Artificial Neural Network Model For Predicting Discharges
Over Weirs of Finite Crest Widths

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ABSTRACT

Multilayers feedforward Artificial Neural Network (ANN) with back propagation learning algorithm is used to develop a computational model to predict discharge over weirs of finite crest widths. A network of size of 4-6-1 is found suitable for this purpose with 3000 iterations and hyperbolic tangent (tanh) activation function. The results of the trained, verified and tested ANN model are compared to the experimental measurements. Also, results from previously developed models based on statistical methods are compared to results of ANN model. The effect of using filter applied on the head over weir on the performance of the ANN model is investigated. The sensitivity analysis that conducted using the ANN model indicated that the most contributing variables to flow rate are the head over the weir and the weir width followed by the length of the weir and its height. Results indicated that the prediction of discharge over broad-crested weirs using ANN is more accurate than predictions offered by other previously developed models.

Keywords: Discharge measurement structures, Flow measurements, Flow modeling, Free surface flow, Artificial neural network, Artificial intelligence

INTRODUCTION AND REVIEW OF LITERATURE

Measurements of irrigation water is important in any irrigation project. Different methods could be used as gates, flumes, current meters, sharp-crested weirs and broad-crested weirs (BCW). Basic information on these devices can be found in BSI (1965), USBR (1967); Ackers et al. (1978), Bos (1976), Bos et al. (1984), and French (1986). This research paper concerns with the BCWs. BCW is a simple measurement structures with the advantage of easy construction, easy installation and structure stability. Figure 1 shows a definition sketch of the flow over BCW. Studies on the characteristics and features of flow over BCW include those of Woodburn (1932).

for suppressed long crested-weirs (LCWs) with $H/L \leq 0.10$:

$$Q = \frac{2}{3} \left( 0.56 \left( \frac{H}{L} \right)^{0.022} \right) \sqrt{2gbH^{1.5}}$$

(1)

in which $Q$ is the discharge, $H$ is the head over the weir, $L$ is the length of weir in flow direction, $b$ is the crest width and $g$ is the gravitational acceleration.

- for suppressed BCWs with $0.1 \leq H/L \leq 0.35$:

$$Q = \frac{2}{3} \left( 0.521 + 0.028 \frac{H}{L} \right) \sqrt{2gbH^{1.5}}$$

(2)

and

- for suppressed narrow crested weirs (NCWs) with $0.45 \leq H/L \leq 1.5$:

$$Q = \frac{2}{3} \left( 0.492 + 0.12 \frac{H}{L} \right) \sqrt{2gbH^{1.5}}$$

(3)

![Figure 1. Definition sketch for flow over weir of finite crest width (BCW)](image)

Swamee (1989) proposed the following set of equations instead of Eqs. (1-3) based on the data due to Rao and Muralidhar (1963):

for suppressed LCWs with $H/L < 0.10$

$$Q = \frac{2}{3} \left( 0.50 + 0.10 \left( \frac{H}{L} \right)^{0.8} \right) \sqrt{2gbH^{1.5}}$$

(4)

for suppressed BCWs with $0.1 \leq H/L \leq 0.40$

$$Q = \frac{2}{3} \left( 0.50 + 0.05 \left( \frac{H}{L} \right)^{0.3} \right) \sqrt{2gbH^{1.5}}$$

(5)

and

- for suppressed NCWs with $0.40 \leq H/L \leq 1.5$

$$Q = \frac{2}{3} \left( 0.50 - 0.11 \frac{H}{L} \right) \sqrt{2gbH^{1.5}}$$

(6)

Then he developed a generalized equation for the discharge coefficients of long, broad and narrow crested weirs. When it is combined with the weir equation it takes the form:
\[ Q = \frac{2}{3} \left[ 0.58 + 0.10 \left( \frac{H}{L} \right)^{0.5} + 1500 \left( \frac{H}{L} \right)^{0.7} \right] \left( \frac{H}{L} \right) \left( \frac{H}{L} \right)^{0.15} \sqrt{2gh^{1.5}} \]  

(7)

Considering the channel width, the contracted BCW has a width smaller than the channel width. Hall (1962) proposed a theoretical relationship for the discharge over the contracted broad-crested weirs based on the boundary layer theory. Muralidhar (1965) studied the characteristics of flow over contracted BCW. Ranga Raju and Ahmed (1973) investigated the flow over suppressed and contracted BCW both experimentally and theoretically.

Recently, Negm and Alshaikh (1997) conducted an experimental investigation to study the characteristics of flow over contracted broad-crested weirs for different contraction ratios in the range of 0.508 ≤ b/B ≤ 1.0 at constant P/L = 0.375 with 0.1 ≤ H/L ≤ 0.5. They developed an expression for estimating the discharge and velocity coefficients in the following form

\[ C_d = \left( \frac{H}{L} \right) \left[ \alpha \right] \]  

(8)

in which:

\[ \chi = \left( \frac{b}{B} \right)^{0.5} \left( \frac{H}{L} + \frac{P}{L} \right)^{0.25} \left( \frac{b}{B} \right) \]  

(9)

B is the channel width and P being the height of the weir. The values of \( \alpha \) and \( \beta \), are given in appendix A.

\[ C_V = b_0 + b_1 \left( \frac{C_s B}{H + P} \right) + b_2 \left( \frac{C_s H}{H + P} \right) \]  

(10)

The values of \( b_0, b_1 \) and \( b_2 \) are the regression coefficients. Their values are given in appendix A.

Using equations (8) and (10), the discharge over the BCW could then be computed using the following equation. Bos (1976)

\[ Q = C_d C_V \left( \frac{2^{0.5}}{3} \right) b \sqrt{\frac{H}{L}} \]  

(11)

Equations (3) and (5) were recalibrated by Saleh et al. (1998) for different values of P/L within the range 0.425 to 0.50 at b/B = 0.607. The related coefficients are also given in appendix A.

Equation (7) due to Swamee (1988) could be slightly modified to fit the data for contracted BCW collected by Negm and Alshaikh (1997) and those collected by Saleh et al. (1998) as follows

\[ Q = \frac{2}{3} \left[ 0.452 + 0.088 \left( \frac{H}{L} \right)^{0.5} + 1500 \left( \frac{H}{L} \right)^{0.7} \right] \left( \frac{H}{L} \right) \left( \frac{H}{L} \right)^{0.15} \sqrt{2gh^{1.5}} \]  

(12)

Again, Eq (7) for contracted NCW could be slightly modified to take the form
\[ Q = \frac{2}{3} \left( 0.432 + 0.083 \left( \frac{H}{L} \right)^{1.5} + 1500 \left( \frac{H}{L} \right)^{1.3} \right) \left( \sqrt{g} b H^{1.5} \right) \]

Furthermore, El-Saiad (2000) provided the following equation for \( C_v \) of Eq. (11) when it is applied to contracted NCW as follows:

\[ C_v = 0.99 + 0.063 \frac{C_d H}{H + P} \]

in which \( C_d \) is defined by Eq. (8) with \( \alpha_s = 0.738 \) and \( \beta_s = 0.512 \).

**COLLECTION OF EXPERIMENTAL DATA**

The data pertaining to contracted BCW that are needed for training, validating and testing the ANN model were collected from experimental investigations conducted by the writers and others. The present study conducted two sets of tests, one using suppressed BCWs and the second using suppressed NCWs. The experimental data are collected using a horizontal rectangular flume 30.5 cm wide, 30.5 cm high, and 9.5 m long. The flume is equipped with a tail gate to control the tailwater depth. A centrifugal pump lifts water from underground sump to the flume inlet. Water runs through the flume and returns back to the sump tank via a measuring tank. The weirs are made from clear perspex with fixed height of 15 cm and variable lengths.

Discharges are measured by a pre-calibrated V-notch installed in a measuring tank located below flume outlet at its downstream end and is connected directly to underground sump tank. Water depths, 40 cm upstream the weir, are measured using a precise point gauge (up to ± 0.1 mm accuracy).

Previous data from other investigations were also collected, namely, data from Hager and Schwalt (1994) covering the range of 0.08 < H/L < 0.4 with P/L = 0.8, data from Negm and Alshaikh (1997) on contracted BCW covered a range of 0.508 ≤ b/B ≤ 1.0 at constant P/L = 0.75 with 0.1 ≤ H/L ≤ 0.5, data from Saleh et al. (1998) on CBCW covered a range of P/L = 0.425, 0.472 and 0.50 at b/B = 0.607 with 0.1 ≤ H/L ≤ 0.4 and data for contracted BCW due to Ranga Raju (1973) with 0.25 ≤ b/B ≤ 0.75, P/L = 0.322 and 0.84 and 0.209 ≤ H/L ≤ 0.330. Observations on the narrow contracted crest weirs were also included in the input data, El-Saiad (2000).

The basic statistics of all the input and target variables being used in this paper (which include minimum, maximum, mean values and standard deviations of the different parameters) are shown in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Min. Value</th>
<th>Max. Value</th>
<th>Mean value</th>
<th>Std. Dev</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>L cm</td>
<td>7.50</td>
<td>40.00</td>
<td>26.167</td>
<td>11.268</td>
<td>Input</td>
</tr>
<tr>
<td>P cm</td>
<td>10.50</td>
<td>25.50</td>
<td>15.305</td>
<td>11.768</td>
<td>Input</td>
</tr>
<tr>
<td>b cm</td>
<td>15.50</td>
<td>68.50</td>
<td>28.438</td>
<td>7.87</td>
<td>Input</td>
</tr>
<tr>
<td>H cm</td>
<td>3.00</td>
<td>15.50</td>
<td>8.175</td>
<td>2.867</td>
<td>Input</td>
</tr>
<tr>
<td>Q L/s</td>
<td>2.78</td>
<td>33.00</td>
<td>11.070</td>
<td>5.654</td>
<td>Target</td>
</tr>
</tbody>
</table>
It should be mentioned that the collected data were normalized according using zero mean unit standard deviation method. The collected data were divided into three subsets. The first subset is of size 253 data records for training the network, the second subset is for the validation and consists of 50 data records and the last one is for testing the network and consists of 34 data records.

EVALUATION OF EXISTING DISCHARGE EQUATIONS

The discharge is computed using the above presented equations and then compared with the available experimental data. The mean relative absolute error is computed as follows:

\[ \text{MRAE} = \frac{\text{abs}(\text{discharge measured} - \text{discharge computed})}{\text{discharge measured}} \]

The results are presented in Table 2. This table indicates that no single equation can be used for all weirs of finite crest widths. Errors are more than 5% in many cases and in some cases errors exceed 10%. This highlights the importance of developing a general computational model for different types of weirs of finite crest width. This is the main aim of this study through the use of ANNs.

Table (2) Evaluation of the performance of the existing common discharge equations of weirs of finite crest widths based on mean relative error

<table>
<thead>
<tr>
<th>Weir</th>
<th>Data Source</th>
<th>Eq. (11)</th>
<th>Eq. (7)</th>
<th>Eq. (12)</th>
<th>Eq. (13)</th>
<th>Eq. (2)</th>
<th>Eq. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCW</td>
<td>Negm et al. [21]</td>
<td>0.168</td>
<td>0.034</td>
<td>-</td>
<td>-</td>
<td>0.056</td>
<td>-</td>
</tr>
<tr>
<td>BCW</td>
<td>Negm et al. [21]</td>
<td>0.076</td>
<td>-</td>
<td>0.059</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CBCW</td>
<td>Ranga, Raju [23]</td>
<td>0.073</td>
<td>-</td>
<td>0.040</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NCW</td>
<td>Present</td>
<td>-</td>
<td>0.111</td>
<td>-</td>
<td>-</td>
<td>0.128</td>
<td>-</td>
</tr>
<tr>
<td>CNCW</td>
<td>El-Saied [8]</td>
<td>0.023</td>
<td>-</td>
<td>0.039</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figures 2a to 2g present the comparison between the predicted discharges using different equations and the measured ones due to the present and other investigators. It is clear that unacceptable discrepancies are found between the measured and predicted results. This highlights the need of one and only one more accurate model to predict the discharge over the weirs of finite crest widths. This is the main objective of this study that will be achieved through the use of artificial neural networks.

BASIC PRINCIPLES OF ARTIFICIAL NEURAL NETWORKS

Artificial Neural Network (ANN) were originated to simulate the structure and function of the brain but are much more simpler than the more complicated brain. All types of ANN have the ability to learn from data. The ANN may learn in supervised or unsupervised way. In the supervised learning, which is adopted in this study, the ANN learned from the given examples of the application including the desired output. The ANN learns and yields outputs that are compared to the desired ones to determine the errors. Then, the learning algorithm modifies the weights of the connections between the processing elements (neurons), so that the errors are reduced at the end of the next iteration. So many iterations are made till the network performed well. One of the most common learning algorithms is the back propagation. More information on this learning algorithm and others are found in specialized textbooks in neural networks as
Figure 2. Comparison of measured discharges and the corresponding predicted ones using different prediction equations. (a) CBCW data [23]. (b) BCW data [10]. (c) present BCW data, (d) present NCW data. (e) BCW data [21]. (f) CBCW data [21, 28] and (g) CNCW data [10].
Figure 3 shows a typical feedforward back propagation network. It has three layers. The input layer where the input data of the input variables are presented to the network. The number of neurons (nodes / cells / neurons or elements) is determined by the number of input variables. For flow over weirs of finite crest width as BCW and NCW both suppressed and contracted, four input variables are involved, namely, the length of the weir in the direction of the flow L, the height of the weir P, the breadth of the weir b, and the head of water over the weir H. The output layer consists of a number of neurons equals the number of dependent variables (output variables) to be predicted. In the present application, the discharge Q, is to be predicted and hence the output layer has one neuron to represent Q. One or more layers in between the input and the output layers may be involved and are called hidden layers because they do not interface with the user and the entire processing is not accessible. The neurons are connected by links or connections that hold the key to learn through iterative adjustments of the weights on these links. The input data to the input layer are combined by the weights on the links between the input layer and the next layer through a combining function as that defined by Eq. (15). Then a nonlinear transfer function is applied to the resulting signal (1) to yield the output of the hidden layer (h). The hyperbolic tangent is used in this study as the nonlinear transfer or the activation function. The output of Eq. (16) (Schalkoff (1997)) is then combined with the weights of the connections between the hidden layer and the output layer by the combination function defined by Eq. (17) (Schalkoff (1997)) to yield the output of the network (O):

\[ I_i = \sum_{j=1}^{m} x_j w_{ij} + b_i \]  \hspace{1cm} (15)

\[ h_j = \frac{\exp(I_j) - \exp(-I_j)}{\exp(I_j) + \exp(-I_j)} \]  \hspace{1cm} (16)

\[ O_k = \sum_{j=1}^{n} h_j c_{jk} + b_k \]  \hspace{1cm} (17)

where \( x_i \) is the input of the neuron i in the input layer with m being the number of neurons in the input layer and \( b_i \) is the bias of the unit. The \( w_{ij} \) is the weights vector of the connections between the neurons of the input layer and the neurons of the hidden layer. \( c_{jk} \) is weight of the connection between neuron j of the hidden layer and neuron k of the output layer, n is the number of neurons in the hidden layer and \( b_k \) is the bias to the neuron k.

**TRAINING THE NETWORK**

Training the network involves the determination of the weights vectors \( w_{ij} \) and \( c_{jk} \) such that the sum of squares of the error between the actual value of the output and the desired value of the output is minimal. The network weights are randomly assumed within a particular range. Then they are updated through training of the network in the direction of minimizing the errors. The range in which weights are assumed should be selected carefully by trial and error. The Neural Connection (1998) software was used to train the developed network model. A range of [-5, 5] seems to be reasonable.

Once the suitable activation function is determined, the range of the weights is selected, the number of neurons is chosen, the network of the best size is calibrated to find the maximum number of iterations that minimized the validation system error. The problem is solved many
times using different sizes of maximum updates and the validation system error is observed. The results of network calibration indicated that 5000 iterations are sufficient to produce reasonable accuracy. The weights of the best network of the present application are listed in the appendix A. The following parameters are adopted for the present application:
The network consisted of 4 neurons at the input layer, 6 neurons at the hidden layer and 1 neuron at the output layer.
Initial weights range is ±5 with seed 1 and uniform distribution.
Activation function: Hyperbolic tangent.
Learning algorithm is back propagation based on conjugate gradient based method.
Maximum iterations are 5000 distributed over the four stages as follows 100-100-100-4700.
Maximum number of observations is 253 distributed randomly with seed 5 into 75% for training, 15% for validation and 10% for testing the network.

EFFECT OF APPLYING FILTER TO THE INPUT DATA

The Neural Connection software (1998) has the capability of using filter (some type of transformation) to transform the values of inputs to enable proper mapping of input and output values and to reduce effect of noise in the data to increase accuracy of prediction. Different filters (square root of $H$, $H$ raised to power of 2 and $H$ raised to a power of 1.5) are used. The use of $H^2$ produces high value of errors, the use of $H^{1.5}$ produces correlated residuals in validation and test data sets while the use of $\sqrt{H}$ produces the lowest system error and uncorrelated residuals.

Therefore, this type of filter is used to transform $H$ to $\sqrt{H}$ before any processing can take place.

PERFORMANCE OF ANN MODEL

Figures 4a, 4b and 4c present the comparison between measured discharges and predicted ones using ANN model for training, validation and test data sets respectively. The correlation coefficients between the measured and the predicted discharges using ANN model are 0.996, 0.988 and 0.986 for training, validation and test data sets respectively. Clearly, very good matching is observed in all cases indicating good performance of the ANN model in predicting the discharge over the BCW. The variations of residuals with the predicted values using ANN model are shown in Figure 5. Clearly, the residuals of the ANN are mostly distributed around the line of zero residuals with mean $=0.0008, 0.003, 0.197$ and standard deviation $=0.406, 0.678, 1.123$ and $r = 0.001, 0.134$ and 0.076 for training, validation and test data sets respectively. The low value of $r$ means that the residuals are un-correlated indicating the validity of the ANN model.

COMPARISON OF ANN MODEL AND OTHER MODELS

Figures 6a to 6g are plotted to compare the predictions of the other previously developed models with or without modifications of their constants with the predictions of the ANN model. In all cases, it is clear that the performance of ANN model is much better than that of others where all models except ANN model shows either overestimation or underestimation compared to ANN results. The correlation coefficients between measured and predicted discharges using all the models included in this study are computed using all the data. The values of the correlation coefficients are 0.993, 0.961, 0.968, 0.969, 0.945 and 0.982 for ANN model, Eqs. (2,3), Eqs. (5,6), Eq (7), Eq (8-11) and Eqs (7,12,13) respectively. These results indicate that the best models in order are ANN, modified Swamee model, Eq (7,12,13), Swamee model, Eq (7), Swamee empirical model, Eqs. (5,6), Rao and Muralidhar empirical model, Eqs (2,3) and Negm-Alshaikh...
Figure 3. The multilayers feedforward backpropagation Artificial Neural Network of size 4-6-1 that adopted to model discharge over weir of finite crest width (BCW)

Figure 4. Results of the developed ANN model as compared with measured discharges for different data sets, (a) training data, and (b) both validation and test data sets

Figure 5. Variations of the residuals with ANN prediction for the different data sets, (a) training data set and (b) both validation and test data set
Figure 6: Comparison of predicted results of ANN and those of other models using data from different sources. (a) CBW data [23], (b) BCW data [10], (c) present BCW data, (d) present NCW data, (e) BCW data [21], (f) CBW data [21, 28] and (g) CNCW data [10].
model, Eq.(8-11). Because Swamee model and the modified Swamee model, Eqs. (7, 12, 13), produce the best results after ANN model, they could be combined in one equation as follows:

\[
Q = \frac{2}{3} \left[ a + b \left( \frac{H}{L} \right)^{1/3} + 1500 \left( \frac{H}{L} \right)^{1/5} \right] \left( \sqrt{2ghH^{1/3}} \right)
\]

in which \( a = 0.5 \) & \( b = 0.1 \) for LCW, BCW and NCW and \( a = 0.452 \) & \( b = 0.088 \) for contracted BCW and \( a = 0.432 \) & \( b = 0.083 \) for contracted NCW.

**SENSITIVITY ANALYSIS**

Figures 7a and 7b show the results of conducting a sensitivity analysis using the ANN model in terms of mean relative error (MRE) and correlation coefficient (r) between measured and predicted discharges. The application is solved several times and in each case an input variable is removed and both MRE and (r) are calculated. The zero on the X-axis means that no input variable is removed. It is clear from Figures 7a and 7b that H affects greatly the discharge more than the other variables. However, removing any of the input variable increases the error of the network and poor prediction is obtained. Table 4 shows the system error due to training and validation data sets for the network with the four input variables and one variable is removed in each case during conducting the sensitivity analysis.

<table>
<thead>
<tr>
<th>Input variables</th>
<th>Removed variable</th>
<th>System error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Training data</td>
</tr>
<tr>
<td>L, P, b &amp; H</td>
<td>None</td>
<td>0.075</td>
</tr>
<tr>
<td>P, b &amp; H</td>
<td>Length L</td>
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</tr>
<tr>
<td>L, b &amp; H</td>
<td>Height P</td>
<td>0.105</td>
</tr>
<tr>
<td>L, P &amp; H</td>
<td>Breadth b</td>
<td>0.569</td>
</tr>
<tr>
<td>L, P &amp; b</td>
<td>Head H</td>
<td>0.897</td>
</tr>
</tbody>
</table>

**STABILITY OF THE NETWORK**

In order to study the performance and stability of the ANN model, various computer experiments were carried out with the training set consisted of 253 data vectors, validation set consisted of 30 patterns and test set of 24 data vectors randomly chosen in the data set. This process is repeated 10 times. The resulting training system error and the validation system error from the 10 random test sets are shown in Figures 8a and 8b. The average correlation coefficient of the 10 random sets are 0.9948, 0.993 and 0.985 for training, validation and test data sets with standard deviation of 0.0025, 0.0043 and 0.0095 respectively. Figures 8a and 8b present the variation of MRE and r in terms of the seed number of random number generation (each seed represent an iteration). The deviation from the mean is very small and therefore the developed ANN is considered stable and could be used safely to predict the discharge over the broad-crested weirs for both suppressed and contracted ones.
Figure 7. Results of sensitivity analysis in terms of (a) MRE and (b) correlation coefficient, r

Figure 8. Testing the stability of the developed ANN model in terms of (a) MRE and (b) correlation coefficient, r
CONCLUSIONS

The Artificial Neural Networks are used to predict the discharge passing over weirs of finite crest widths such as broad-crested weirs, narrow crested weirs, contracted broad-crested weirs and contracted narrow crested weirs. The previously developed models for computing the discharges over these weirs are presented and their performance are evaluated compared to both experimental results and the results of ANN. Both mean relative error and correlation coefficient are used in the evaluation processes. Results of ANN model are compared to the results of previously developed models. It is proved that the ANN model is more accurate than other models developed by other investigators in predicting the discharge over the weirs of finite crest widths which could be followed by modified Swamee model, Eq (18). The sensitivity analysis indicates that the discharge over the broad-crested weirs is affected greatly by the head over the weir and the width of the weir while other factors such as the weir length and weir height have smaller effect on the discharge.

REFERENCES

Appendix A:

Regression Coefficients for Eq. (3)

<table>
<thead>
<tr>
<th>b/B</th>
<th>α_0</th>
<th>β_1</th>
<th>P/L</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.0139</td>
<td>-0.4856</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.902</td>
<td>0.9694</td>
<td>-0.4915</td>
<td>0.375</td>
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</tr>
<tr>
<td>0.803</td>
<td>0.3178</td>
<td>-0.4945</td>
<td>0.375</td>
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</tr>
<tr>
<td>0.705</td>
<td>0.3528</td>
<td>-0.4584</td>
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</tr>
<tr>
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<td>0.8667</td>
<td>-0.4811</td>
<td>0.375</td>
<td>[20]</td>
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<tr>
<td>0.508</td>
<td>0.3297</td>
<td>-0.4345</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.607</td>
<td>0.8929</td>
<td>-0.4983</td>
<td>0.425</td>
<td>[28]</td>
</tr>
<tr>
<td>0.607</td>
<td>0.9157</td>
<td>-0.3178</td>
<td>0.472</td>
<td>[28]</td>
</tr>
<tr>
<td>0.607</td>
<td>1.0032</td>
<td>-0.4449</td>
<td>0.500</td>
<td>[28]</td>
</tr>
<tr>
<td>0.700</td>
<td>1.496</td>
<td>-0.622</td>
<td>0.33 &amp; 0.84</td>
<td>Present</td>
</tr>
<tr>
<td>0.700</td>
<td>1.108</td>
<td>-0.624</td>
<td>0.33 &amp; 0.84</td>
<td>Present</td>
</tr>
<tr>
<td>0.700</td>
<td>0.880</td>
<td>-0.553</td>
<td>0.33 &amp; 0.84</td>
<td>Present</td>
</tr>
</tbody>
</table>

*The coefficient is proposed by the present study, based on the data due to Ranga Raju and Ahmed [23].

Regression Coefficients for Eq. (5)

<table>
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<tr>
<th>b/B</th>
<th>b_0</th>
<th>b_1</th>
<th>b_2</th>
<th>P/L</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>1.00229</td>
<td>-0.02430</td>
<td>0.28756</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.902</td>
<td>0.96015</td>
<td>-0.04094</td>
<td>0.27335</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.803</td>
<td>0.93520</td>
<td>0.02353</td>
<td>0.12172</td>
<td>0.275</td>
<td>[20]</td>
</tr>
<tr>
<td>0.705</td>
<td>1.10124</td>
<td>-0.07514</td>
<td>0.23806</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.607</td>
<td>0.99330</td>
<td>0.04411</td>
<td>0.01156</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.508</td>
<td>1.02938</td>
<td>-0.17853</td>
<td>0.32087</td>
<td>0.375</td>
<td>[20]</td>
</tr>
<tr>
<td>0.607</td>
<td>0.99999</td>
<td>-0.00092</td>
<td>0.08582</td>
<td>0.425-0.50</td>
<td>[28]</td>
</tr>
<tr>
<td>0.25-0.75</td>
<td>1.00443</td>
<td>-0.03078</td>
<td>0.75032</td>
<td>0.33-0.84</td>
<td>[28]</td>
</tr>
</tbody>
</table>

*The data are due to Ranga Raju and Ahmed [23].

APPENDIX B:

Final Weights of the Network Connections

The network of the present application has a size of 4-6-1 with two bias neurons. The activation function of the hidden layer is sigmoidal (tanh) and that of the output layer is linear. The back propagation learning algorithm uses the conjugate method and maximum updates of 5000. A total of 37 weights (with a range [-5,5]) to initiate the weights matrix) are used with final values as follows:

| Hidden Node 1 | Bias = 0.100365 | 0.124456 | +0.842717 | 0.496950 |
| Hidden Node 2 | Bias = -0.568488 | +0.311877 | -0.943300 | +0.147464 | +0.978383 |
| Hidden Node 3 | Bias = -5.949222 | -1.099471 | +0.903719 | -0.087911 | -0.29384 |
| Hidden Node 4 | Bias = +7.373214 | -2.213851 | -4.356189 | -0.655442 | +0.839031 |
| Hidden Node 5 | Bias = +1.003305 | +0.013087 | +0.032355 | +0.144703 | +0.312202 |
| Hidden Node 6 | Bias = +5.644442 | +1.170394 | -1.168358 | +0.439576 | -0.344340 |

Output Node 0 | Bias = +7.967912 | -1.062083 | -1.221662 | -0.386286 | +0.480576 | +0.234425 | +0.284521 |