RELIABILITY CALCULATION FOR GENERATION - INTERCONNECTED POWER SYSTEMS

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ABSTRACT:

This paper introduces a proposed technique to calculate reliability indices of interconnected power systems. The technique is based upon the sequential path supplementation, state space methods and power flow computations. The aspects of component outage reliability data, operating constraints and interconnected power system configurations are taken into considerations. The general steps of the method are: preparation of component reliability data, constructing the reliability data for each subsystem according to the connection of units and subsystems, enumerating possible operating paths and formulating path connection matrix, and then state transition matrix according to assumed initial states.

The application of the proposed algorithm on three different interconnected power systems shows accurate and valuable results. Also, it reduces the mathematical calculations, because of using path word instead of component words when defining the power system state connection matrix and using network flow to define the condition of each state.

The proposed algorithm is useful in case of planning field, when comparing different plans or alternative designs. It is also valuable for operating engineers when they decide to add new between substations, and it helps them to compare between the location of two and associated increase in reliability indices.

KEY WORDS:

Reliability, state space, topology, sequential path supplementation, and network flow.
1- INTRODUCTION:

Two or more power subsystems are often connected via tie lines to form an interconnected system in order to improve the system reliability and economical benefits.

It was proposed in [1] to use network flow to investigate the load flow calculations with network flow calculations. The substitution is deemed to be a kind of permissible approximation in reliability evaluations and the failure criterion is not taken into considerations, although, the capacity limit and the control over the network flow are considered. However, only the probability indices could be calculated, while no frequency index was touched upon.

Wang, Yifan and Q. Sun [2] proposed an “sifting method” algorithm based upon path combination which reduces the number of network flows that must be calculated. For the interconnected power system reliability evaluations, the involved sources (subsystems) are multi-state components and the ties are dual state components. All the different state combinations of these components comprise the basic event space under investigation. For the time being the multi-state components are also treated as dual-state components (connection and disconnection), the sifting of network configuration states starts with sequentially supplementing paths.

There are many methods to find minimum paths. The path enumeration method presented in [3] is a simple and direct one. However, the sparsity of the component path conjugate matrices is not taken into consideration and therefore it is not very suitable for large scale load flow system calculations. The sparse matrix should be condensed for reliability evaluation in order to reduce the storage space and increase the calculation efficiency[4-7].

At present, there are two methods for interconnected system reliability calculations. One is the probability array method [8] and the other is the equivalent supporting unit method [9-10].

2- PROPOSED METHOD:

In this paper the interconnected power system reliability indices can be calculated depending upon the basic reliability data of each component and the network structure.
The general steps of the proposed method are:

1. Tabulate the reliability data for each component (failure rate and average outage rate per failure).
2. Calculate the reliability indices of each subsystem according to its component connections and component reliability indices.
3. Calculate the reliability indices of the lines by using the components failure rate and down time per failure.
4. Use set and set theory to enumerate main paths and path connections matrix.
5. By the use of component outage and repair rates, calculate path reliability indices.
6. Consider the first possible path $P_1$ and calculate the corresponding component set involved $E_1$ and path space set. After this step supplement the second path $P_2$ and calculate the space set path set, and not supplemented path set.
7. Repeat the steps 1 to 6 of possible supplemented paths $P_n, P_{n+1} \ldots P_m$.
   - From steps 5, 6 & 7 determine the possible configuration path states.
   - Perform the state-transition matrix $[A]$ to obtain the probabilities and frequencies of the different possible configuration states.
   - Combine failure states set probabilities to get the overall interconnected power system failure and success probabilities and frequencies according to the assumed failure criterion and operating constraints.

3. CASE STUDY:

Three configurations of generation-interconnected power systems are illustrated in Fig. 1-(a, b, c).

Fig. 1-(a) 3 single subsystems without any interconnecting links, while Fig. 1-(b) is supported by a link "L_1" to tie buses $B_1$ and $B_2$, also Fig. 1-(c) is supported by $L_2$ to tie $B_3$ with $B_4$. 
The proposed technique is applied on three cases to calculate reliability indices: probability, frequency and mean duration of failure, $P_F$, $F$ and $T_F$ in order to illustrating the variations of these values according to the addition of links $L_2$ or $L_3$.

![Diagram of interconnected power systems]

Fig. (1 - a) and Fig. (1 - b) illustrates two different configurations of interconnected power systems. Fig. (1 - c) shows a third configuration.

Considering the power system without any additional interconnecting links, the only paths are $P_1$, $P_2$ and $P_3$ but when the link "L_1" is added, two paths are created $P_4$ & $P_5$. Also, when the link $L_2$ is added, another two main paths are created $P_6$ and $P_7$. The all possible paths $P_1$ through $P_7$ are shown in the following component path contour matrix, Table (1).

The first step is to calculate path-reliability indices, based on the components failure outage rate and their connections.

Consider, the path "P_1", it consists of substation $G_1$, 1.2 mile transmission line, two medium voltage circuit breakers, bus circuit breaker and transformer $T$. By use of the failure outage data shown in Table (2), and the connection of these components to form $P_1$, the path failure rate is given by
Table (1): Component path congate matrix.

<table>
<thead>
<tr>
<th>Component</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>L1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>L2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>L3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>L4</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>L5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \lambda_{P_i} = \sum_{i=1}^{n} \lambda_i \]

Where:

\[ \lambda_i = \text{the failure rate of component } i \text{ in the path } P_i \]

\[ n_{S1} = \text{No. of series components forming path } P_1. \]

\[ \lambda_{P_i} = \lambda_i 	imes \lambda_{S1} - 2\lambda_{CB} + \lambda_{ba} \]

\[ = 0.9005 \times 1.20 \times 0.007 - 2 \times 0.002 + 0.002 = 0.004 \text{ (yr)} \]

and the average outage duration per failure is given by

\[ r_{\lambda} = \frac{1}{\lambda_{P_i}} \left\{ r_{i_1} \lambda_{i_1} + r_{i_2} \lambda_{i_2} + \ldots + r_{i_n} \lambda_{i_n} \right\} \]

\[ = \frac{1}{0.0045} \left( 0.0005 \times 0.0001 \times 3 + 1.20 \times 0.007 \times 3 + 2 \times 0.002 \times 3.50 + 0.002 \times 3.50 \right) \]

\[ = 3.164 \text{ hr/failure} \]
Table (2): Component outage rate and repair time [6]

<table>
<thead>
<tr>
<th>No.</th>
<th>Component</th>
<th>Failure rate λi/year</th>
<th>Repair time ri/ hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Supply (subsystem)</td>
<td>0.0005</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>46 kv-11 ky bus.</td>
<td>0.0002</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>46 kv-11 ky disconnecting switch</td>
<td>0.0001</td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td>46 kv-13.8 kv Trans.</td>
<td>0.0040</td>
<td>5.00</td>
</tr>
<tr>
<td>5</td>
<td>13.8 kv C.B.</td>
<td>0.0100</td>
<td>3.50</td>
</tr>
<tr>
<td>6</td>
<td>13.8 kv (enclosed)</td>
<td>0.0020</td>
<td>1.20</td>
</tr>
<tr>
<td>7</td>
<td>13.8 kv feeder breaker</td>
<td>0.0020</td>
<td>3.50</td>
</tr>
<tr>
<td>8</td>
<td>1.8 kv feeder</td>
<td>0.0150/mile</td>
<td>10.0</td>
</tr>
<tr>
<td>9</td>
<td>13.8 kv bus C.B.</td>
<td>0.0020</td>
<td>3.50</td>
</tr>
<tr>
<td>10</td>
<td>11 kv feeder</td>
<td>0.0700/mile</td>
<td>3.00</td>
</tr>
</tbody>
</table>

The reliability indices of all paths are calculated and tabulated in Table (3).

Table (3): Calculated path reliability indices

<table>
<thead>
<tr>
<th>Path</th>
<th>λpi</th>
<th>rpi</th>
<th>μpi</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.0945</td>
<td>3.1164</td>
<td>0.32088</td>
</tr>
<tr>
<td>P2</td>
<td>0.1155</td>
<td>3.0952</td>
<td>0.3227</td>
</tr>
<tr>
<td>P3</td>
<td>0.1292</td>
<td>3.0849</td>
<td>0.32415</td>
</tr>
<tr>
<td>P4</td>
<td>0.1895</td>
<td>3.0686</td>
<td>0.32588</td>
</tr>
<tr>
<td>P5</td>
<td>0.1685</td>
<td>3.0771</td>
<td>0.32498</td>
</tr>
<tr>
<td>P6</td>
<td>0.2055</td>
<td>3.0680</td>
<td>0.32594</td>
</tr>
<tr>
<td>P7</td>
<td>0.1893</td>
<td>3.0686</td>
<td>0.32588</td>
</tr>
</tbody>
</table>

The second step is to apply sequential path supplementing method to obtain all possible states of the considered interconnected power system as follow:

1. Consider the first possible path "P1" and calculate component set "E1" and path space set "S1" with respect to Fig. (1-9)

   \[ E_1 = \{ G_1, L_1 \} \]

   i.e., when path \( P_1 \) is supplemented, the component set is \( E_1 \). This means that \( G_1 \) & \( L_1 \) are in operation and the path set is \( S_1 = \{ P_1 \} \) and commutative state

   \[ S_1^* = \{ S_{O1}, P_1 \} \]

   Where:

   \( S_{O1} = \overline{P_1} \) is the not yet supplemented path set.
When the path $P_2$ is supplemented, the component set and complement elemental set are:

$E_2 = \{ G_2, L_2 \}$

$E_2^* = \text{Commutative component set when } P_2 \text{ is supplemented.}$

$= E_2 \cup E_2$

$= \{ G_1, L_1 \} \cup \{ G_2, L_2 \}$

$= \{ G_1, G_2, L_1, L_2 \}.$

$\bar{E}_2 = \text{Complemented element set when } P_2 \text{ is supplemented}$

$= E_2 - E_2$

$= \{ G_1, L_1 \}$

each of $G_1 \& L_1$ has two states. Table (3) illustrates the states when $P_2$ is added.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_2$</th>
<th>$\bar{E}_2$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$G_2$</td>
<td>$L_2$</td>
<td>$G_1$</td>
</tr>
<tr>
<td>2</td>
<td>$G_2$</td>
<td>$L_2$</td>
<td>$G_1$</td>
</tr>
<tr>
<td>3</td>
<td>$G_2$</td>
<td>$L_2$</td>
<td>$G_1$</td>
</tr>
<tr>
<td>4</td>
<td>$G_2$</td>
<td>$L_2$</td>
<td>$G_1$</td>
</tr>
</tbody>
</table>

From table (3),

$S_2 = \{ P_2, P_1 \bigcap P_2 \}$ and $S_2^* = S_1 \cup S_2$

$= \{ S_{01}, P_1 \bigcap P_2 \}$

$= \{ S_{01}, P_1, P_2, P_1 \bigcap P_2 \}$

Where:

$S_{01}$ is the not yet supplemented path set = $P_1 \bigcap \bar{P}_2$

When path $P_3$ is added, $E_3$ and $S_3^*$ are:

$E_3 = \{ G_3, L_3 \}$

$E_3^* = E_3 \cup \bar{E}_2 = \{ G_1, G_2, G_3, L_1, L_2, L_3 \}$
\[ S_3 = E_3^* - E_3 = \{ G_1, G_2, L_1, L_2 \} \]

\[ S_3^* = S_3^* \cup S_3 \]

\[ = \{ S_{33}, P_1, P_2, P_1 \cap P_2 \} \cup \{ P_3, P_1 \cap P_2, P_2 \cap P_3, P_1 \cap P_2 \cap P_3 \} \]

Where:

\[ S_{33} = \overline{P_1} \cap \overline{P_2} \cap \overline{P_3} \]

[4] When path \( P_4 \) is added, its elements are \( E_4 \) and \( S_4^* \) are:

\[ E_4 = \{ G_1, L_4, L_1 \} \]

\[ E_4^* = E_4 \cup E_1 \]

\[ = \{ G_1, G_2, G_3, L_1, L_2, L_3 \} \cup \{ G_1, L_4, L_2 \} \]

\[ = \{ G_1, G_2, G_3, L_1, L_2, L_3, L_4 \} \]

\[ E_4 = E_4^* - E_4 = \{ G_1, G_2, G_3, L_1, L_4 \} \]

\[ S_4 = \{ P_4, P_1 \cap P_4, P_2 \cap P_4, P_3 \cap P_4, P_1 \cap P_2 \cap P_4, P_2 \cap P_3 \cap P_4, P_1 \cap P_2 \cap P_3 \cap P_4 \} \]

\[ S_4^* = S_4^* \cup S_4 \]

\[ = \{ S_{44}, P_1, P_2, P_1 \cap P_2, P_1 \cap P_4, P_2 \cap P_4, P_3 \cap P_4, P_1 \cap P_2 \cap P_4, P_2 \cap P_3 \cap P_4, P_1 \cap P_2 \cap P_3 \cap P_4 \} \]

Where:

\[ S_{44} = \overline{P_1} \cap \overline{P_2} \cap \overline{P_3} \cap \overline{P_4} \]
When path \( P_i \) is supplemented, \( E_4 \) and \( S_5^* \) are:
\[
E_4 = \{ G_1, L_1, L_2 \}
\]
\[
E_5^* = E_1^* \cup E_4 = \{ G_1, G_2, G_3, L_1, L_2, L_3, L_4 \}
\]
\[
S_5 = \{ P_1, P_1 \cap P_5, P_2 \cap P_5, P_2 \cap P_3 \cap P_5, P_1 \cap P_3 \cap P_5, P_1 \cap P_2 \cap P_5, P_1 \cap P_2 \cap P_3 \cap P_5, P_1 \cap P_2 \cap P_3 \cap P_5 \cap P_2 \cap P_3 \cap P_5 \}
\]
\[
S_5^* = S_4 \cup S_5
\]

The number of components are seven, i.e., the base events are \( 2^7 = 128 \) states. These states are reduced to \( S_5^* \) states = 21 state that should be calculated to get probability, frequency and duration for each state of the considered interconnected power system.

![State-space diagram](https://example.com/state-space-diagram.png)

**Fig. (2): State-space of paths-sets after merging states.**
The transitions rates between states are shown in the transition matrix \([A]\) given below.

The interconnected power system that represented in Fig. (a) is described by its states and by the possible transitions between them as seen in Fig. (2). The main advantage of the state-space approach is that in most cases a Markov model can be applied to describe the process of the system travelling through the states.

The major application of the state-space approach is the reliability calculation of repairable systems, that is, of systems where all the components are repairable.

If only the long term values of the state probabilities \(P_t(0)\) are of interest, they can be obtained by solving the set of linear equations. \([8]\):

\[
[P]\ [A] = [O]
\]

Where:

- \([P]\) = row - vector contains state probabilities.
- \([O]\) = row - vector contains zeros.
- \([A]\) = transition intensity matrix.

The solution for \(P\) requires an additional equation which is provided by the fact that the summation of probabilities of all states equals to 1, i.e.

\[
\sum_{i=1}^{n} P_i = 1
\]

\[
\begin{array}{c|cccc|c}
\text{State} & 1 & 2 & 3 & 4 & 5 \\
\hline
5.991 & 0.1295 & 3.718 & 3.446 & 2.798 \hline
\end{array}
\]

\[
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
P_4 \\
P_5 \\
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]
\[ \begin{array}{c|c|c|c|c|c}
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array} \]

Matrix: \( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \)
\[ \sum_{i=1}^{2} p_i = 1.0 \]

\[ d = 6.8254 \times 10^{05} \quad d_1 = 1.2343 \times 10^{05} \]
\[ d_1 = 1.6482 \times 10^{05} \quad d_2 = 1.2879 \times 10^{05} \]
\[ d_3 = 1.5576 \times 10^{05} \quad d_3 = 1.1182 \times 10^{05} \]
\[ P_1 = 0.2415 \quad P_2 = 0.2253 \]
\[ P_3 = 0.1208 \quad P_4 = 0.1886 \]
\[ P_5 = 0.1838 \]

After solving the state-space transition equation we have to define the failure criterion and perform the system path states according to such a failure criterion, then calculate reliability indices as follow:

- Combine all path states in subset \( W \) and also states in \( F \) subset.

Then \( P_f \) = probability of failure

\[ P_f = \sum_{i \in F} P_i \]  \hspace{1cm} \text{(1)}

\( F_f \) = frequency of failure

\[ \sum_{i \in F} \sum_{j \in W} \lambda_{ij} \]  \hspace{1cm} \text{(2)}

\( \lambda \) is the system failure frequency is the sum of the system failure state probabilities, each multiplied by the rate of transitions from the respective state to the success domain [5].

\( T_f \) = mean duration of stays in combined state \( F \); therefore

\[ T_f = \frac{P_f}{F_f} = \frac{\sum_{i \in F} P_i}{\sum_{i \in F} \sum_{j \in W} \lambda_{ij}} \]  \hspace{1cm} \text{(3)}

If we choose the failure criterion as, “the system is considered failed if only one path is in operation or less (FC-1)”, e.g.

\( P_1 \) only is in failure domain

\( P_1, P_2, P_3, \) and \( P_4 \) are in working domain.
\[ \rho_k = \sum_{i \in F} \beta_i = 0.1638 \]

\[ \bar{\Gamma} = \sum_{i \in F} \sum_{j \in W} \lambda_{ij} \]

\[ = 0.1638 \times 6.51264 - 0.1638 \times 3.91203 \]

\[ = 0.1638 \times 25.2358 + 0.1638 \times 2.5966 = 9.8209 \]

\[ T_F = \frac{\bar{\Gamma}}{\rho_k} = 0.01667 \]

If failure criterion is chosen as the system is considered failed if two or less paths are only working (FC-2),

\[ \rho_k = \sum_{i \in F} \beta_i = \beta_1 + \beta_4 = 0.1638 - 0.2253 = 0.3891 \]

\[ \Gamma_F = 0.1638 \times (25.2358 - 6.512 + 3.912) - 0.3891 \times (7.848 + 6.5808) = 92.4641 \]

\[ = 0.1638 \times 25.6994 - 9.3891 \times 16.862 = 5.84117 + 18.23 = 24.075 \]

\[ T_F = \frac{\Gamma_F}{\rho_k} = 0.01646 \]

If the failure criterion is chosen as the system considered failed when three paths or less are working, therefore (FC-3),

\[ \rho_k = \sum_{i \in F} \beta_i = \beta_1 + \beta_4 + \beta_3 \]

\[ = 0.1638 + 0.1886 - 0.1808 = 0.5532 \]

\[ \Gamma_F = \sum_{i \in F} \sum_{j \in W} \lambda_{ij} \]

\[ = 0.1638 \times (6.51264 - 3.91203) - 0.1886 \times (7.81839 + 6.58081 + 0.1808 + 4.23528 + 4.2313) \]

\[ = 1.7075 + 2.71508 - 1.5307 = 5.95528 \]

\[ \Gamma_F = \frac{\bar{\Gamma}}{\rho_k} = 0.5532 \]

\[ \Gamma_F = 0.00895 \]

If (FC4) the system is considered failed when 4 paths or less are working (FC-4).
\[ P_F = \sum_{i \in F} P_i = P_1 + P_3 + P_4 + P_2 \]

\[ = 0.163 \]

\[ S = 0.1866 + 0.1808 + 0.2233 = 0.7585 \]

\[ \lambda_F = \sum_{i \in F} \sum_{j \in W} \lambda_{ij} = 3.38 \]

\[ T_F = \frac{P_F}{\lambda_F} = \frac{0.7585}{3.38} = 0.2244 \]

Table 4: Fault criterion (FC) versus reliability indices

<table>
<thead>
<tr>
<th>FC</th>
<th>( P_F )</th>
<th>( \lambda_F )</th>
<th>( T_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1638</td>
<td>0.8269</td>
<td>0.1667</td>
</tr>
<tr>
<td>2</td>
<td>0.3891</td>
<td>14.0750</td>
<td>0.0261</td>
</tr>
<tr>
<td>3</td>
<td>0.5332</td>
<td>5.9528</td>
<td>0.0895</td>
</tr>
<tr>
<td>4</td>
<td>0.7585</td>
<td>3.3800</td>
<td>0.2244</td>
</tr>
</tbody>
</table>

With the same way, repeat the calculations, when link \( L_a \) is added, and the paths \( P_1 \) & \( P_2 \) are supplemented, the following reliability indices are calculated in Figs. (3-a, 5 and c).

**CONCLUSIONS**

This paper introduces an accurate algorithm to calculate the reliability indices of interconnected power systems based on state space method, the sequential path supplementation and power flow computations. The algorithm takes into account outage reliability data, operation constraints and system configuration.

The proposed algorithm reduces the mathematical calculation because of using path word instead of component word.

The proposed algorithm employed state space method to calculate reliability indices of an interconnected power system at different failure criteria as seen from figures.

The proposed technique is useful for planning engineers to check design alternatives according to the desired degree of reliability indices.
Fig. (3-a, b, and c): Reliability indices for the chosen three interconnected power systems.
REFERENCES:


