STABILITY REGION ESTIMATION OF HYBRID MULTI-MACHINE POWER SYSTEM

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Abstract

The concept of producing electric energy utilizing the energy embedded in wind is a real practice on a wide scale. At different locations where wind potential is so promising, the energy planer is talking about wind parks connected to existing utilities. The expansion of interconnected utilities with non-conventional plants should be investigated regarding different topics, among which is the stability of the hybrid system under expected operating conditions.

In this paper, the stability problem is investigated using the decomposition aggregation technique. Attempts to overcome some of the drawbacks given in other methods have led to the application of the new technique based on Bellman's concept of vector Lyapunov function. An advanced approach is proposed in this work; triple-wise decomposition-aggregation for multi-machines hybrid power system considering the transfer conductance and uniform damping. For this algorithm the system is decomposed into (n-1)/2 three-machine subsystems for odd number of machines, or (n-2)/3 three-machine plus one two-machine subsystems for even number of machines. Six non-linearities are considered for each free subsystem. The domain of attraction is estimated to study the effect of introducing non-conventional energy source.

1. Introduction

Today the concept of producing electric energy utilizing the energy embedded in wind is a real practice on a wide scale; few kW to MW-rated wind energy conversion systems, autonomous as well as grid connected systems. At different locations where wind potential is so promising, the energy planer is talking about wind farms connected to existing utilities.

Basically, a design phenomenon is chosen to select the optimum configuration of the system components for each field of application. One of the standard configurations of wind energy conversion systems (WECS) is considered in this study. It consists of a variable speed wind turbine (WT) connected to synchronous generator, equipped with a gearbox and frequency converter. Variable speed is considered because it can increase the energy captured by the turbine and it also reduces some loads. Moreover, the frequency converter can control the generator power and thereby reduce the demands.

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on efficient damping. On the average, the variable speed systems are as efficient as the directly grid connected systems, because both the generator and gearbox no-load losses are much reduced.\[1\][2][3]

Extension of interconnection of utilities with non-conventional plants should be investigated regarding the added value, efficiency, cost and availability of the system, and stability of the hybrid system under expected abnormal operating conditions. For power systems, the stability problem is concerned with the property that enables the synchronous machine to respond to a disturbance so as to move from one to another stable operating condition.

In this paper, the stability problem of hybrid power system is investigated using the triple-wise decomposition aggregation technique. This technique is proposed to overcome some of the drawbacks given in other methods; e.g., in the pair-wise scheme of work the complex power system is decomposed into \((n-1)\) two-machine interconnected subsystems, whereas the non-linearities are considered for each free subsystem. Although this technique is more powerful than some old techniques, it fails to produce stable aggregation matrix for large-scale power systems. Therefore, the pair-wise scheme is applicable to medium scale systems and should be performed in such a way to assure decomposing the complex system into weakly coupled subsystems. One of the attempts to overcome such drawbacks has led to the application of the new the triple-wise decomposition aggregation technique based on Bellman's concept of vector Lyapunov function.\[4\]

2. System Description and Modeling

The hybrid power system under consideration consists mainly of conventional synchronous generating plants, wind parks composed of certain number of wind turbines coupled with synchronous generators and finally interconnected transmission lines. To carry out the stability study, each of these components will be modeled.

2.1 Wind Turbine Model

The mechanical power \(P_m\) produced by a wind turbine is given by:

\[
P_m = 0.5 \rho C_p A U^3
\]  
(1)

where

- \(\rho\) air density,
- \(U\) instantaneous wind speed,
- \(A\) rotor swept area,
- \(C_p\) power coefficient.

The mechanical rotational speed \(\omega\) of the wind turbine is expressed by the swing equation:

\[
\frac{d\omega}{dt} = \frac{a_{ou}}{2H} (T_m - T) - \frac{D}{\omega} \omega
\]  
(2)

where

- \(T_m\) mechanical, \(\omega\) electrical,
- \(T_m = P_m/S(\omega/\omega_p)\), \(S\) KVA rating.

2.2 Utility Model

For a hybrid power system, the differential equations describing the dynamics of the system are:

\[
M_i \delta_i = D_i \delta_i = P_m_i - P_e_i \quad i=1,2,..,n
\]  
(3)

Where

\[P_e = \sum \epsilon_i \delta \epsilon_i + \sum \gamma_i \delta \epsilon_i \]

It is also assumed that for all machines the damping to inertia ratio is constant, that is:

\[\frac{D_i}{M_i} = \lambda, \quad i=1,2,..,n\]

Selecting the nth machine to be the reference one, and introducing \(3(n-1)\) state vector \(x\) as:

\[x = \begin{pmatrix} \omega & \delta & \omega & \delta & \cdots & \omega & \delta \end{pmatrix}^T\]

Where

\[\delta = \delta_i - \delta - \delta = \omega - \omega - \omega = \delta\]

3. Decomposition Aggregation Control Technique

The domain of attraction of the equilibrium point is the set of all points such that trajectories initiated at these points eventually converge to the origin. Since the stability question of how the non-conventional renewable energy systems would affect existing utilities should be accurately investigated, a powerful tool is to be implemented so as not to disregard a number of operating points as unstable. One of the most powerful tools is the decomposition aggregation technique, which is based on Bellman's concept of vector Lyapunov functions. It consists of decomposing a large-scale system into a set of subsystems. The stability properties for the disconnected free subsystems are derived, again aggregated to describe the domain of attraction of the complex system.

Two approaches could be proposed in estimating the domain of attraction of large-scale power systems: pair-wise and triple-wise decomposition aggregation techniques. In the pair-wise scheme of work the complex power system is decomposed into \((n-1)\) two-machine interconnected subsystems where two non-linearities are considered for each free subsystem. Although this technique is more powerful than some old techniques due to the increased number of non-
linearities, it fails of producing stable aggregation matrix for large-scale power systems. Therefore, it is applicable to medium scale systems and should be performed in such a way to assure weakly coupled subsystems. [18][paper1] In the next section, the triple-wise technique, which is a step forward in the application of the decomposition aggregation technique, will be in detail discussed.

4. Triple-wise Decomposition Aggregation Technique

4.1 Power System Decomposition

Stability considered For the triple-wise algorithm the interconnected system is decomposed into (n-1)/2 three-machines systems for odd number of machines, or (n-2)/2 three-machines plus one two-machine subsystems for even number of machines with six non-linearities order to obtain the largest asymptotic for each free subsystem. Figure (1) shows the schematic diagram of the decomposed system.

![Schematic diagram of the decomposed system](image)

Figure (1): Schematic diagram of the decomposed system

Adopting the state vector \(x_i\) as

\[x_i = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ \vdots & \vdots & \vdots \end{bmatrix}^T \]

where \((2l-1), 2l\) are the elements of the set \(J\), the whole system is decomposed into \((n-1)/2\) fourth-order interconnected subsystems. Each of the \(s\) subsystems may be written in the general form

\[
\dot{x}_i = \sum_{j \in J} P_{i,j} x_j + B_{i,j} y_j + C_{i} h_i(x)
\]

(7)

where \(P_{i,j}, B_{i,j}, C_{i}\), and \(h_i(x)\) are defined as follows

\[P_i = \begin{bmatrix} -\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \]

\[B_i = \begin{bmatrix} -M_{i,1} & A_i & 0 & 0 \\ 0 & -M_{i,2} & A_i & 0 \\ 0 & 0 & -M_{i,3} & A_i \\ 0 & 0 & 0 & -M_{i,4} \end{bmatrix} \]

\[h_i(x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]
\[ h_i(x) = \begin{bmatrix} \sum_{i=1}^{m} (-M_{ij}^* A_{ij}^* \Phi_i(y_{ij}) + M_{ij}^* A_{ij} \Phi_i(y_{ij})) \\ \sum_{i=1}^{m} (-M_{ij}^* A_{ij}^* \Phi_i(y_{ij}) + M_{ij}^* A_{ij} \Phi_i(y_{ij})) \\ 0 \\ 0 \end{bmatrix} \]

In domain estimate for the considered power system, the subsystem of equation (3) is decomposed of non-linearities, i.e., the vector \( \Phi_i(y_{ij}) \) is defined such that the free subsystem contains the largest number (eq. 8): \[ \Phi_i(y_{ij}) = \begin{bmatrix} \Phi_1(y_{ij}) \\ \Phi_2(y_{ij}) \\ \Phi_3(y_{ij}) \\ \Phi_4(y_{ij}) \\ \Phi_5(y_{ij}) \\ \Phi_6(y_{ij}) \end{bmatrix} \]

where the six non-linearities \( \Phi_i(y_{ij}) \) are defined as (eq. 9):

\[ \begin{align*}
\Phi_1(y_{ij}) &= \cos(y_{ij} + \delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) - \cos(\delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) \\
\Phi_2(y_{ij}) &= \cos(y_{ij} + \delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) - \cos(\delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) \\
\Phi_3(y_{ij}) &= \cos(y_{ij} + \delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) - \cos(\delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) \\
\Phi_4(y_{ij}) &= \cos(y_{ij} + \delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) - \cos(\delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) \\
\Phi_5(y_{ij}) &= \cos(y_{ij} + \delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) - \cos(\delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) \\
\Phi_6(y_{ij}) &= \cos(y_{ij} + \delta_{ij} + \theta_{2i,j} - \theta_{2i,j}) - \cos(\delta_{ij} + \theta_{2i,j} - \theta_{2i,j})
\end{align*} \]

Now, we can decompose each of these subsystems into a free (disconnected) subsystem and interconnections. The free subsystem has the general form (eq. 10): \[ \dot{x}_i = P_i x_i + B_i \Phi_i(y_{ij}) \]

\[ y_i = C_i^T x_i, i = 1,2,\ldots,s \]

### 4.2 Free Subsystem Analysis

For the free subsystem eq. (8), we adopt a Lyapunov function in the form "quadratic form plus sum of integrals of the six non-linearities (eq. 11)"

\[ V_i(x_i) = x_i^T H_i x_i + \sum_{i=1}^{m} d_i \int_0^t \phi_i'(y_{ij}) d\tau_i \]

In this expression, \( H_i \) is a fourth-order symmetric positive definite matrix, \( d_i \) are positive numbers. The time derivative of \( V_i(x_i) \) for the free subsystem is derived as (eq. 12):

\[ \dot{V}_i(x_i) = x_i^T (-G_i) x_i + 2 \Phi_i^T B_i^T H_i x_i + \sum_{i=1}^{m} d_i \phi_i'(y_{ij}) \dot{y}_{i,j} \]

and equ. 13.

\[ -G_i = P_i^T H_i + H_i P_i \]

It is computed in the form

\[ G_i = \begin{bmatrix} 2(\lambda_1 b_1^2 - b_1^2) & 2\lambda_1 b_1^2 - b_1^2 & \lambda_1 b_1^2 - b_1^2 & \lambda_1 b_1^2 - b_1^2 \\
2\lambda_2 b_2^2 - b_2^2 & 2(\lambda_2 b_2^2 - b_2^2) & \lambda_2 b_2^2 - b_2^2 & \lambda_2 b_2^2 - b_2^2 \\
\lambda_3 b_3^2 - b_3^2 & \lambda_3 b_3^2 - b_3^2 & 0 & 0 \\
\lambda_4 b_4^2 - b_4^2 & \lambda_4 b_4^2 - b_4^2 & 0 & 0 \end{bmatrix} \]

Selecting \( p_i \) as an arbitrary positive number, the following relations will yield a \( G \) positive definite matrix, where (eq. 14):
\[ h_{11}^2 = \frac{(1 + \rho_1)}{\lambda} h_{11}^1, \quad h_{12}^2 = \frac{(1 + \rho_2)}{\lambda} h_{12}^1 \]
\[ h_{23}^2 = \lambda h_{23}^1, \quad h_{34}^2 = \lambda h_{34}^1 \]

Under the conditions \( h_{11}^2, h_{22}^1 \geq 0 \) the corresponding matrix \( \mathbf{H} \) is positive definite and given as (equ. 15):

\[
\mathbf{H} = \begin{bmatrix}
\frac{(1 + \rho_1)}{\lambda} h_{11}^1 & 0 & h_{12}^1 & 0 \\
0 & \frac{(1 + \rho_2)}{\lambda} h_{12}^1 & 0 & h_{13}^1 \\
0 & 0 & \lambda h_{23}^1 & 0 \\
0 & 0 & 0 & \lambda h_{34}^1
\end{bmatrix}
\]

Selecting the constants \( d_{ii} \) as (equ. 16):

\[
d_{11} = 2 M_{11}^{-1} A_i h_{11}^1, \quad d_{12} = 2 M_{12}^{-1} A_i h_{12}^1, \quad d_{13} = 2 M_{13}^{-1} A_i h_{23}^1, \quad d_{14} = 2 M_{14}^{-1} A_i h_{34}^1, \quad d_{15} = 2 M_{15}^{-1} A_i h_{15}^1
\]

equation (10) becomes (equ. 17):
\[
\psi_i(x_i, y_i) = -2 k_i h_{11}^1 x_i^2 - 2 k_i h_{12}^1 y_i x_i - 2 M_{11}^{-1} A_i h_{11}^1 (y_i) x_i - 2 M_{12}^{-1} A_i h_{12}^1 (y_i) x_i - 2 M_{13}^{-1} A_i h_{23}^1 (y_i) x_i - 2 M_{14}^{-1} A_i h_{34}^1 (y_i) x_i + h_{11}^1 x_i^2 + h_{12}^1 y_i x_i + h_{13}^1 x_i + h_{14}^1 y_i x_i
\]

Now, let us introduce the positive constants \( \xi_{ij} \in [0, \xi_{ij}^0] \) which satisfy the following condition
\[
y_{\xi, \phi_i}(y_i) \geq e_{\phi_i} y_i^2, \quad i = 1, 2, \ldots, 6
\]

on a compact interval \( \Omega_{\phi_i} \) of \( y_i \).

\[ U_{\phi_i} = \{ \sqrt{\bar{U}_{\phi_i}}, \sqrt{\bar{U}_{\phi_i}} \}, \quad i = 1, 2, \ldots, 6
\]

where \( \Omega_{\phi_i}, \bar{U}_{\phi_i} \) are respectively the negative and positive solutions of the following equation
\[
\psi_i(x_i, \bar{U}_{\phi_i}) = \xi_{\phi_i} y_i^2, \quad i = 1, 2, \ldots, 6
\]

Based on inequality (18) and by adding to the right-hand side of equation (17) the non-negative expression
\[
2 M_{11}^{-1} A_i h_{11}^1 (y_i) \phi_i(B_i(x_i)) - \xi_{\phi_i} h_{11}^1 (y_i)
\]

we obtain (equ. 22)
\[
\psi_i(x_i) \leq -\psi_{\phi_i} \xi_{\phi_i}^0 y_i^2 \quad \forall \alpha = 1, 2, \ldots, 5
\]

where \( \psi_{\phi_i} \) is the minimum eigenvalue of the sixth-order symmetric positive definite matrix \( M_{\phi_i} \).

### 4.3 Power System Aggregation

The desired domain of attraction for the overall hybrid power system is generated using aggregation matrix of order (n2) for triple-wise technique. The stability criterion of the equilibrium \( x = 0 \) of the overall system is based on the construction of an aggregation matrix \( \mathbf{W} = \left[ w_{ij} \right] \), where its elements (real numbers) obey the following inequality (equ. 23):
\[ V_i(x)_{(s)} = \left[ \text{grad} V_i(x_i) \right]^T \left( P_i x_i + B_i \Phi_i(x_i) + h_i(x) \right) \leq \sum_{i=1}^{\infty} w_i u_i(x_i) u_i(x_i) \quad \forall i = 1, 2, \ldots, s \]

where \( V_i(x)_{(s)} \) is the same derivative of the function \( V_i \) of the decomposed subsystem of equation (5), and \( u_i \) are positive definite functions:

\[ u_i(x_i) = \left[ x_i \right]^{\infty} = (x_i^T x_i)^{\infty} \quad \forall i = 1, 2, \ldots, s \]

Thus \( V_i(x)_{(s)} \) can be expressed as (equ. 34):

\[ V_i(x)_{(s)} = V_i(x_i)_{(s)} + \left[ \text{grad} V_i(x_i) \right]^T h_i(x) \quad \forall i = 1, 2, \ldots, s \]

where \( V_i(x_i)_{(s)} \) is the total derivative of \( V_i \) along motions of the free subsystem.

Using the following relations (equ. 25):

\[ \phi_{11-14}(y_{11-14}) = \xi_{11-14}(y_{11-14}) + \xi_{14-11}(y_{11-14}) \]

\[ \xi_{11-14} = \sin(\theta_{11-14}) - \delta_{11-14} \]

\[ \phi_{21-14}(y_{21-14}) = \xi_{21-14}(y_{21-14}) + \xi_{14-21}(y_{21-14}) \]

\[ \xi_{21-14} = \sin(\theta_{21-14}) - \delta_{21-14} \]

We get (equ. 26):

\[ \left[ \text{grad} V_i(x_i) \right]^T h_i(x) \leq \overline{w}_i \quad \forall i \]

\[ + 2 \sum_{i=1}^{\infty} Z_{ii} \left\| x_i \right\| \left\| x_i \right\| \]

where

\[ Z_{ii} = Z_i(Z_i + \overline{w}_i) \quad (27) \]

\[ Z_{in} = Z_i((M_{ii} A_{ii-n} + \delta_{ii-n}) + M_{ii-n} A_{ii-n+1} + \delta_{ii-n+1}) X_{i}(h_i') \]

\[ Z_{ni} = Z_i((M_{ii} A_{ii-n} + \delta_{ii-n}) + M_{ii-n} A_{ii-n+1} \delta_{ii-n+1}) X_{i}(h_i') \]

\[ \overline{w}_i = Z_i((M_{ii} A_{ii-n} + \delta_{ii-n}) + M_{ii-n} A_{ii-n+1} \delta_{ii-n+1}) X_{i}(h_i') \]

and \( \overline{w}_i \) is the maximum eigenvalue of the fourth-order symmetric matrix \( Q_{ii} \), whose elements are defined as (equ. 28):

\[ q_{00} = q_{11} = q_{01} = q_{11} = q_{10} = q_{10} = 0 \]

\[ q_{ii} = M_{ii} h_i' \sum_{i=1}^{\infty} (A_{ii-n} A_{ii-n+1} + \delta_{ii-n} \delta_{ii-n+1}) \]

\[ q_{ij} = M_{ij} h_i' \sum_{i=1}^{\infty} (A_{ij} A_{ij} + \delta_{ij} \delta_{ij}) \]

Hence, the elements of the \( s \times s \) aggregation matrix \( W \) can be defined in the following form (equ. 29):

\[ w_{ii} = \begin{cases} -1 & \text{if } i = 1 \\ \frac{1}{2} Z_{ii} & \text{if } i \neq 1 \end{cases} \quad \forall i = 1, 2, \ldots, s \]

Therefore, the stability criteria of the whole system can be defined as the equilibrium state \( x = 0 \) [equation (5)] is asymptotically stable if the aggregation matrix \( W \) [equation (29)] has eigenvalues with negative real parts.
4.4 Stability Domain Estimate For The Complex System

In order to determine a stability domain estimate for the complex system, we proceed the following systematic step:

Assuming that the complex system is asymptotically stable having domain $E$ which is to be estimated as:

$$E = \{ x: V(x) \leq \gamma \}$$

where the Lyapunov function for the complex system is

$$V(x) = \sum_{i=1}^{n} \beta_i V_i(x)$$

$$\beta = B^T$$

is a matrix (with zero off diagonal) chosen such that $W^T \beta + \beta W$ is negative definite matrix, equs. 31,32:

$$\gamma = \min \left( \beta_i V_i^*: k = 1,2,...,s \right)$$

$$V_i^* = \min \left( \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_i} \right) \left\{ \sum_{i=1}^{n} \frac{d_i}{e_i} \int_{\gamma_i}^{\gamma_i^*} \left[ \phi_i(y_i) dy_i \right] \right\}$$

$$\forall i = 1,2,...,s$$

$V_i^*$ is the estimate of asymptotic stability domain for the $i$th free subsystem.

5. Verification of the control algorithm

The triple-wise decomposition aggregation technique is applied to estimate the domain of attraction of a hybrid power system containing a wind park. As a start point, the operation of wind parks is to be analyzed.

First, a WECS supplies electrical power in the electrical network as long as the wind velocity lies within its operating range. If the wind velocity falls out the cut-in / cut-out speed range, the WECS is to be disconnected, which is an extreme condition. For a wind park containing a number of WECSs located on a wide area, the diversity of wind speed will maximize the probability of the extreme of disconnecting all WECSs due to falling outside the operating range. More often the WECSs are sectionalized. Thus the wind park can be theoretically considered as one bus-bar (3.B.1), but practically connected to the utility through more than one 6.B.

Second, based on equation (32), the analysis of the decomposition aggregation technique has come to the following conclusion: The triple-wise decomposition for odd number of machines is more powerful than for even number, since the estimate of the two machines sub-system will dictate the domain of attraction of the whole system. Therefore, combine these two conclusions establishes the basis of estimation the domain of attraction of the hybrid system; increment of the power injected by the wind park will be in wide range of the whole system through the estimation of the 6.B.1. The new approach is verified using one of the IEEE standard systems [6]. The parameters of the system under study are shown in tables (1), (2).

The power generated on B.B.2 by the wind park is increased in steps. Figure (2) shows the incremental change in each step considering case #1 as the base case. The incremental change reached 26.7% of the base value. In figure (3), wind park generation (WQ) is given in percentage of the whole system generation, starting at 13.23% to 17.22%. In this case, the corresponding stability function $V$ (domain estimate) is calculated, showing a monotone increasing trend. Contour representation of the $V$ function is given in figure (4) showing non-intersected contours with increasing value of $x$.

An interesting point to be mentioned is B.B.46 was tested to connect the WECS, the $V$ function showed the same monotonically increasing trend but slightly saturated in cases 5,6,7. This remark could lead to the possibility of using the technique to optimally locate the new WECSs.

6 Conclusion

As stated in previous work [4], estimation of the domain of attraction applying pair-wise decomposition aggregation technique implies that the decomposition of the system model should be performed in such a way that the resulting subsystems are weakly coupled. This means that any strong interconnection between system machines (except reference one) may lead to unstable aggregation matrix. However, the triple-wise decomposition scheme, proposed in this paper, is more suitable for real power systems than the pair-wise decomposition. This technique allows new interconnections among machines to be included in the subsystems instead of exploiting them as interconnections and subsystems. Real power systems are almost invariably composed of weakly connected groups of tightly interconnected machines. It is also clear that in the triple-wise case Lyapunov function of any subsystem contains six nonlinearities only, since in the case of triple-wise decomposition it is a function of only nonlinearities. This means that larger stability domain estimates can be obtained by increasing the number of the nonlinearities included in each free subsystem. The second property of the triple-wise decomposition is: the technique is more powerful for odd number of machines than for even number, since the estimate of the two machines sub-system will dictate the domain of attraction of the whole system.
Hybrid power systems containing unconventional units require advanced techniques to evaluate the system operation under different conditions. Wind energy conversion systems imply dynamic conditions are set under study, utilizing the properties of the triple-wise decomposition aggregation technique. The work presented in this paper approved that the new technique is providing much better estimate for the domain of attraction, and the increased energy of the WECS also improves this domain. This technique can be applied to select the optimum allocation of planned WECS's.

References

Table (1): System parameters

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Table (2): T.L. parameters (p.u.deg)

| Y12 | 0.75 | -82 |
| Y16 | 0.001 | -82 |
| Y17 | 0.62 | -80 |
| Y23 | 0.0008 | -82 |
| Y25 | 0.001 | -82 |
| Y27 | 0.61 | -77 |
| Y34 | 0.7 | -81 |
| Y37 | 0.65 | -75 |
| Y45 | 0.0008 | -82 |
| Y47 | 0.6 | -30 |
| Y56 | 0.55 | -82 |
| Y57 | 0.6 | -77 |
| Y67 | 0.5 | -80 |

Figure 2: % change of W
Figure 3: Effect of changed WT on stability function V

Figure 4: Contour representation of V function