Experimental Investigation of Forced Convection Heat Transfer from Outer Surface of Annular Tube with Rotating Inner Tube

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Abstract
The paper presents an experimental study of laminar forced convection heat transfer in different rotational isothermal annular tubes. The inner tube of the annulus is rotated with rotational speeds varies from 100 to 628 r.p.m to give rotational Reynolds number in the range 2000 \( \leq Re_\theta \leq 29400 \). The inner tube of the annulus is varied so as to give radii ratios of 0.3879, 0.614 and 0.8842. Water is flowed axially through the annular space with velocities ranged from 0.00414 to 0.27 m/s to give axial Reynolds number in the range 80 \( \leq Re_\phi \leq 2700 \) to cover the laminar flow regime. The outer tube of the annulus is cooled under uniform temperature via the evaporation of refrigerant R 22 flowing through a refrigeration circuit while the inner tube of the annulus is thermally insulated. In the present work the effect of radius ratio, axial Reynolds number and rotational Reynolds number on the heat transfer are subjects of major interest.

The experimental results of this work show that the heat transfer of the rotational as well as the stationary annular tubes increase with the increase of the radius ratio of the annulus. The results show also that heat transfer of the rotational annular tubes is higher than this of the stationary ones at the same radius ratio and axial Reynolds number. There are peak values of heat transfer at a rotational Reynolds number of nearly 12725 (rotational speed of 270 rpm nearly) for different radius ratio of the annulus. An increase as much as 44% in the heat transfer is reported for annular tube of radius ratio of 0.8842 and rotational speed of 270 rpm. Excellent correlation is established between Nusselt number and axial Reynolds number, rotational Reynolds number and radius ratio of the annulus for both rotational and stationary annular tubes.
Keywords: Laminar forced convection, heat transfer, annular tube, rotating inner tube, radius ratio

Nomenclature

\begin{align*}
A & \quad \text{Surface area, m}^2 \\
A_c & \quad \text{Cross section area, m}^2 \\
C_p & \quad \text{Specific heat at constant pressure, J/kg.K} \\
d_i & \quad \text{Annulus inner diameter, m} \\
d_o & \quad \text{Annulus outer diameter, m} \\
d_h & \quad \text{Hydraulic diameter, m} \\
g & \quad \text{Gravitational acceleration, m/s}^2 \\
h & \quad \text{Convective heat transfer coefficient, W/m}^2\text{K} \\
N & \quad \text{Rpm of the rotating inner annulus} \\
K & \quad \text{Thermal conductivity, W/m K} \\
L & \quad \text{Test section length, m} \\
m & \quad \text{water mass flow rate, kg/s} \\
n & \quad \text{No. of thermocouples} \\
P & \quad \text{Pressure, Pa} \\
Q & \quad \text{Heat transfer rate, W} \\
q & \quad \text{Heat flux, W/m}^2 \\
T & \quad \text{Temperature, K} \\
u & \quad \text{Average velocity, m/s} \\
v & \quad \text{tangential velocity, m/s} \\
\end{align*}

Dimensionless Groups

\begin{align*}
\text{Nu} & \quad \text{Nusselt number} \\
\text{Pr} & \quad \text{Prandtl number} \\
\text{Re} & \quad \text{Reynolds number} \\
\end{align*}

d & \quad \text{Based on annulus outer diameter} \\
e & \quad \text{Exit} \\
h & \quad \text{Based on hydraulic diameter} \\
l & \quad \text{Inner} \\
o & \quad \text{Outer} \\
s & \quad \text{wall} \\
w_i & \quad \text{Inlet water} \\
w_o & \quad \text{Outlet water} \\
o & \quad \text{Based on angular velocity} \\
\end{align*}

Greek letters

\begin{align*}
v & \quad \text{Kinematic viscosity, m}^2/\text{s} \\
\omega & \quad \text{Angular velocity, rad/s} \\
\end{align*}

Subscripts

\begin{align*}
c & \quad \text{Cross-sectional area} \\
\end{align*}

1. Introduction

Several forms of rotating flows present themselves in chemical and mechanical mixing and separation devices, electrical and turbomachinery, combustion chambers, pollution control devices, swirl nozzles, rocketry, fusion reactors and in atmospheric and geophysical phenomena. Rotation of the flow in these examples can be induced through boundary motion, as an initial swirl, or through the action of body force field.

The straight axial flow with no boundary rotation and the inner cylinder rotation with no axial flow constitutes the two limiting cases of annular flow heat transfer studies. There exists a large body of work in the literature associated with both categories. A representative list of the former category includes references [1-4]. The last one authored by Kays and Leung is perhaps the most comprehensive treatment of the straight annular flow heat transfer. In that work, the authors presented an analytic solution complemented by extensive experimental data. Also, an empirical relationship in the form $\text{Nu} = 0.022 \text{Re}^{0.8} \text{Pr}^{0.3}$ is proposed to fit the experimental data.

Heat transfer in an annulus with inner cylinder rotation, but in the absence of an axial “carrier” was studied in references [5-8]. These studies established the various possible flow regimes in the annulus and their stability limits as a function of the rotational taylor number. Four flow regimes were recognized: laminar, laminar with
Taylor vortices, turbulent with Taylor vortices and fully turbulent regimes. The heat transfer studies referenced previously spanned all the four regimes. For the fully turbulent case the Nusselt is correlated with the rotational Taylor number as follows [7]:

\[ Nu = 0.409(Ta)_{m}^{0.241} \]

Here \( Ta \) is a modified Taylor number, the conventional rotational Taylor number devices by a complicated radius ratio geometric factor [5].

The mixed mode case with inner cylinder in rotation in the presence of axial flow was studied by Luke (9) in connection with the cooling of the electric motors. This work was later expanded by Gazley [10] who inferred from this data and those of Luke’s, the form \( Nu \sim (Re)_{eff}^{0.8} \). In this relationships \( Re \) was constructed by an effective velocity defined as:

\[ v_{eff} = \left[ u^2 + \left( \frac{v}{2} \right)^2 \right]^{1/2} \]  

(1)

Tachibana and Fukui [11] did similar work to Gazley’s and, again, using Gasley’s effective velocity for the narrow gap annulus offered a heat transfer correlation in the form:

\[ Nu = 0.015 \left( 1 + 2.3 \frac{D_{H}}{L} \right) \left( \frac{D_{H}}{D_{i}} \right)^{0.15} (Re)_{eff}^{0.8} Pr^{1/2} \]  

(2)

In this equation \( L \) is the test section length and short section used in the experiments warranted inclusion of the entrances effect.

Molozhen and Polyak [12] reviewed the work of references [9-11]. They proposed a lengthy expression for Nusselt number, which spans the two limiting cases of pure rotational and pure axial, fully turbulent heat transfer in annuli with arbitrary gap ratio.

The present work attempts to fill a gap in the annular flow heat transfer. This is the case of different radii ratio, comparatively small speed of rotation, long, annular channel with laminar axial flow in the presence of inner cylinder rotation. The rotation speed is varied from 100 to 628 rpm.

2. Experimental Facility

The experimental facility used to study the forced convection flows through the horizontal annular tube is designed to encompass as wide range of the relevant parameters as possible, so that a systematic investigation of the effect of both the axial flow and rotational speed (centrifugal forces) on the overall heat transfer can be carried out.

A schematic diagram showing the major components of the experimental facility is shown in Fig (1). The individual components can be loosely divided into three systems:

1. The actual test section, 2- the drive mechanism and 3- the refrigeration circuits and will be discussed below.

The actual test section is composed mainly of an outer annular shell (6) and a rotor (9) as shown in Fig (1). The annular shell is made of two galvanized steel tubes with inner and outer diameters of 54.4 and 60.33 mm for the inner tube and 78.08 and 88.9 mm for the outer one. The two tubes of the shell are assembled coaxially via two steel flanges (4) by welding so that the inner diameter of the flanges equals the inner diameter of the inner tube of the annular shell. The two flanges also serve as seals for the test section. Four holes are drilled in each flange at a pitch circle diameter of 69.0 mm for the sake of assembly. Two copper tubes diameter 6.0 mm are welded on the outer peripheral of the outer tube of the annular shell at the beginning and end of it. These copper tubes are connected to the refrigeration circuit, the evaporator of which is the annular shell itself.

Two other flanges (13, 18) are machined from steel in order to hold the
rotor concentrically with the inner surface of the annular shell and to prevent water leakage from the test section. The left flange (13) is machined so that it has a cylindrical end cap serves as a base of bearing when assembled with one end of the rotor. The right flange (18) is machined to hold a short shaft (16) provided with a ball bearing and O-ring seal in order to allow smooth shaft rotation and prevents water leakage from the test section.

The inside end of the short shaft is machined so as to have an outside diameter equal to the inside diameter of rotor end. The outside end of the short shaft is provided by a keyway in order to hold bully used to transmit motor rotation with variable speed gearbox (20) and v-belt (21).

Due to the substantial machining requirements of the annular shell, only one was constructed and used with each of rotors. The effective length of the test section is 500 mm in all cases. Thus the radii ratios (inner to outer cylindrical gap diameters) are 0.3879, 0.614 and 0.8842.

The rotor (9) of the test section is made of plastic (PVC) pipe of 22.1, 33.4 and 48.1 mm outer diameters. It is provided with a copper short rod (15) laid inside the end cup of the left flange, while the other end is fixed the short shaft by means of a through bolt passing perpendicular to the axis of the rotor.

The drive mechanism is consisted of a 220V, 1/4 kW, 1420 rpm AC electric motor (19) coupled with variable gearbox (20), the motion is transmitted from the gearbox shaft to the pulley using a key to achieve the transfer of torque to the rotor via V-belt (21).

The refrigeration circuit consisted mainly of an evaporator and condensing unit. The annular shell of the test section is used directly as the refrigeration circuit evaporator. The condensing unit (22) type DKSJ-100 Copeland is consisted of a 1.0 kW semi-hermetic compressor working with refrigerant R22, connected to an air-cooled condenser. The circuit is provided also with a receiver and thermostatic expansion valve type TF-0.5 with orifice no. 2 (10) from Danfoss company to control the refrigerant passing through the annular gap of the annular shell.

The surface temperature of the inner cylinder of the annular shell is measured by four thermocouples (8) impeded within the cylinder wall at various axial locations. An additional thermocouple is used to measure the ambient room temperature. Two thermocouples (2) are used to measure water inlet and outlet temperature. Water is drawn from constant level tank (11) through control valve (1) and the flow rate is measured by a calibrated rotameter (12). In order to facilitate water flow through the test section under investigation, two circular disks (14) made of Teflon, 10mm thickness, having the same inner and outer diameter of the flanges welded with the annular shell. Four holes of diameter 5mm distributed equally through the outer peripheral of the disk perpendicular to its axis and provided with four ports connected to the main water intake and discharge through a rubber tubes. These ports are used to insure well distribution of water through the annular gap under test.

The entire arrangement is wounded by a 50 mm fiberglass self-adhesive tape. (7) Thermocouples in the test section are all type K copper constantan. A total of 7 thermocouples were placed in the system and connected to a 12 point self switch temperature recorder type Yokogawa (17) capable of reading up to 200°C with a sensitivity of 0.1°C. The rotational speed can be detected by a photoelectric tachometer.
3. Experimental Operation and Procedure

Three input parameters can be varied independently in this system: the speed of rotation of the inner cylinder, the velocity of water flowing through the annular space and gap thickness (inner cylinder diameter). For a given combination of these parameters, the outlet temperature of water will eventually reach a certain fixed value, when this value is reached, the system is in a state of equilibrium. These conditions are ascertained by measuring water temperature at selected time intervals and observing any changes. When these changes become small (within ±0.1°C), the steady state condition is assumed. For every case studied, the equilibrium time was approximately 30 minutes. To be certain that the values
recorded are representative at the steady-state calibration 30 minutes beyond the transient setting periods and the average of these values is used in subsequent calculations.

Determining the heat transfer results in stationary annular tubes of different radius ratio and comparing them with the available correlation first standardized the experimental set up. Steady state values of mean Nusselt numbers for uniform temperature cooling of water then determined with each of the different radius ratio of annular tube for different speed of rotation of its inner tube. The characteristics of stationary annular tube as well as rotational one are shown in Table 1.

<table>
<thead>
<tr>
<th>Tube Number</th>
<th>(d_i) mm</th>
<th>(d_o) mm</th>
<th>(d_h) mm</th>
<th>(L) mm</th>
<th>(N) rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.1</td>
<td>54.4</td>
<td>33.3</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>33.4</td>
<td>54.4</td>
<td>21.0</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>48.1</td>
<td>54.4</td>
<td>6.30</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>21.1</td>
<td>54.4</td>
<td>33.3</td>
<td>500</td>
<td>100-628</td>
</tr>
<tr>
<td>5</td>
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<td>21.0</td>
<td>500</td>
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</tr>
<tr>
<td>6</td>
<td>48.1</td>
<td>54.4</td>
<td>6.30</td>
<td>500</td>
<td>100-628</td>
</tr>
</tbody>
</table>

4. Flow and Heat Transfer Characteristics

From the detailed constructions of the annular tubes described in Table 1, the cross sectional area of the stationary as well as the rotational annular tubes can be set as:

\[
A_c = \frac{\pi}{4} (d_o^2 - d_i^2)
\]

The volumetric diameter defined as four times the volume for flow divided by the area of the witted surface was used as the hydraulic diameter (\(d_h\)) for the sake of comparison with the available literatures. The hydraulic diameter can be expressed as:

\[
d_h = d_o - d_i
\]

Based on the hydraulic diameter as a characteristic length, Reynolds number of the annular tubes \(Re_h\) can be calculated from:

\[
Re_h = \frac{u \times d_h}{v}
\]

Where \(u\) is the mean velocity of water flowing through the annular space. Correspondingly, Nusselt number of the annular tubes \(Nu_h\) based on the hydraulic diameter can be written as:

\[
Nu_h = \frac{h_o \times d_h}{k}
\]

Where \(h_o\) is the heat transfer coefficient at the outer surface of the annular tube that can be calculated as follows: The heat transfer rate between Refrigerant R-22 and water flows across the annular space were calculated by:

\[
Q = mc_p (T_{wo} - T_{wi}) = h_o A_o (LMTD)
\]

Where \(LMTD\) is the logarithmic mean temperature difference defined as:

\[
LMTD = \frac{(T_{wi} - T_s) - (T_{wo} - Ts)}{\ln[(T_{wi} - T_s) / (T_{wo} - T_s)]}
\]
Where $T_{in}$ and $T_{out}$ are inlet and outlet water temperatures while $T_s$ is the mean wall temperature defined as:

$$ T_s = \left( T_{s1} + T_{s2} + T_{s3} + T_{s4} \right) / 4 $$  \hspace{1cm} (9)

Where $T_{s1}$ to $T_{s4}$ are the local temperatures of the outer tube of the annulus.

In view of the different values of inner tube diameter and consequently the different values of the hydraulic diameter, the outer diameter of the annulus is used as a characteristic length. Thus Reynolds number as well as Nusselt number of the stationary as well as the rotational annular based on its outer diameter can be expressed as:

$$ Re_d = \frac{u \times d_o}{\nu} $$  \hspace{1cm} (10)

$$ Nu_d = \frac{h_o \times d_o}{k} $$  \hspace{1cm} (11)

The effect of rotational speed of the inner tube may be accounted for by the rotational Reynolds number $Re_o$, based on the tangential velocity of the rotated tube $\nu$ and the hydraulic diameter of the annular tube as a characteristic length. Then the rotational Reynolds number can be set as:

$$ Re_o = \frac{\omega \times d_i \times d_h}{\nu} $$  \hspace{1cm} (12)

Where $\omega$ is the angular velocity of the rotating inner tube (rad/sec).

5. Results and Discussion

Laminar forced convection of water flowing through annular tubes having rotational inner tubes is studied experimentally. The outer tube of the annul is subjected to uniform temperature due to the evaporation of Refrigerant R22 flowing through the outer annular shell used as the evaporator of a refrigeration circuit while the inner wall is insulated. The effect of axial Reynolds number $Re_h$, Rotational Reynolds numbers $Re_o$ and radii ratios of the annulus tube dido on the heat transfer rates are discussed in this section. In the present work radii ratios are equal to 0.3879, 0.614 and 0.8842, axial Reynolds numbers range from 80 to 2700 while the rotational Reynolds numbers range from 2000 to 29400.

5.1. Validation of Experimental Results

Before reporting to the main results of the present work, mention will be made of relevant auxiliary experiments. To demonstrate the validity of the experimental apparatus, heat transfer experiments were performed with three stationary annular tubes in order to enable comparison with heat transfer results in the literature. The relation between Nusselt number and Reynolds number based on the hydraulic diameter of the annular tubes with radii ratios of 0.3879, 0.614 and 0.8842 are presented in Fig. (2) as well as three straight lines representing the correlation of reference [17] for different radii ratios. It is concluded from the figure that the present experimental results for stationary annular tubes with different radii ratios are in fair agreement with the literature.

![Fig. (2) Nusselt number versus Reynolds number based on hydraulic diameter ($h_o$) at different radii ratios for stationary annulus.](image)
The diameter of the outside tube of the annulus rather than its hydraulic diameter are used in Nusselt and Reynolds numbers in order to best show the variation obtainable due to the variation of radii ratios. The average Nusselt numbers $N_u$ of the stationary annular tubes with radii ratios of 0.3879, 0.614 and 0.8842 are shown in Fig. (3) as functions of axial Reynolds number $Re_a$. As can be expected the Nusselt number increases with Reynolds number for the same radius ratio. The figure also shows that, the Nusselt number increases with the increase of radius ratio at the same Reynolds number.

5.2 Stationary Annular Tubes

Correlation

As seen from Fig. (2) the empirical correlation of Ref [17] when represented for different radii ratios gives different straight lines each of them corresponding to a finite radius ratio. From these plots it is easily perceived that relations of the type $N_u = c (Re)^n$ will represent the experimental data of constant radius ratio.

![Fig. (3) Nusselt number versus Reynolds number based on annulus outer diameter ($d_o$) at different radii ratios for stationary annulus.](image)

An empirical correlation was suggested to correlate the data of stationary annular tubes of different radius ratio. A least square fit of the present experimental data of stationary annular tube covering a wide range of $Re_h$ and different radii ratios gave the following correlation:

$$N_u = 1.175 Re_h^{0.4} (1 + 0.676 (d_i / d_o))$$

The above correlation is valid for $80 \leq Re_h \leq 2700$, $0.3879 \leq d_i / d_o \leq 0.8842$ and $15 \leq L / D_h \leq 80$ and predicts the values of Nusselt number, which agree with the experimental results within ±8 % as shown in Fig (4).

5.3 Rotational Annular tubes Results

Attention will now be turned to the heat transfer results of rotational annular tubes. In this connection the sequence in which the experiments were performed was chosen in order to facilitate the presentation of results. For a specified value of radius ratio $(d_i / d_o)$, successive experiments were carried out with different values of rotational Reynolds number $Re_o$ one at a time. The axial Reynolds number $Re_h$ was changed for each radii ratio and rotational Reynolds number in order to cover its specified range each time. Therefore three groups of experiments were performed and discussed in the following sections.

![Fig. (4) Representation of general equation of Stationary annular tubes](image)
The heat transfer coefficient at the outer surface of the annular tubes in case of laminar flow of water through the annular space for three different radii ratios were analyzed in terms of \( \text{Nu}_h - \text{Re}_h \) relationships. Figures (5-7) show the variation of Nusselt number \( \text{Nu}_h \) with Reynolds number \( \text{Re}_h \) for annulus of radii ratios of 0.3879, 0.614 and 0.8842 respectively. From the over view of these figures it may be concluded that Nusselt number for the rotational annular tubes are higher than those of the stationary ones in spite of the value of radius ratio.

Fig (5) shows that for a radius ratio of 0.3879 the rotational speed of nearly 270 rpm (\( \text{Re}_o = 11142 \) nearly) produce a maximum increase in Nusselt number of about 40%, whereas the rotational speed of 628 rpm yielded an increase of only 12% in Nusselt number, compared to the results of the stationary annular tube of the same radius ratio. The same trend can be seen from Figures (6 and 7) with maximum increase in Nusselt numbers of 28% and 44% and minimum increase in Nusselt numbers of 4% and 12% at radii ratios of 0.614 and 0.8842 respectively in comparison with the results of stationary annular tubes with the same radius ratio.

To clearly show the effect of rotational speed on the heat transfer rate, elimination of the effect of axial Reynolds number and radious ratio has been done. The complex \( \text{Nu}_h / [(1 - 0.676(\text{d}/\text{d}_o))^{0.4}] \) is plotted versus the rotational Reynolds number \( \text{Re}_o \) as shown in Fig (8). It is concluded from Fig (8) that the Nusselt number is increased with the increases of the rotational Reynolds number (rotational speed) up to a maximum value at a rotational Reynolds number of nearly
11000 (rotational speed $\pm 270$ rpm) and then decreases with further increase in rotational Reynolds number.

The above mentioned trend may be due to the effect of centrifugal force created due to the rotation of the inner surface of the annular tube and consequently the rotation of fluid layer in contact with the rotated surface. Increasing the rotational speed to a certain value increases the centrifugal force and consequently mixing of fluid and boundary layer disturbance occurs. This in turn increasing the heat transfer rate At higher values of rotational speed fluid layer adjacent to the rotating inner tube may be slipped over it. This means less rotation of water inside the annular space and consequently minimum centrifugal force prevails from one hand and heat is generated due to friction between the rotating tube and the adjacent fluid from the other hand. This leads to decrease the heat transfer rate with further increase in rotational speed.

Nusselt number $Nu_d$ was correlated with the other relevant governing parameters such as axial Reynolds number $Re_h$, rotational Reynolds number $Re_\omega$ and radius ratio $d/d_o$ of the rotational annular tubes including the results of stationary annular tubes. With the aid of the least square method three steps were made to eliminate $Re_h$, $d/d_o$ and $Re_\omega$ using computer program (GRAPHER), the following correlation was obtained:

$$Nu_d = 1.199 Re_\omega^{0.4} f_1(Re_\omega) f_2(d_1/d_o)$$

(14)

Where:

$$f_1(d_1/d_o) = 1 + 0.676(d_1/d_o)$$

(15)

$$f_1(Re_\omega) = 1 + 4.69 \times 10^{-3} Re_\omega - 1.786 \times 10^{-2} Re_\omega^2$$

(16)

This correlation is valid for the following ranges parameters:

$80 \leq Re_h \leq 2700, \quad 2000 \leq Re_\omega \leq 29400$ and $0.3879 \leq d_1/d_o \leq 0.8842$

The above correlation predicts the Nusselt number values within an error of $\pm 18\%$. The above correlation is presented together with the experimental results in Fig (9).

![Fig. (8) Effect of rotational Reynolds number on Nusselt number for different radii ratios.](image)

![Fig. (9) Effect of rotational Reynolds number on Nusselt number for different radii ratios.](image)

6. Correlation

An attempt was made to correlate the experimental results obtained in the present study. Such correlation is quit useful for the designers to calculate the heat transfer coefficient for similar cases. The average
7. Conclusion

Experiments were conducted to evaluate the laminar flow forced convection heat transfer characteristics of water flowing through the annular space of horizontal rotational annular tubes with different values of radii ratios under uniform temperature. The following conclusions can be drawn from the results of this investigations:

1. Nusselt number increases with both axial Reynolds number and the decrease of radius ratio for both stationary and rotational annular tubes.

2. Nusselt number of rotational annular tube is higher than that of stationary annular tubes at the same axial Reynolds number and radius ratio.

3. Nusselt number of rotational annular tubes increases with rotational Reynolds number (rotational speeds) and then decreases with further increase in rotational Reynolds number.

4. Compared to the stationary annular tube results, an increase as high as 44% in heat transfer has been achieved in rotational annular tube having diameter ratio of 0.8842 and rotational speed of 270 r.p.m.

References


