MATCHING OF PV POWER SYSTEMS WITH ELECTRICAL LOADS USING GRAPH THEORETIC MODELING APPROACH

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EXTRACTED TEXT:

خاتمة:

تعتبر الموائمة بين الخلايا الفوتولفولتية والأعمال الكهربائية التي تخذلها من أهم الأعمال التي تؤثر في كفاءة أداء المنظومة الفوتولفولتية والاستفادة القصوى من الطاقة الناتجة عن الخلايا، وفي حالة الموائمة الجيدة بين الخلايا والأعمال يكون منحنى التحويل قريبًا من منحنى الفضاء الحولية الفردي عن الخلايا عند قيم إشعاع مختلفة.

ويعد هذا البحث طريقة جديدة لموائمة نظام شمسي مكون من منظومة خلايا فوتولفولتية مستقلة مترابط مع MATLAB 6

تتمثل المكونات المختلفة للنظام بطريقة منهجية متكاملة النظرية. ومن ثم تطبق هذا البرنامج على عدد من دراسات الحالة لأزواج مختلفة من محركات التيار المستمر والأعمال السينتوباكية والمطاربة بينها للوصول إلى أفضل موائمة بين الخلايا وكل نوع من أنواع الأعمال.

ABSTRACT

The quality of load matching in a photovoltaic power system (PVPS) determines the system performance and the degree or the solar cells utilization. In a good matched system, the operation of the load-line is close to the maximum power-line of the solar cell (SCA) generator. This paper presents a new methodology for modeling a stand alone PVPS using the Graph Theoretic Modeling (GTM) Approach. The studied system composed of a directly connected SCA generator-dc motor supplying different load types. A computer program (written in MATLAB 6) has been developed to model the studied system. The computer program is then applied to nine case studies for different types of dc-motors and various mechanical loads. A comparison between these cases is introduced to indicate the appropriate motor type for each load.

1. INTRODUCTION

The quality of load matching in a photovoltaic power system (PVPS) determines the system performance and the degree or the solar cells utilization. In a good matched system, the operation of the load-line is close to the maximum power-line of the solar cell (SCA) generator. Computer simulation for the operation and sizing of photovoltaic (PV) components is a very thorough method of determining the behavior of PVPS. Using simulation methods the electrical power output can be optimized with respect to component sizes. A method of detailed modeling and simulation must be available before the issue of optimum sizing is adequately pursued. The topics of modeling and simulating PV powered systems which, include PV array, dc motor and mechanical load was well documented in [1], [2], and [3]. An investigation of directly coupled photovoltaic pumping connected to a large absorber field through modeling and simulating PV array, dc motor,
centrifugal pump and the absorber field was presented in [4]. Detailed modelings of Photovoltaic system components using TRANSYS program were analyzed in [5]. Whereas, Ref. [6] applied the TRNSYS program to a large scale PVPS used for water heating and expanded the model to add more components to the program. Ref [7] investigated the long-term performance of PV pumping system with a maximum power point tracker. The operation characteristics of dc-permanent, series motor and shunt motor with a centrifugal load was presented and compared in [8]. A mathematical methodology for the optimum configuration of photovoltaic pumping system in a solar domestic hot water was investigated in [9]. One of the widely accepted network theories is the Graph Theoretic Modeling (GTM) Approach. The GTM approach has been in use since 1955. It has emerged as a method that allows the modeling and solving of systems that are hydraulic, pneumatic, structural, mechanical and thermal or combination thereof [10]. A method for modeling the directly-connected stand-alone PVPS using GTM method was introduced in Ref. [11]. The paper presented an approach to reduce the computational efforts needed in setting up the required equations. The GTM approach was applied to a PV powered ventilator system and a PV powered pumping system [11].

This paper applies a comprehensive study of applying the GTM approach to a directly connected SCA generator to supply different load type. The load may be constant, ventilator or centrifugal pump. Whereas, the used dc motor may be series, separately excited or shunt motor. The system components are modeling using GTM approach to generate a nonlinear system of equations called Newton-Raphson Mixed Nodal Tableau (NR-MNT). The equations are then solved numerically by developing a computer program written in MATLAB 6. A comparison between the different case studies is achieved based on the quality of load matching for each case.

2- SOLVING NONLINEAR SYSTEMS USING GTM APPROACH

The GTM approach provides a set of equations that characterize the individual components of the system. The component is the smallest indivisible element of our interest within a system. The behavior of a component can be characterized by using two types of variables [11].

- **Through variables (q):** quantify flow process such as current, torque and flow rate.
- **Across variables (x):** quantify the stimulus causing flow such as voltage, speed and hydraulic head.

For modeling a combined PV-dc motor system, components of electrical, mechanical and hydraulic are encountered. When the components are replaced by their terminal graphs, the resulting diagram is called **system graph.** The system graph depicts the connectivity of components without any ambiguity. The system graph can be easily converted into a matrix form (incident matrix) to indicate the connectivity of the system components. The incident matrix performs this task through the use of +1, -1 & 0's. Rows and columns represent the individual nodes and edges, respectively. With the help of the incident matrix both the **vertex** and the **nodal transformation** equations are evolved. The vertex equations stem from a generalization of Kirchoff's current law (to incorporate systems other than electrical networks). This matrix ensures the conservation of flow at each node and appears as:
\[
\begin{bmatrix}
Y_o \\
A_e \\
A_x \\
A_r
\end{bmatrix}
\begin{bmatrix}
Y_o \\
Y_r \\
Y_i
\end{bmatrix} = [0]
\]

(1)

Where, \(A_e\) and \(A_v\) are across and variable sources respectively. \(A_x\) and \(A_r\) are conductive and resistive portions of constitutive components respectively. The vectors with \(Y\) represent the set of all the through variables \(y\) partitioned analogous to the incidence matrix \(A\) corresponding to the component types. The nodal transformation equations provide the relationship between component across variables \((X_o, X_r, X_e, X_i)\) to the nodal across variables \((Y_o)\). The vectors with \(X\) represent the collection of across variables \(X\) partitioned analogous to the incidence matrix \(A\) corresponding to the component type.

\[
\begin{bmatrix}
X_o \\
X_r \\
X_e \\
X_i
\end{bmatrix}
= \begin{bmatrix}
A_o^T \\
A_e^T \\
A_x^T \\
A_r^T
\end{bmatrix} \ast X_o
\]

(2)

These two sets of equations give rise to a system of equation known as Mixed Nodal Tableau (MNT) [11]. The MNT system of equations is formulated to find a set of variables that once solved, is sufficient to calculate all other system variables. For linear systems the MNT is a system of equations summarizing the behavior of the system. For nonlinear systems the same set of equations is used to formulate the Newton Raphson Mixed Nodal Tableau (NR MNT), which have to be solved iteratively, explained later.

Expanding Eq. (1) and substituting for \(Y_o\) with the conductive vector \(G_e\), the vertex equation can be rewritten as:

\[
A_o^T X_o + A_e G_e + A_x Y_r + A_r Y_i = 0
\]

(3)

Where the values of through sources \(Y_i\) are known. Using the identity for \(X_e\) in Eq. (2), and substituting \(X_e\) for the resistive vector \(P_r\), the following can be written as:

\[
A_e^T X_e - P_r = 0
\]

(4)

By using the identity of \(X_r\) from Eq. (3), a third set of equations using a generalization of Kirchoff’s voltage law, can be written as:

\[
A_o^T X_o - X a = 0
\]

(5)

Where the values of the across sources \(X_o\) are known. Equations (3)-(5) can consolidated and designated as the MNT \(F(z)\) which represent a set of nonlinear equations in which there are the same number of unknown variables \(Z = [X_o^T, Y_r^T, Y_i^T]\) as the number of equations:

\[
F(z) = \begin{bmatrix}
f_1(z) \\
f_2(z) \\
f_3(z)
\end{bmatrix} = \begin{bmatrix}
A_o G_e + A_e Y_r + A_x Y_o + A_r Y_i \\
A_o^T X_o - P_r \\
A_o^T X_o - X a
\end{bmatrix}
\]

(6)

The Newton–Raphson method is used to solve this system of non linear equations as follows:

\[
\frac{\partial F(z)}{\partial z} \bigg|_{z^{k+1}} \Delta z^{k+1} = -F(z)_{z^{k+1}}
\]

(7)

where \(\Delta z^{k+1}\) is given by \((z^{k+1} - z^k)\) in which \(z^k\) is the \(k^{th}\) iterative guess for \(z\). The Jacobian can be written as:
\[ F^{1z} = \frac{\partial F(z)}{\partial z^T} = J(F(z)) = \begin{bmatrix} \frac{\partial f_1(z)}{\partial X_n} & \frac{\partial f_1(z)}{\partial Y_r} & \frac{\partial f_1(z)}{\partial Y_a} \\ \frac{\partial f_2(z)}{\partial X_n} & \frac{\partial f_2(z)}{\partial Y_r} & \frac{\partial f_2(z)}{\partial Y_a} \\ \frac{\partial f_3(z)}{\partial X_n} & \frac{\partial f_3(z)}{\partial Y_r} & \frac{\partial f_3(z)}{\partial Y_a} \end{bmatrix} \]

Where \( F^{1z} \) is a notation for the first order partial derivative of the MNT equations \( F(\theta) \) with respect to the transpose of the vector \( z \).

Throughout this paper a similar notation is used in which \( a^{1z} \) is the first order partial derivative of \( a \) with respect the transpose of vector \( b \). Using this notation (7) can be expanded and written as:

\[ F^{1z} |_{z=a}^k = - F(z)|_{z=a}^k + F^{1z} |_{z=a}^k z_k \]

By performing the operation indicated in Eq. (9) on Eq. (6), the NR MNT system of equations is found as:

\[
\begin{bmatrix}
A_0G_c^{1z}A_r\tilde{\alpha} + A_0G_c^{1z}A_r - P_{s}^{1z}A_r \\
A_r - P_{s}^{1z}A_r - P_{r}^{1z}X_a + A_0G_c^{1z}Y_{r} - A_0Y_{r} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
X_{n}^{r+1} \nu \nY_{r}^{r+1} \nu \nY_{a}^{r+1} \nu
\end{bmatrix}
= \begin{bmatrix}
X_{n}^{r} \nu \nY_{r}^{r} \nu \nY_{a}^{r} \nu
\end{bmatrix}
\]

Where [NR MNT] or [NR R] is used, it refers to the left side matrix and the right side vector, respectively. When NR MNT is mentioned, it refers to the entire system of equations.

Eq. (10) can be solved iteratively by evaluating:

\[
\begin{bmatrix}
X_{n}^{r+1} \nu \\
Y_{r}^{r+1} \nu \\
Y_{a}^{r+1} \nu
\end{bmatrix} = [\text{NR MNT}]^{-1}[\text{NR R}]
\]

3- APPLYING THE GTM APPROACH TO COMBINED PV-DC MOTOR SYSTEMS

Any system consists of different components, each component has its behavior and its special characteristics that can be replaced by a stamp on behalf this component. By modeling any component using the GTM approach we can create a stamp of each one by identifying the following items:

- Schematic diagram,
- Component type,
- System terminal graph,
• System incident matrix,
• System terminal equation,
• Newton Raphson Mixed Nodal Tableau (NR MNT) and
• Newton Raphson Reduction (NR R).

3-1 Steps of Applying GTM Approach to a System

In this paper we will apply the GTM approach to a combine PV-dc motor system
supplying different mechanical load types to obtain the optimum utilization of solar
radiation. Following are the main steps required to apply the GTM approach on any
system:

1. **Draw the terminal graph of the system.**
   • The “component terminal graph” represents the combination of all edges (line
     segments) and nodes (end points of edges) associated with a single component are
drawn. The components are then replaced by terminal graph, and the resulting
diagram of nodes and edges express “system graph”.
   • The “incident matrix” is formulated using the system graph, which depicts the
     connectivity of components without any ambiguity.
   • The nodes are designated by letters and the edges are designated by numbers.

2. **Identify the system terminal equations.**
   • The across and through variables mathematical relations for each component are
     expressed.
   • The number of terminal equations of the system is equal to the dimension of
     “NR MNT” matrix.
   • “Across variables” denote the stimulus causing flow such as (V, H, ω)
   • “Through variables” denote flow process such as (I, Q, T).

3. **Create incident matrix:**
   • The incident matrix connects the system components through the use of +1, -1, and
     0s. The individual a_{ij} th element of the incident matrix A is given a value as
     follows:

   \[
   a_{ij} = \begin{cases} 
   0 & \text{if the } j \text{th edge is not incident on} \\
   1 & \text{incident and towards} \\
   -1 & \text{incident and away from} 
   \end{cases} 
   \text{the } i \text{th node}
   \]

   • If the component does not exist, the corresponding sub-matrix altogether is omitted
     from the matrix; the datum nodes are omitted from the matrix because they offer
     redundant information.

   \[
   A = [A_o \ A_c \ A_r \ A_i ]
   \]

4. **Get the variables**

   The variables include the across, through, and nodal across variables:
   • Across variables \([X_o : X_c : X_r : (P) : X_i ]\)
   • Through variables \([Y_o : Y_r (g_r) : y_r : y_i ]\)
   • Nodal across variable \([X_o ]\)
Where

\((X_a, Y_a)\) complementary pair associated each other \((X_a)\) presents across source such as voltage source and static head \([V_{in}], [H_{static}]\), \((Y_a)\) complement pair of \([X_a]\).

\((X_c, Y_c)\) (motor or pump) conductive portion of constitutive component \(Y_c\) complement pair \(X_c\) motor or the pump, \((X_c)\) often not included.

\((P_r, Y_r)\) (motor or pump) resistive portion of constitutive component such as the input voltage of a motor or the input speed of a pump, \((Y_r)\) complement pair \(P_r\).

\((X_t, Y_t)\) \((Y_t)\) through sources such as current source \((I_{eb})\) or constant torque source \((T_{const})\) \((X_t)\) complement pair \((Y_t)\) not included across variable \((X_t)\) at each node such as \((V_a, \omega, \mathbf{H})\).

5. **Formulate the [NR MNT] and [NR R]**

Use the system equations and the variables of the system to substitute in the Jacobian matrix Eq. (8) to get the [NR MNT]. Then substitute in Eq. (9) to get Newton Raphson Reduction [NR R].

6. **Numerically Solve the problem**

The [NR MNT] and [NR R] are solved numerically by developing a MATLAB computer program to get the sufficient parameters to characterize the system \((V, I, T, \omega, H, Q)\). These parameters are then used to compute the electrical output power of SCA generator \((P_{elech})\), the mechanical output power of the dc motors \((P_{mech})\), the hydraulic power of the pump \((P_{hyd})\), and the system efficiency \((\eta)\) and utilization \((\mu)\) as we will explain in the following sections.

3-2 Modeling of Combined PV-DC Motor Systems Using GTM Approach

Load types may be connected to the SCA generator–dc motor combination according to the required application. The main purpose of this paper is to indicate the quality of load matching for different mechanical load types connected to dc motors and powered by SCA generator for optimum utilization of solar radiation using (GTM) technique. The dc motors can be classified, according to the type of field excitation, into: series, permanent magnet, shunt and separately excited motors. Permanent magnet motors have similar equations as separately excited ones in case of constant field current. The widely used mechanical load types can be categorized as: constant load, ventilator load and centrifugal pump. The following section will analyze each of the three load connected to different motor types to compromise suitable matching. The results are used to compare between different case studies.

3-2-1 SCA generator modeling

The SCA generator can be modeled as a current source in parallel to a diode. Series resistance \((R_s)\) accounts for the total resistance at the PV cell/wire contact interface as shown in Fig. 1-a. The current through the diode can be expressed as:

\[
I_1 = I_o (e^{V/R_s} - 1)
\]

Where:

- \(\sigma = KT/q\)
- \(\sigma\): Thermal voltage, Volts
- \(K\): Boltzmann constant
- \(T\): Absolute temperature in Kelvin
- \(q\): Electron charge, Coulomb
- \(I_o\): Reverse saturation current, A

The current through the series resistance is \(I_2 = V_2 / R_s\) as shown in Fig. 1-a.
3-2-2 Terminal equations of dc motors

The (GTM) methodology deals with the motor as a 4-terminal constitutive component due to the transducing nature of the motor which, converts electrical energy to mechanical energy. The input of the motor is the voltage gained from SCA should be in resistive form i.e. \([P_i] = [V_i]\) whereas, the output of the motor (the electromagnetic torque delivered to the load) should be in conductive form i.e. \([G_e] = [T_e]\). The subscript (3) and (4) denotes to voltage and torque edges. The voltage and torque equations representing the dc motors can be drawn as [8]:

\[
\begin{align*}
V_3 &= M_{af} I_3 \omega_4 + R_a I_3 \\
T_4 &= -M_{af} I_3^2 + B \omega_4 
\end{align*}
\] (14)

\[
\begin{align*}
V_3 &= c_e \omega_3 + R_a I_3 \\
T_4 &= -c_e I_3 + B \omega_4 
\end{align*}
\] (15)

\[
\begin{align*}
V_3 &= \left( \frac{R_a R_f}{R_a + R_f - M_{af} \omega_4} \right) I_3 \\
T_4 &= \left( \frac{M_{af} (R_f - M_{af} \omega_4)}{R_a + R_f - M_{af} \omega_4} \right)^2 I_3^2 
\end{align*}
\] (16)

Where

- \(I_3, V_3\) : Motor input current and voltage (at edge 3),
- \(R_a, R_f\) : Armature and field resistances of the dc motor, \(\Omega\)
- \(M_{af}\) : Mutual inductance between armature and field, \(H\)
- \(B\) : Viscous Torque constant of rotational losses, \(N.m/\text{rad/s}\)
- \(c_e\) : Flux coefficient, \(V/\text{rad/sec}\)
- \(\omega_4, T_4\) : Motor output speed and torque (at edge 4),

3-2-3 Modeling of different load types

\(a\) Constant Load Torque

In steady state operation, the electromagnetic torque value is the summation of the constant load torque plus mechanical losses so that: \(T_e = \text{constant}\). The schematic diagram of the constant load torque and the system graph is shown in Fig 1-a, 1-b.

![Schematic diagram and system graph of constant load powered by SCA generator](image)

\(1\) Schematic diagram and \(2\) System graph

**Fig. 1 Schematic diagram and system graph of constant load powered by SCA generator**
The incident matrix of this system is:

\[
\begin{bmatrix}
A_c & A_r & A_i
\end{bmatrix}
\begin{bmatrix}
1 & 2 & 1 & 4 & 3 & 5 & 6
\end{bmatrix}
\]

\[
A = b
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
\]

The variables required to solve the NRT MN system of equations are:

\[
P_r = [V_1] ; \quad G_s = \begin{bmatrix} T_s \\ J_s \end{bmatrix} ; \quad X_s = \begin{bmatrix} V_s \\ \omega_s \end{bmatrix} ; \quad y_s = \begin{bmatrix} f_{PM} \\ T_s \end{bmatrix} ; \quad y_i = \begin{bmatrix} J_s \\ \omega_s \end{bmatrix}
\]

The resistive portion variable and the conductive portion variable of each constitutive component such as the motor or the pump \([P_r], [G_s]\), respectively, should be identified because they are the heart of the Jacobean matrix (as illustrated by Eq. (6)). The other variables \([X_s], [\omega_s]\) and \([J_s]\) are the sufficient variables needed to characterize the system.

b) Ventilator load torque

This load consists of a dynamic torque \(T_s\) and a static torque \(T_5\) and is given by:

\[
T_s = \text{b} \cdot \omega_s^2 \quad T_5 = \text{a}_f
\]

where:

- \(\omega_s, T_s\): Input speed and torque to the centrifugal pump (at edge 5),
- \(\text{a}_f\): Static torque constant
- \(\text{b}_f\): Dynamic torque constant
- \(\text{c}_f\): C factor

The schematic diagram and system graph are shown in Fig. 2.

*a* Schematic diagram

*b* System graph

Fig. 2 Schematic diagram and system graph of ventilator load powered by SCA generator

The incident matrix of this system is:

\[
\begin{bmatrix}
A_c & A_r & A_i
\end{bmatrix}
\begin{bmatrix}
1 & 1 & 2 & 4 & 5 & 3 & 6 & 7
\end{bmatrix}
\]

\[
A = b
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
\]
The variables required to solve the NRT MN system of equations are:

\[
P, \omega [I, J] : G_x = \begin{bmatrix} l_x \\ l_y \\ J \end{bmatrix} : X_u = \begin{bmatrix} I_u \\ T_u \end{bmatrix} : y_x = [l_x] : y_t = [I_y] \]

c- Centrifugal pump load

The centrifugal pump is the most popular type used in a PV-pumping system. The motor converts the electrical energy into mechanical energy. The pump converts mechanical energy into hydraulic energy with static head. The terminal equations to represent the operation of the pump can be given by:

\[
\omega_s = \omega_{orf} \sqrt{\frac{H_0}{a_{st}}} \quad Q_0 = \frac{\eta \omega_s T_s}{\rho g H_0}
\]

(18)

where:
- \(H_0\) Rated head, m
- \(Q_0\) Rated flow charge, liter/min
- \(\eta\) Pump efficiency, %
- \(\rho\) Fluid Density, KG/m³
- \(g\) Gravitational constant
- \(a_{st}\) Static factor

The pictorial representation, schematic diagram and system graph are shown in Fig. 3.

![Pictorial representation](image)

b- Schematic diagram:

c- System graph

*Fig. 3 Pictorial representation, schematic diagram and system graph of centrifugal pump powered by SCA generator*
The *incident matrix* of this system is:

\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The *variables* required to solve the NRT MN system of equations are:

\[
[p_t^r] = \begin{bmatrix} V_j \\ \omega_j \end{bmatrix}, \quad [G_t^r] = \begin{bmatrix} I_j \\ T_j \\ Q_j \end{bmatrix}, \quad [X] = \begin{bmatrix} V_s \\ \omega_s \end{bmatrix}, \quad [Y] = \begin{bmatrix} h_{t_{1{\text{ath}}}^r} \\ \omega_t \end{bmatrix}, \quad [Y] = \begin{bmatrix} \Omega_j \\ \Omega_t \end{bmatrix}
\]

### 4- ANALYSIS OF LOAD MATCHING WITH DIFFERENT MOTOR TYPES USING GTM APPROACH

The quality of load matching for PVPS is a measure of performance quality and degree of the solar cell utilization efficiency. In good matched systems the operation of the load line is close to maximum power line. In this paper the quality of load matching, \( Q_t \) is defined as the area under the mechanical load curve with respect to the maximum power line curve. Nine case studies are presented in this section. For each case study, the system components are modeled using GTM approach. Then they are converted to NR MNT and NR R matrices and numerically solved to get \((V, I, T, \omega)\) which are sufficient to characterize the system. These parameters are then used to compute the electrical output power of SCA generator, the mechanical output power of the dc motors and the system efficiency and utilization. A computer program is developed to solve these equations numerically using MATLAB 6. The motor, load and SCA generator data used for this study are given in the Appendix.

Following are the final forms of NR MNT and NR R matrices, the input parameter used for each case study, and the results obtained after applying the computer program. Tables 1-3 illustrate the output results, whereas, Figs. 4-9 show the characteristics and the utilization curve for different case studies.

#### 4-1 A Series Motor Connected to Different Load Types

##### 4.1.1 Constant load

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{bmatrix}
\frac{i_{in}}{a} & e^{\frac{-i_{in}}{a}} & -g & 0 & 0 & 0 \\
g & g & 0 & 1 & 0 & 0 \\
0 & 0 & B & -2M_d I_2^{*} & 0 & 0 \\
0 & 1 & -M_d I_2^{*} & -R_d & M_d a_t & 0 \\
\end{bmatrix}
\begin{bmatrix}
V_{in}^{r-1} \\
V_{in}^{t-1} \\
T_d \\
L_2^{*+1} \\
\end{bmatrix}
= \begin{bmatrix}
I_{ph} + I_o + \frac{e^{i_{in}}}{a} \left( \frac{V_{in}^{r}}{a} - 1 \right) \\
0 \\
T_c - M_d (L_2^{*})^2 \\
T_2 - M_d (L_2^{*})^2 \omega_c \\
0 \\
\end{bmatrix}
\]
4.1.2 Ventilator load

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{align*}
I_e & \quad V_e - g \quad 0 \\
-g & \quad g \quad 0 \\
0 & \quad 0 \quad B + bj\omega_e \quad -2M_{e}I_e^2 \\
0 & \quad 1 \quad -M_{e}I_e^2 - R_e - M_{e}\omega_e \\
\end{align*}
\]

\[I_{ph} + I_e + I_e e^{\frac{s}{a}} \left( e^{\frac{v_e}{a}} - 1 \right) = 0\]

4.1.3 Centrifugal pump load

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]
\begin{align*}
\left[ \begin{array}{c}
I_0 e^{\nu_{10}} g + \frac{g}{a} & 0 & 0 & 0 & 0 & 0 \\
-g & g & 0 & 0 & 1 & 0 \\
0 & 0 & B & 0 & -2M_{af} I_s^b & 1 \\
0 & 0 & 0 & \frac{\eta_0 \omega^b}{\rho g h_d^b} & 0 & \frac{\eta_0 \omega^b}{\rho g h_d^b} \\
0 & 1 & -M_{af} I_s^b & 0 & -R_o - M_{af} \omega^s & 0 \\
0 & 0 & 1 & -\frac{\omega_{ref}}{2 \sqrt{a_{in} h_d^b}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array} \right] \\
\left[ \begin{array}{c}
V_{a^1} \\
V_{b^1} \\
\omega_{a^1} \\
\omega_{b^1} \\
I_{a^1} \\
I_{b^1} \\
T_{a^1} \\
T_{b^1} \\
Q_{a^1} \\
\end{array} \right] =
\end{align*}

Table (3-a) Parameters of a series motor connected to a centrifugal pump load

<table>
<thead>
<tr>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a$</td>
<td>148</td>
<td>140</td>
<td>130</td>
<td>117</td>
<td>99</td>
<td>75</td>
<td>48.9</td>
<td>28.6</td>
</tr>
<tr>
<td>$V_b$</td>
<td>140</td>
<td>132</td>
<td>122</td>
<td>109</td>
<td>91.9</td>
<td>68.9</td>
<td>44.1</td>
<td>25.1</td>
</tr>
<tr>
<td>$a_{in}$</td>
<td>194</td>
<td>184</td>
<td>174</td>
<td>158</td>
<td>142</td>
<td>121</td>
<td>88.1</td>
<td>60</td>
</tr>
<tr>
<td>$h_d$</td>
<td>57.7</td>
<td>51.6</td>
<td>46.4</td>
<td>38.5</td>
<td>30.9</td>
<td>22.5</td>
<td>11.9</td>
<td>5.5</td>
</tr>
<tr>
<td>$L_s$</td>
<td>9.15</td>
<td>9.05</td>
<td>8.78</td>
<td>8.45</td>
<td>7.8</td>
<td>6.65</td>
<td>5.4</td>
<td>4.01</td>
</tr>
<tr>
<td>$T_s$</td>
<td>5.65</td>
<td>5.3</td>
<td>5.16</td>
<td>4.82</td>
<td>4.1</td>
<td>2.98</td>
<td>1.97</td>
<td>1.09</td>
</tr>
<tr>
<td>$Q_s$</td>
<td>116</td>
<td>120</td>
<td>119</td>
<td>112</td>
<td>115</td>
<td>98.5</td>
<td>89.3</td>
<td>72</td>
</tr>
</tbody>
</table>

Table (3-b) Results of a series motor connected to a centrifugal pump load

<table>
<thead>
<tr>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{mea}$</td>
<td>1490</td>
<td>1245</td>
<td>1090.7</td>
<td>937.6</td>
<td>785.97</td>
<td>636.57</td>
<td>490.3</td>
<td>348.56</td>
</tr>
<tr>
<td>$P_{test}$</td>
<td>1281</td>
<td>1195</td>
<td>1067</td>
<td>921</td>
<td>717</td>
<td>488.2</td>
<td>238</td>
<td>100.4</td>
</tr>
<tr>
<td>$P_{nom}$</td>
<td>1096</td>
<td>1017</td>
<td>897.8</td>
<td>761.6</td>
<td>582.2</td>
<td>360.6</td>
<td>173.6</td>
<td>65.4</td>
</tr>
<tr>
<td>$P_{load}$</td>
<td>602.8</td>
<td>560</td>
<td>493.8</td>
<td>418.9</td>
<td>320.2</td>
<td>198</td>
<td>95.5</td>
<td>36</td>
</tr>
<tr>
<td>$\mu$</td>
<td>91.5</td>
<td>96</td>
<td>97.9</td>
<td>98.3</td>
<td>91.2</td>
<td>72</td>
<td>48.5</td>
<td>28.8</td>
</tr>
<tr>
<td>$\eta$</td>
<td>85.5</td>
<td>92.3</td>
<td>84.1</td>
<td>82.7</td>
<td>81.2</td>
<td>78.7</td>
<td>72.9</td>
<td>65</td>
</tr>
</tbody>
</table>
4.2 A Separately Excited Motor Connected to Different Load Types

4.2.1 Constant load

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{bmatrix}
\frac{I_a}{a} & \frac{v_a}{a} + g & -g & 0 & 0 \\
g & \frac{v_b}{a} & -c_e & 0 & 0 \\
-1 & -c_e & -R_a & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
V_{a}^{k+1} \\
V_{b}^{k+1} \\
\omega_c^{k+1} \\
T_l \\
\end{bmatrix}
= 
\begin{bmatrix}
I_{PH} + \frac{v_b}{a} \left( - \frac{v_a}{a} - 1 \right) \\
0 \\
0 \\
T_l \\
0 \\
\end{bmatrix}
\]

Table (4-a) Parameters of a separately excited motor connected to a constant load

<table>
<thead>
<tr>
<th></th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a$</td>
<td>157.78</td>
<td>155.79</td>
<td>149.12</td>
<td>141.97</td>
<td>134.73</td>
</tr>
<tr>
<td>$V_b$</td>
<td>151.19</td>
<td>149.12</td>
<td>143.79</td>
<td>138.41</td>
<td>132.12</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>225.82</td>
<td>216.94</td>
<td>208.41</td>
<td>200.09</td>
<td>191.92</td>
</tr>
<tr>
<td>$I_x$</td>
<td>7.3082</td>
<td>7.2821</td>
<td>7.2582</td>
<td>7.2378</td>
<td>7.2184</td>
</tr>
<tr>
<td>$T$</td>
<td>4.838</td>
<td>4.5221</td>
<td>4.4053</td>
<td>4.3055</td>
<td>4.2386</td>
</tr>
</tbody>
</table>

Table (4-b) Results of a separately excited motor connected to a constant load

<table>
<thead>
<tr>
<th></th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{max}$</td>
<td>1400</td>
<td>1240</td>
<td>1096.76</td>
<td>937.588</td>
<td>785.97</td>
</tr>
<tr>
<td>$P_{elec}$</td>
<td>1105.13</td>
<td>1060.5</td>
<td>1000.8</td>
<td>918.59</td>
<td>779.33</td>
</tr>
<tr>
<td>$P_{mech}$</td>
<td>1075</td>
<td>981.07</td>
<td>922.37</td>
<td>841.98</td>
<td>704.57</td>
</tr>
<tr>
<td>$M%$</td>
<td>78.93</td>
<td>85.18</td>
<td>91.75</td>
<td>97.97</td>
<td>99.155</td>
</tr>
<tr>
<td>$H%$</td>
<td>92.75</td>
<td>92.51</td>
<td>92.16</td>
<td>89.8</td>
<td>90.4</td>
</tr>
</tbody>
</table>

4.2.2 Ventilator load

\[
\begin{bmatrix}
\frac{L_d \omega_i}{a} + g & -g & 0 & 0 \\
-g & g & 0 & 1 \\
0 & 0 & B + k_c(\omega_e)^{\gamma_{p-1}} - c_e & -c_e \\
0 & 1 & \omega_e & -R_e
\end{bmatrix}
\begin{bmatrix}
\nu_e^{p-1} \\
\omega_e^{p-1} \\
\nu_i^{p-1} \\
\omega_i^{p-1}
\end{bmatrix}
= \begin{bmatrix}
I_{pm} + I_d + I_0 e^{-\left(\frac{\nu_i}{a} - 1\right)} \\
0 \\
\omega_f + (b_f(c_f - 1)\omega_i^{p-1}) & 0
\end{bmatrix}
\]

**Table (5-a) Parameters of a sep. excited motor connected to a ventilator load**

| \( V_e \) | 148.819 | 143.679 | 137.717 | 130.74 | 122.5 | 112.703 | 101.025 | 87.106 | 70.429 | 49.835 |
| \( V_0 \) | 140.443 | 135.765 | 130.336 | 123.966 | 116.415 | 107.412 | 96.638 | 83.707 | 68.111 | 48.662 |
| \( c_e \) | 203.697 | 197.415 | 190.922 | 181.455 | 171.157 | 158.772 | 143.819 | 125.68 | 103.465 | 75.212 |
| \( T \) | 5.774 | 5.4526 | 5.089 | 4.671 | 4.255 | 3.647 | 3.033 | 2.345 | 1.5977 | 0.899 |

**Table (5-b) Results of a separate excited motor connected to a ventilator load**

| \( P_{max} \) | 1400 | 1245 | 1090.76 | 937.588 | 785.97 | 636.57 | 490.3 | 318.56 | 213.67 | 90.3 |
| \( P_{rest} \) | 1385.82 | 1192.07 | 1067.82 | 932.433 | 797.67 | 630.89 | 471.88 | 376.89 | 175.24 | 63.25 |
| \( P_{nwb} \) | 1176.15 | 1076.42 | 967.13 | 847.57 | 728.27 | 579.64 | 436.146 | 294.7 | 165.3 | 60.85 |
| \( \mu \% \) | 93.27 | 95.75 | 97.9 | 99.45 | 101.49 | 99.71 | 96.24 | 90.68 | 82.2 | 70 |
| \( \% \) | 90.07 | 90.3 | 90.57 | 90.9 | 91.3 | 91.78 | 92.42 | 93.23 | 94.32 | 95.2 |

**4.2.3 Centrifugal pump**

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{bmatrix}
\frac{I}{a} \nu_i^{\gamma_{11}} + g & -g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-g & g & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & B & 0 & -C_e & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\eta \omega_e^{\gamma_{11}}}{\rho g(h_i^*)^3} & \frac{\eta \omega_i^{\gamma_{11}}}{\rho g h_i^*} & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & -C_e & 0 & -R_e & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\omega_{ref}}{2\sqrt{a_e * h_i^*}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\nu_i^{p+1} \\
\omega_i^{p+1} \\
\nu_e^{p+1} \\
\omega_e^{p+1} \\
\nu_{ref}^{p+1} \\
\omega_{ref}^{p+1} \\
\nu_i^{p+1} \\
\omega_i^{p+1}
\end{bmatrix}
= \begin{bmatrix}
I_{pm} - I_0 e^{-\left(\frac{\nu_i}{a} - 1\right)} + I_0 \\
0 \\
0 \\
-\frac{\eta \omega_e^{\gamma_{11}}}{\rho g h_i^*} & \frac{\eta \omega_i^{\gamma_{11}}}{\rho g h_i^*} & 0 \\
0 & \omega_{ref}^{p+1} & -\frac{\omega_{ref}^{p+1} h_i^*}{2\sqrt{a_e * h_i^*}} & \frac{\omega_{ref}^{p+1} h_i^*}{H_{nwb}}
\end{bmatrix}
\]
### Table 6-a: Parameters of a sep. excited motor connected to a centrifugal pump load

<table>
<thead>
<tr>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
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<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>146</td>
<td>138</td>
<td>132</td>
<td>122</td>
<td>114</td>
<td>104</td>
<td>90.6</td>
<td>73.5</td>
<td>52.4</td>
</tr>
<tr>
<td>$V_2$</td>
<td>137</td>
<td>130</td>
<td>124</td>
<td>115</td>
<td>108</td>
<td>98</td>
<td>86</td>
<td>70</td>
<td>50</td>
</tr>
<tr>
<td>$n_1$</td>
<td>197</td>
<td>186</td>
<td>178</td>
<td>166</td>
<td>157</td>
<td>143</td>
<td>126</td>
<td>103</td>
<td>74.2</td>
</tr>
<tr>
<td>$n_2$</td>
<td>59.2</td>
<td>53.3</td>
<td>48.8</td>
<td>42</td>
<td>37.6</td>
<td>31.3</td>
<td>24.4</td>
<td>16.4</td>
<td>8.43</td>
</tr>
<tr>
<td>$J_1$</td>
<td>9.9</td>
<td>9.5</td>
<td>8.8</td>
<td>8.1</td>
<td>7.15</td>
<td>6.1</td>
<td>5.1</td>
<td>3.87</td>
<td>2.62</td>
</tr>
<tr>
<td>$T_1$</td>
<td>6.14</td>
<td>5.83</td>
<td>5.46</td>
<td>5.05</td>
<td>4.44</td>
<td>3.8</td>
<td>3.17</td>
<td>2.4</td>
<td>1.62</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>124</td>
<td>125</td>
<td>121</td>
<td>121</td>
<td>113</td>
<td>106</td>
<td>100</td>
<td>93</td>
<td>87.7</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>159</td>
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</table>

### Table 6-b: Results of a separately excited motor connected to a centrifugal pump load

<table>
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<tr>
<th>100%</th>
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<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{m}=1400$</td>
<td>1245</td>
<td>1090.7</td>
<td>937.58</td>
<td>785.97</td>
<td>636.57</td>
<td>490.3</td>
<td>348.56</td>
<td>213.67</td>
<td>90.3</td>
</tr>
<tr>
<td>$P_{elec}=1256$</td>
<td>1221</td>
<td>1089</td>
<td>931.5</td>
<td>772.2</td>
<td>543.4</td>
<td>423.6</td>
<td>271</td>
<td>131</td>
<td>20.1</td>
</tr>
<tr>
<td>$P_{net}=1290.6$</td>
<td>1084.4</td>
<td>971.88</td>
<td>835</td>
<td>697</td>
<td>372.4</td>
<td>272.6</td>
<td>247.2</td>
<td>250</td>
<td>17.43</td>
</tr>
<tr>
<td>$M%=665.28$</td>
<td>542.2</td>
<td>534.34</td>
<td>459.25</td>
<td>383.35</td>
<td>294.8</td>
<td>150.1</td>
<td>135.99</td>
<td>66.11</td>
<td>10.5</td>
</tr>
<tr>
<td>$H%=86.35$</td>
<td>98.15</td>
<td>99.1</td>
<td>99.25</td>
<td>98.25</td>
<td>85.63</td>
<td>89.45</td>
<td>77.75</td>
<td>61.3</td>
<td>22.26</td>
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</tbody>
</table>

### Fig. 6: I-V Characteristics for a sep. excited motor connected to different load types

### Fig. 7: Utilization curves for a sep. excited motor connected to different load types
4.3. A Shunt Motor Connected to Different Load Types

4.3.1 Constant load

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{bmatrix}
\frac{f_a}{a} e^s + g - g & 0 & 0 \\
g & g & 0 \\
0 & 0 & B + \frac{M_q R_s I_3^2}{(R_a + R_f - M_q \omega_s)^2} \frac{2M_q R_s I_3 (R_f - M_q \omega_s)}{(R_a + R_f - M_q \omega_s)^2} \\
0 & 1 & \frac{R_f R_s I_3}{(R_a + R_f - M_q \omega_s)^2} \\
\end{bmatrix}
\begin{bmatrix}
v^{k+1} \\
v^{k+1} \\
\omega^{k+1} \\
I^{k+1} \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

For the shunt motor connected to constant load torque the motor does not produce sufficient torque to start the system even for the full insolation.

4.3.2 Ventilator load

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{bmatrix}
\frac{f_a}{a} e^s + g - g & 0 & 0 \\
g & g & 0 \\
0 & 0 & \frac{b_j \omega^s}{a} + \frac{M_q R_s I_3^2}{(R_a + R_f - M_q \omega_s)^2} \frac{2M_q R_s I_3 (R_f - M_q \omega_s)}{(R_a + R_f - M_q \omega_s)^2} \\
0 & 1 & \frac{R_f R_s I_3}{(R_a + R_f - M_q \omega_s)^2} \\
\end{bmatrix}
\begin{bmatrix}
v^{k+1} \\
v^{k+1} \\
\omega^{k+1} \\
I^{k+1} \\
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
I_{in} + I_a + e^s \left(\frac{b_j \omega^s}{a} - 1\right) \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\frac{c_f (\omega_s - 1) \omega^s}{a} \\
\end{bmatrix}
\begin{bmatrix}
\frac{M_q R_s R_f I_3}{(R_a + R_f - M_q \omega_s)^2} + (b_j (\omega_s - 1) \omega^s) \\
R_f R_s I_3 \omega_s \\
\end{bmatrix}
\]
4.3.3 Centrifugal pump

Newton Raphson Mixed Nodal Tableau [NR MNT] & [NR R]

\[
\begin{bmatrix}
\frac{\partial P}{\partial x} + g - g \\
\frac{\partial m}{\partial x} + g - g \\
\frac{\partial J}{\partial x} + g - g \\
\frac{\partial I}{\partial x} + g - g \\
\frac{\partial P_{\text{mech}}}{\partial x} + g - g \\
\frac{\partial P_{\text{inds}}}{\partial x} + g - g \\
\frac{\partial P_{\text{load}}}{\partial x} + g - g \\
\frac{\partial P_{\text{rev}}}{\partial x} + g - g \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial P}{\partial x} + g - g \\
\frac{\partial m}{\partial x} + g - g \\
\frac{\partial J}{\partial x} + g - g \\
\frac{\partial I}{\partial x} + g - g \\
\frac{\partial P_{\text{mech}}}{\partial x} + g - g \\
\frac{\partial P_{\text{inds}}}{\partial x} + g - g \\
\frac{\partial P_{\text{load}}}{\partial x} + g - g \\
\frac{\partial P_{\text{rev}}}{\partial x} + g - g \\
\end{bmatrix}
\]
Table 8-a Parameters of a shunt motor connected to a centrifugal pump load

<table>
<thead>
<tr>
<th></th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a$</td>
<td>162.7</td>
<td>154.9</td>
<td>16.4</td>
<td>32.24</td>
<td>25.77</td>
</tr>
<tr>
<td>$V_b$</td>
<td>157</td>
<td>150</td>
<td>27.5</td>
<td>22.5</td>
<td>18.5</td>
</tr>
<tr>
<td>$\omega_a$</td>
<td>181.3</td>
<td>180.1</td>
<td>81.78</td>
<td>73.36</td>
<td>69.5</td>
</tr>
<tr>
<td>$h_f$</td>
<td>67.7</td>
<td>53</td>
<td>46.4</td>
<td>38.5</td>
<td>30.9</td>
</tr>
<tr>
<td>$I_f$</td>
<td>7.6</td>
<td>7.8</td>
<td>10.35</td>
<td>9.4</td>
<td>8.08</td>
</tr>
<tr>
<td>$T_f$</td>
<td>5.84</td>
<td>5.65</td>
<td>1.67</td>
<td>1.01</td>
<td>0.763</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>110</td>
<td>121</td>
<td>102</td>
<td>118</td>
<td>121</td>
</tr>
</tbody>
</table>

Table 8-b Results of a shunt motor connected to a centrifugal pump load

<table>
<thead>
<tr>
<th></th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{max}$</td>
<td>1400</td>
<td>1245</td>
<td>199.76</td>
<td>937.6%</td>
<td>785.97</td>
</tr>
<tr>
<td>$P_{act}$</td>
<td>1158</td>
<td>1145</td>
<td>292.6</td>
<td>218.6</td>
<td>149.48</td>
</tr>
<tr>
<td>$P_{erch}$</td>
<td>1020.3</td>
<td>991.3</td>
<td>122.3</td>
<td>81.6</td>
<td>52.3</td>
</tr>
<tr>
<td>$P_{load}$</td>
<td>590</td>
<td>545.215</td>
<td>71.1</td>
<td>47.14</td>
<td>26.15</td>
</tr>
<tr>
<td>$\mu$</td>
<td>96.1</td>
<td>83.9</td>
<td>25.1</td>
<td>21.7</td>
<td>18.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>87.2</td>
<td>85.8</td>
<td>41.4</td>
<td>37.21</td>
<td>35</td>
</tr>
</tbody>
</table>

Fig. 8 1-V characteristics of a shunt motor connected to different load types

Fig. 9 Utilization curves for a shunt motor connected to different load types

4-4 COMPARISON BETWEEN CASE STUDIES

A comparison between the nine case studies is presented in Table 9 and it shows that:

- The separately excited motor is the appropriate type for a different mechanical load types. Its efficiency and matching quality are better than other motors.
- Separately excited motor coupled to a ventilator load type is found to be the most suitable combination for SCA generator. The utilization of SCA output energy is 96.07% of its maximum output power.
- It is not preferable to use shunt motors with constant mechanical loads. However, the SCA generator can produce sufficient torque to start up the shunt motor to
supply both ventilator and centrifugal pump loads (at 59.6%, and 59% of insolation level respectively).

<table>
<thead>
<tr>
<th>Motor type</th>
<th>Start up insolation level</th>
<th>Matching quality</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series motor</td>
<td>57.94%</td>
<td>62.73%</td>
<td>89.96%</td>
</tr>
<tr>
<td>Separately motor</td>
<td>49.68%</td>
<td>65.84%</td>
<td>92.05%</td>
</tr>
<tr>
<td>Shunt motor</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor type</th>
<th>Start up insolation level</th>
<th>Matching quality</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series motor</td>
<td>19.99%</td>
<td>83.84%</td>
<td>84.17%</td>
</tr>
<tr>
<td>Separately motor</td>
<td>5.9%</td>
<td>96.07%</td>
<td>92%</td>
</tr>
<tr>
<td>Shunt Motor</td>
<td>59.6%</td>
<td>32.25%</td>
<td>81%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Motor type</th>
<th>Start up insolation level</th>
<th>Matching quality</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series motor</td>
<td>20%</td>
<td>81.73%</td>
<td>82.75%</td>
</tr>
<tr>
<td>Separately motor</td>
<td>6%</td>
<td>93.5%</td>
<td>90.7%</td>
</tr>
<tr>
<td>Shunt Motor</td>
<td>59%</td>
<td>29.66%</td>
<td>81.08%</td>
</tr>
</tbody>
</table>

5- CONCLUSION

- The systematic mathematical methodology known as Graph Theoretic Modeling (GTM) was presented in details.
- The GTM technique has been applied to PVPS composed of different dc-motor types (series, separately excited and shunt motors) connected to different load types (constant, ventilator and pump loads) supplied by a SCA generator.
- A computer program using MATLAB 6 was developed to numerically solve the system matrix equations.
- The quality of load matching for each case was investigated as a measure of performance and system utilization efficiency.
- A comparison between different case studies was introduced to indicate the appropriate motor type for each load.
- Applying the GTM technique to PV systems results in many advantages:
  - GTM methodology is global and flexible enough to include linear or nonlinear components and replace them with a stamp.
  - GTM analyzes electrical and mechanical characteristics for combined PV-dc motor in addition to hydraulic characteristics in case of centrifugal pump.
  - GTM technique is a general integral promising methodology capable of providing PVPS with ready simulated stamps for any component.
  - GTM technique as a mathematical methodology can be used as a measure of matching quality, which reflect the performance of the system and exhibit the appropriate load connected to a dc motor powered by SCA generator.
The paper presents a method for modeling SCA generator, different types of dc-motors and various mechanical loads. It provides an evaluation for any motor type connected to a mechanical load powered by a stand-alone PV power system.

6- REFERENCES

## APPENDIX

### Parameter Values

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV Cell</td>
<td>( I_c )</td>
<td>Reverse Saturation current</td>
<td>8.1e^3 A</td>
</tr>
<tr>
<td></td>
<td>( I_{ph} )</td>
<td>Cell photocurrent</td>
<td>0.756 A</td>
</tr>
<tr>
<td></td>
<td>( R_s )</td>
<td>Cell series resistance</td>
<td>0.05 ohm</td>
</tr>
<tr>
<td></td>
<td>( \alpha )</td>
<td>Thermal voltage ( (A=KT/q) )</td>
<td>1/13.68 l/Volt</td>
</tr>
<tr>
<td></td>
<td>( K_B )</td>
<td>Boltzman constant</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( T )</td>
<td>Absolute temperature</td>
<td>300 Kelvin</td>
</tr>
<tr>
<td></td>
<td>( q )</td>
<td>Electron charge</td>
<td>1.6e^19 coulomb</td>
</tr>
<tr>
<td>Output Motor Data</td>
<td>( V )</td>
<td>Terminal voltage</td>
<td>120 Volt (nom)</td>
</tr>
<tr>
<td></td>
<td>( I_a )</td>
<td>Armature current</td>
<td>9.2 A (rated)</td>
</tr>
<tr>
<td></td>
<td>( \omega )</td>
<td>Shaft speed</td>
<td>157.1 rad/sec</td>
</tr>
<tr>
<td></td>
<td>( T_e )</td>
<td>Electromagnetic torque</td>
<td>5.7 N.m</td>
</tr>
<tr>
<td></td>
<td>( A )</td>
<td>Torque constant of rotational losses</td>
<td>0.2 N.m</td>
</tr>
<tr>
<td></td>
<td>( B )</td>
<td>Viscous Torque constant of rotational losses</td>
<td>2.387e^-5 N.m/rad/s</td>
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<tr>
<td>Series Motor</td>
<td>( R_s )</td>
<td>Combined field and armature resistance</td>
<td>2.2 ohm</td>
</tr>
<tr>
<td></td>
<td>( M_{La} )</td>
<td>Mutual inductance between field and armature</td>
<td>6.75e^-5 H</td>
</tr>
<tr>
<td>Sep. Excited Motor</td>
<td>( R_a )</td>
<td>Armature resistance</td>
<td>1.5 ohm</td>
</tr>
<tr>
<td></td>
<td>( C_r )</td>
<td>Flux coefficient</td>
<td>0.621 V/mrad/sec</td>
</tr>
<tr>
<td>Shunt Motor</td>
<td>( M_{La} )</td>
<td>Mutual inductance between field and armature</td>
<td>0.518 H</td>
</tr>
<tr>
<td></td>
<td>( R_a )</td>
<td>Armature resistance</td>
<td>1.5 ohm</td>
</tr>
<tr>
<td></td>
<td>( R_f )</td>
<td>Field resistance</td>
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<td>Constant Load</td>
<td>( T_0 )</td>
<td>Load torque</td>
<td>4.0 N.m</td>
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<td>Ventilator Load</td>
<td>( a_0 )</td>
<td>Static torque constant</td>
<td>3.0 e^4 N.m</td>
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<td>( b_0 )</td>
<td>Dynamic torque constant</td>
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<td></td>
<td>( c_0 )</td>
<td>C factor</td>
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<td>Centrifugal Pump</td>
<td>( H )</td>
<td>Rated head. m</td>
<td>37.795</td>
</tr>
<tr>
<td></td>
<td>( Q )</td>
<td>Rated flow charge, liter/min</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>( \eta )</td>
<td>Pump efficiency, %</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>( p )</td>
<td>Fluid Density, KG/m3</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>( g )</td>
<td>Gravitational constant</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>( \theta_{st} )</td>
<td>Static a factor</td>
<td>37.795</td>
</tr>
</tbody>
</table>