An Experimental Study of Drag Reduction of Polymer Solutions in Capillary Tubes

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ABSTRACT

This paper introduces the concept of the modified drag reduction envelope for flow of polymer solutions in capillary tubes. An experimental work studied drag reduction of polymer solutions in capillary tubes with diameters of 0.45, 0.55 and 0.75 mm. The experimental study were carried out under constant head for Reynolds number range between $10^3$ and $10^4$. The polymer used in the test was polyacrylamide. Four concentrations, 10, 50, 100 and 250 ppm were investigated. The experimental results showed that the friction drag reduction differs significantly among different diameters of capillary tubes at various concentration of polymer solutions. The results of the present work showed also that, for pure water flow, the friction factor values are lower than the conventional tubes because the flow in capillary tubes exhibits the rarefied phenomena due to the extremely small dimension of flow passages. Friction factor of polymer solution decreases with the increase of Reynolds number, tube diameter and solution concentration. Drag reduction ratio increases with solution concentration and the decrease of tube diameter.

Key words: Drag reduction, polymer, Capillary tubes.

1. Introduction

Pressure drop in flow can be drastically reduced by adding small quantities of certain long
chain polymers to solvent such as water. This phenomenon is called drag reduction. Generally, credit is given to Toms [1] for being the first to observe the phenomenon. Therefore, drag reduction is also called Toms phenomenon. Toms investigated the mechanical degradation of high polymer solutions in pipe flow. It was found that a solution of polymethyl methacrylate in monochlorobenzene required a lower pressure gradient than the solvent alone to produce the same flow rate.

Applied investigations have been concerned with the type and concentrations of the most effective polymer additives, while theoretical researches have been directed towards the determination of the constitutive equations or rheological models capable of describing the polymer solution behavior in various simple flow conditions. As a step in this direction it is of importance to assess the nature of the parameters characterizing the solution properties. Numerous investigations have been carried out which show the existence of a characteristic fluid time parameter [2-6].

One most extensive review on drag reduction was published by Virk [7]. It addressed the drag reduction fundamentals of dilute solutions. It covered broad areas of drag reduction studies including gross flow, mean velocity profile, turbulence structure, and mechanisms of drag reduction. In the discussion of the gross flow of dilute polymer solutions, Virk proposed that the concept of maximum drag reduction envelope was bounded by two universal asymptotes. Virk and Wagger [8], using the concept of drag reduction envelope, further discussed two extreme forms of drag reduction behavior.

The effect of capillary tube diameters in laminar flows of polymer solutions was published by Oulibrahim [9]. It has been shown that the behavior of polymer (Polyox 301) solutions in small down to 0.026 mm, diameter capillary tubes was influenced by the flow length scale (diameter). The relative apparent viscosity of solutions decreased with increasing tube diameter and reached a minimum for a particular value of the tube diameter which depends on the polymer concentration.

The polymer induced turbulent drag reduction in a rotating disk apparatus was investigated by Chul, et al [10]. It has been shown that the drag reduction induced by polymer additives was found to be applicable to the Ocean thermal energy conversion plant to reduce the pumping energy cost.

Subhash, et al [11] investigated the drag reduction performance of several polymer solutions in coiled tubing with diameters 1, 1-1/2 and 2-3/8 inch. Experimental results showed that the amount of friction drag reduction differs significantly among the different types of polymer at various concentrations. Data interpretation and analysis revealed that the coiled diameter is an important geometrical parameter affecting drag reduction.

Choi, et al [12] measured friction factors of nitrogen flow for 0.1mm diameter micro-tubes. The measured friction factors for laminar and turbulent flows were found to be consistently smaller than those predicted by the macro scale correlation in macro tube. For laminar flow (Re<2300), the friction constant, was 19 to 27% smaller than the conventional one, with an average friction constant of 52, instead of 64. Utilizing the same experimental apparatus, by Yu, et al [13], experimental investigation of nitrogen and water flows in micro-tubes has been reported. They could obtain friction factors falling into approximately the same range as in the previous research. For laminar flow (Re<2000), nitrogen and water friction factors about 19% lower than the conventional value of 64. The relation, \( f = 50.13/Re \) was proposed for this regime.
Due to the rapid development of Micro-Electro-Mechanical systems and micro-fluidics such as micro-tubes heat sinks for cooling micro-chips, chemical and biomedical analyses, and micro fluid pumps, etc., it is highly desirable to understand the fundamentals. The objective of the present work is to investigate experimentally drag reduction of polymer solutions in capillary tubes with different diameters and at various polymer concentrations. The experimental studies are carried out under constant head for Reynolds number range between $10^3$ and $10^4$. The polymer used in the test was Polyacrylamide. Three different capillary tube diameters namely 0.45, 0.55 and 0.75 mm are used. Four concentrations, 10, 50, 100 and 250 ppm were investigated.

2. Experimental Test Rig:

A schematic diagram of the experimental test rig is illustrated in figure (1). The test rig shown in figure (1), consists of reservoir discharging into a feed chamber to which the capillary tubes is attached. The tube discharges freely to atmospheric pressure. Solution flow rates were determined by weighing the volume of fluid collected during a given time. Due to the small dimensions of the tubes, no pressure taps were used to measure the pressure drop instead it will be measured according to the method explained later. The diameters of the tubes were measured using a travelling microscope.

![Fig. (1) Experimental test rig.](image-url)
The polymer used in the test was Polyacrylamide. Four concentrations, 10, 50, 100 and 250 ppm were investigated. All polymer solutions were prepared by diluting in demineralized and filtered water and the solution was mixed for 24 hours in advance.

3. Theory and Definition

As a result of the small diameter of the tested capillary tubes investigated in this work, it is difficult to measure the pressure drop across a certain length using the conventional methods, i.e., measuring the pressures at inlet and outlet of the tube using pressure taps connected to any differential manometer. Accordingly, a non-conventional method can be used to measure the pressure drop through the tube. This suggested method can be carried out by measuring the liquid flow rates through two tubes of different lengths, $L_1$ and $L_2$. The two tubes have the same inner diameter and of the same material and connected in parallel to the same junction under a known pressure head, $H$ as shown in Fig. (1).

Applying Bernoulli's equation to the tubes yields:

\[
H = \Delta H + \frac{(f)(L_i)(Q_i^2)}{12d^3}, \quad \text{and} \quad H = \Delta H + \frac{(f)(L_2)(Q_2^2)}{12d^3}.
\]

Where, $\Delta H$ is the secondary losses through the test section, which can be set equal.

The relations between $H$, $Q_1$, and $Q_2$ for the two tubes of lengths $L_1$ and $L_2$ can be drawn as shown in Fig. (2). From this figure at a certain liquid flow rate $Q$ one can determine the difference in pressure drops through the two tubes ($\Delta H$). This difference is equal to the pressure drop across a tube of length ($\Delta L = L_1 - L_2$).

Knowing the pressure drop ($\Delta H$) across a tube of length ($\Delta L$) at a certain liquid flow rate $Q$, the friction factor of the flow through the tube can be set as:

\[
f = \frac{12d^3(\Delta H)}{Q_i^3(\Delta L)} = \frac{12d^3(\Delta P)}{\rho gQ_i^3(\Delta L)}
\]

**Drag ratio (DR) and drag reduction ratio (DRR):**

The drag ratio is defined as the observed pressure gradient for a polymer solution to the observed pressure gradient for the solvent, both measured under the same flow conditions:
The drag reduction ratio can be defined as:

\[ DRR = 1 - \left( \frac{\frac{\partial P}{\partial l}}{\frac{\partial P}{\partial l}} \right)_p \]  (4)

If the density of the polymer solution is the same as the density of the solvent, the following equation is applicable:

\[ DRR = 1 - \frac{j_p}{j_s} \]  (5)

Where, \( j_p \) and \( j_s \) are the friction factors of the polymer solution and solvent. DRR is usually expressed in percentage.

4. Results and Discussions
4.1. Water Flow Results

To prove the reliability and validity of the experimental test rig, some pure water tests were conducted using all sizes of capillary tubes under test. The relation between the pressure head drops (\( \Delta H \)) and liquid flow rate (\( Q \)) for tubes with inside diameters of 0.45, 0.55 and 0.75 mm and tube lengths 26 and 90 mm are illustrated in Fig. (2). From the foregoing figure, the difference in pressure head drops (\( \Delta H \)) as a function of the flow rate (\( Q \)) flowing through a tube of length (\( \Delta L = 64 \text{mm} \)), at different tube diameters can be determined. Then, the friction factor can be determined using equation (2).

Figure (3) demonstrates the relation between friction factor and Reynolds number based on the tube inner diameters. Comparison between the present experimental results and the conventional correlation for smooth circular tubes is illustrated in Fig. (3). Also, the correlation of Yu et al. [13] for 0.104 mm diameter microtubes are presented in this figure (\( f = 0.14/R^0 \)). It is seen from the figure that the friction factor is lower than the conventional correlation by about 28.7%, while it is lower than Yu et al. correlation by about 9.77%. These results clearly demonstrate the reliability of the experimental results and the validity of the test rig. The figure shows also that the experimental friction factor for capillary tube of diameter 0.75 mm is nearly the same as in the work of Yu et al. An attempt was made to correlate the present experimental friction factor data as a function of Reynolds number as follows:
4.2. Polymer Solution Results

A total of 135 experiments were made to show the effect of tube diameter, solution concentration and Reynolds number on friction factor of polymer solution flowing inside capillary tubes. In these experiments, tube diameter were 0.45, 0.55 and 0.75 mm, while polymer concentrations were in the range 10 to 250 ppm. The flow velocities of polymer solutions were chosen to satisfy Reynolds numbers ranging from $10^3$ to $10^4$.

All the experimental results are demonstrated in Figs. (4-10) as a relation between friction factor and Reynolds number at different discussed parameters.

4.2.1. Effect of Tube Diameter on Friction Factor

Figures (4-7) illustrate the relation between friction factor and tube diameter for different solution concentrations. It can be seen from the overview of figures that the friction factor decreases with the increase of Reynolds number regardless of the effect of tube diameter and solution concentration.

Figure (4), drawn for solution concentration of 10 ppm, shows that the friction factor decreases with the increase of tube diameters. While at solution concentration equal 50 ppm or higher, the friction factor decreases with the decrease of tube diameter as seen from Figs. (5-7).

From the previous discussion it may be concluded that the degree of solution concentration greatly affects the dependence of friction factor on tube diameter.

4.2.2. Effect of Solution Concentration on Friction Factor

Figures (8-10) illustrate the effect of solution concentration on friction factor as a relation between friction factor and Reynolds number for different tube diameters. The figures show that the friction factor decreases with the increase in solution concentration. It is also seen that the rate at which friction factors decrease as the solution concentration increases and with the increase of tube diameter. One can also conclude that the use of polymer solutions with different concentration decreases the friction factor with respect to that of pure water flow.

4.3. Drag Reduction Ratio (DRR)

The drag reduction ratio (DRR) defined by equation (5) represents the decrease in friction coefficient of the polymer solution with different concentration with respect to that of the pure water flowing inside a capillary tube. Figures (11-13) demonstrate the relation between drag reduction ratio and Reynolds number for different diameter of capillary tubes and different concentration of polymer solutions. From the overview of these figures it is observed that the drag reduction ratio increases with Reynolds number and solution concentration. Comparison of figures (11, 12 and 13) for tube diameter of 0.45, 0.55 and 0.75 mm respectively shows that the drag reduction ratio increases with the decrease of capillary tube diameter. Generally, there is a
greater drag reduction in the smaller diameter tube. Of course, the larger diameter reduces the average velocity of the main flow and therefore, delays the onset of turbulence regime can only be explained as drag related effect of diameter. Mechanistically, since drag reduction of a polymer solution occurs in the boundary layers, therefore, drag reduction polymers would be more effective in a smaller tube. It is also observed that for a given polymer concentration (100ppm), the onset of drag reduction shifts toward greater with increasing tube size.

5. CONCLUSIONS

Drag reduction experiments through capillary tubes were conducted for polyacrylamid-water solution. Tube diameter of 0.45, 0.55 and 0.75mm, solution concentration of 0, 10, 50, 100 and 250 ppm were discussed for a Reynolds number range of 1000 to 10000. From the experiment results the following conclusions can be drawn:

1. For pure water flow, the friction factor values of flow through capillary tubes are lower than the values for conventional tubes. Two effects from current experimental investigation. First, the frictional resistance in capillary tubes was affected by compressibility effect, which was identified by the fact that the friction constant starts to deviate from the prediction of the incompressible theory even at very low pressure ratio. Secondly, the flow in capillary tubes exhibits the rarefied phenomena due to the extremely small dimension of flow passages.

2. The following relation correlates the present experimental friction factor data as a function of Reynolds number for pure water:

\[ f = 45.23 / Re, \quad (1000 \leq Re \leq 10000, \text{ and } d \leq 0.75\text{mm}) \]

3. Friction factor of polymer solution decreases with the increase of Reynolds number, tube diameter and solution concentration.

4. Drag reduction ratio increase with solution concentration and the decrease of tube diameter.
Fig. (2) Relation between mass flow rate and pressure head for different diameters at concentration (0 ppm).

Fig. (3) Friction factor versus Reynolds number at different size.
(concentration 0 ppm)

Fig. (4) Friction factor versus Reynolds number at different sizes.
(concentration 10 ppm)

Fig. (5) Friction factor versus Reynolds number at different sizes.
(concentration 50 ppm)
Fig. (6) Friction factor versus Reynolds number at different sizes, (concentration 100 ppm)

Fig. (7) Friction factor versus Reynolds number at different sizes, (concentration 250 ppm)

Fig. (8) Friction factor versus Reynolds number at different polymer concentration, (d=0.45 mm).

Fig. (9) Friction factor versus Reynolds number at different polymer concentration, (d=0.55 mm).
Fig. (10) Friction factor versus Reynolds number at different polymer concentration, (d=0.75mm).

Fig. (11) Effect of concentration on drag reduction ratio at 0.045mm diameter.

Fig. (12) Effect of concentration on drag reduction ratio at 0.055mm diameter.

Fig. (13) Effect of concentration on drag reduction ratio at 0.075mm diameter.
Fig. (18) Effect of diameter on drag reduction ratio at 10 ppm concentration.

Nomenclature

\( d \) Capillary diameter, mm
\( \text{DRR} \) Drag reduction ratio, %
\( f \) Darcy friction factor
\( L \) Capillary length, mm
\( H \) Head, m
\( Q \) Flow rate, g/s
\( U \) Mean velocity, m/s

Dimensionless Groups

\( f \) Friction factor (Darcy friction factor)
\( \text{Re} \) Reynolds number \( (= \frac{U d}{\nu}) \)

Greek Symbols

\( \rho \) Fluid density, \( g/m^3 \)
\( \nu \) Kinematics viscosity, \( m^2/s \)
\( \Delta p \) Pressure drop, \( N/m^2 \)

Subscripts

\( p \) Polymer solution
\( s \) Solvent
References