and Hasegawa, S., "Flow Patterns of Natural Convection in Horizontal Cylindrical Annuli", *Int. J. Heat Mass Transfer*, Vol.28, 1985.
13. Glakpe, E. K., Watkins, C. B and Cannon, J. N., "Constant Heat Flux Solutions for Natural Convection between Concentric and Eccentric Horizontal Cylinders", *Numerical Heat Transfer*, 10, 279-, 1986
14. Kumar, R. and Keyhani, M., 'Visualization studies of Natural Convective flow in Horizontal Cylindrical Annulus", *J. Heat Transfer*, 112, 1990.
15. Hamad. F. A., "Experimental Study of Natural Convection Heat Transfer in Inclined Cylindrical Annulus", *Solar and Wind Technology*, 6, 1989.
16. Hamad, F. A., and Khan, M. K., "Natural Convection Heat Transfer in Horizontal and Inclined Annulus of Different Diameter Ratios", Energy Conversation, Mgmt., Vol.39, No.8, pp.797-807, 1998.
17. Chakir, A., Souli, M., Aquelet, N., "Study of a turbulent natural convection in cylindrical annuli of gas-insulated transmission lines 400 kV", *Applied Thermal Engineering*, 23, pp.1197–1208, 2003.
18. Kreith, F., *Radiation Heat Transfer*, Int. Textbook, 1962.
19. McAdams, W. H., Heat Transmission, 3rd Ed., Mc Graw-hill, New York, 1954.
20.Grigul, U., and Hauf, W., "Natural convection in horizontal cylindrical annuli", Proceeding of the Third International Heat Transfer Conference, Paper no. 60, vol. 2, 1966, pp. 182–195.
21. Kuehn, T.H., Goldstein, R.J., "A parametric study of Prandtl number and diameter ratio effects on natural convection heat transfer in horizontal cylindrical annuli", *Trans. ASME, J. Heat Transfer*, Vol.102, pp. 768-770.
22. Cengel, Yunus, A., Introduction to thermodynamics an heat transfer, McGraw-Hill, New York, pp. 590-592, 1997.
23. *VDI Heat Atlas*, Fe3, Association of German Engineers, Germany, 1993.
25. Fishenden, M., and Saunders, O. A., An Introduction to Heat Transfer, Oxford University Press, 1950.
26. Morgan, V. T., "The Overall Convective Heat Transfer from smooth Circular Cylinders," in Irvine, T. F., and Hartnett, J. P., Eds., *Advances in Heat Transfer*, Vol. II, Academic Press, New York, 1975.

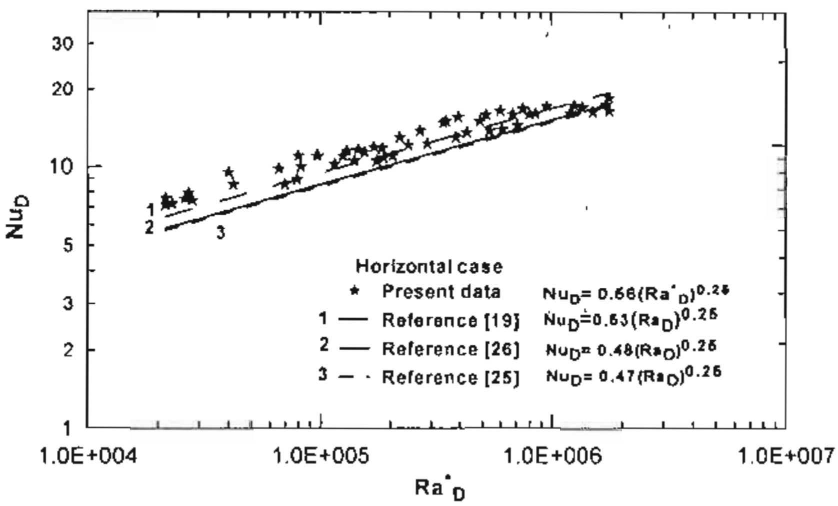

Fig. 3 Comparison between present experimental results for outer cylinder and several literature correlations.

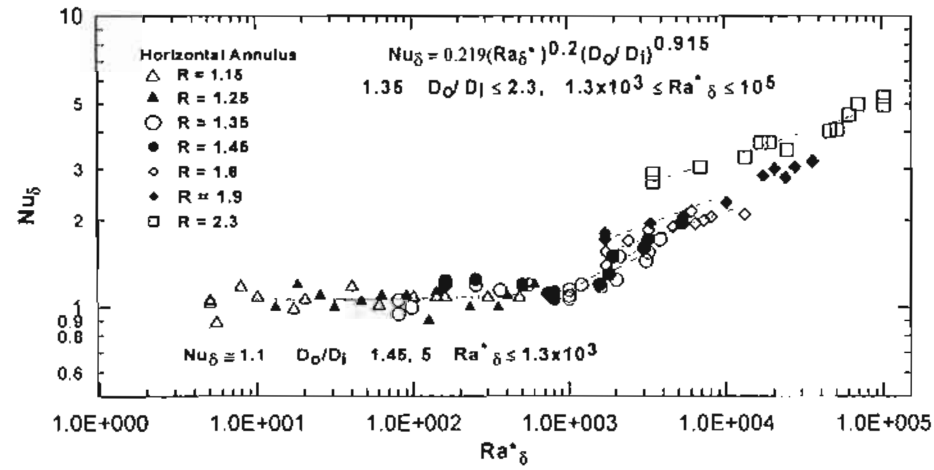

Fig.4 Nusselt number versus modified Rayleigh number for horizontal annulus of different diameter ratio.

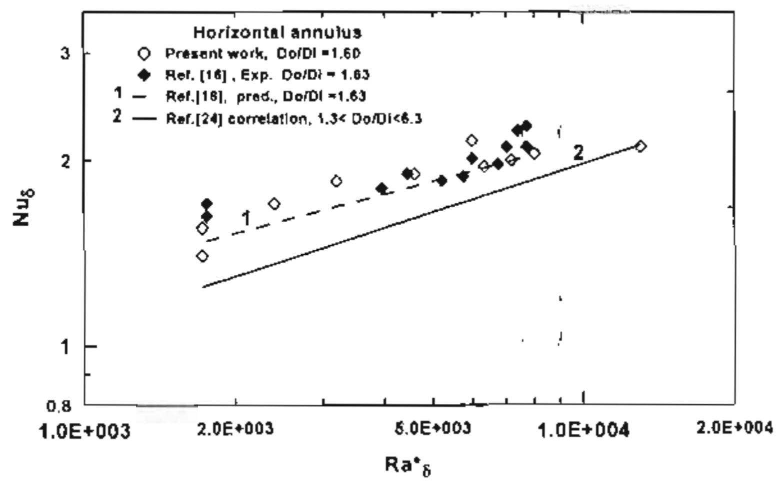

Fig. 5 Comparison between the present experimintal data and available correlations for horizontal annulus.

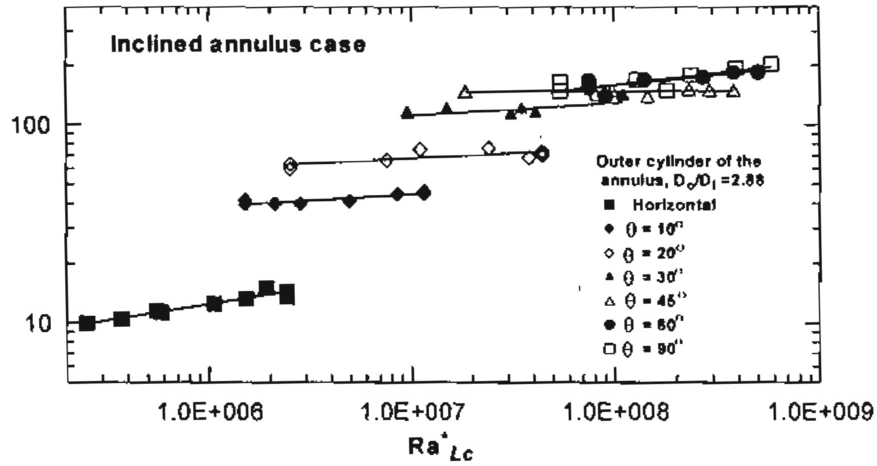

Fig. 6 Nusselt number versus modified Rayleigh number for the outer annulus cylinder at various inclination angle.

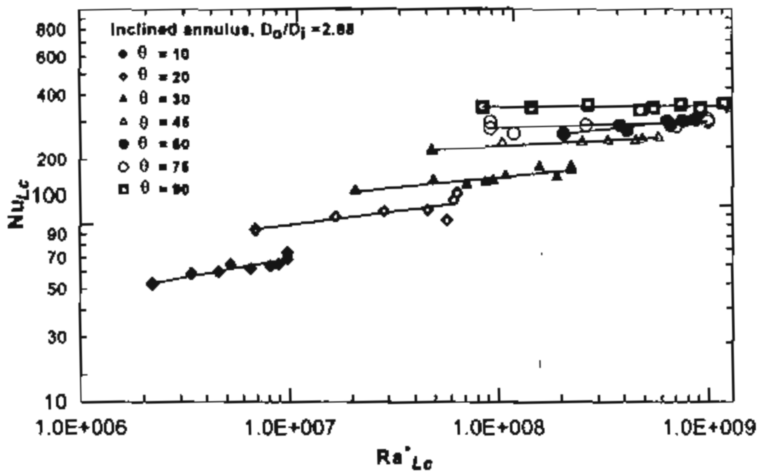

Fig. 7 Nusselt number versus modified Rayleigh number for annulus at different inclination angle.

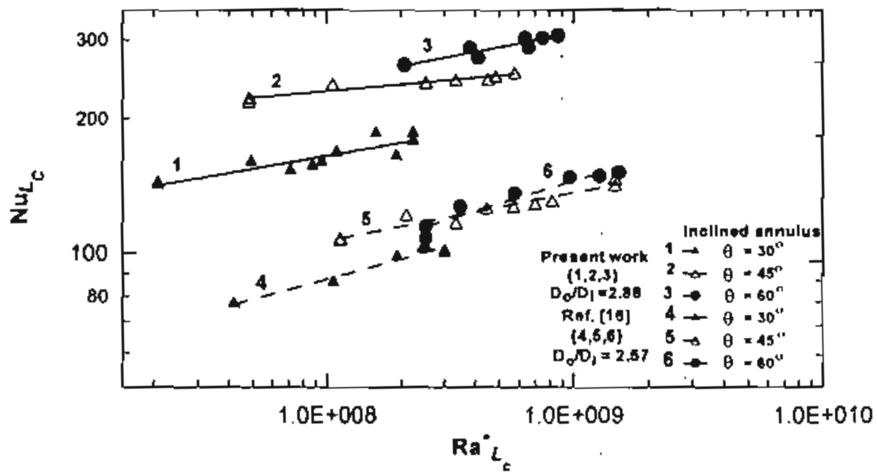

Fig. 8 Comparison between the experimental resuls for annulus at variable inclination angle with the avilable literature.