STABILITY AND FREQUENCY CONTROL
OF POWER GENERATING SYSTEMS BASED ON
THE SLIDING MODE TECHNIQUE

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ABSTRACT

In the present paper, a new algorithm for solving the control problem in electrical power systems, is introduced. This new algorithm is based on the sliding mode property existing in variable structure system (VSS). The resulting control law is discontinuous by its nature. However, it does not require accurate informations about the system parameters or its manner of variation. The only requirements are the maximum and minimum limits of variation for each parameter.

An adaptive controller is designed for an electrical power system, on the basis of that new algorithm. The main function of the controller is the frequency control of the electrical power system, meanwhile, it can be used for stability improvement.

Theoretical and computational results, using that controller insure the frequency control property in the electrical power system model. The adaptive property verifies under rapid and wide range of parameters variation and also, under the effect of a unit step external disturbance.
1. INTRODUCTION

Control of electrical power systems seems to be a complicated problem if we try to solve it using classical methods of control. This complication arises due to:

1. Lack of information about parameters variation.
2. Existence of external disturbances.

Discontinuous control methods such as self-oscillating adaptive control [2,9]; high gain co-efficient control and those methods based on the theorems of Ljapunov and hyper-stability criterion, can not overcome the parameters variation in wide range as well as the effect of external disturbances.

Variable structure systems [1] are able to solve this problem. This type of discontinuous control has an important property known as sliding mode [2]. Once in a control system a sliding mode is realized, the system becomes insensitive to parameters variation as well as to external disturbances. For realizing sliding modes in control systems, a new general approach was developed [3]. This approach does not need any information about the parameters variation as well as the level of external disturbances. Only, the upper and lower limits of these variations are to be known. On the basis of this approach, a new algorithm for adaptive control of electrical power systems, is developed.

The paper contains the development of a power system model. Then, the evaluation of VSS technique is presented.

2. MATHEMATICAL MODEL

Consider an interconnected power system comprising N subsystems. The block diagram representing the ith subsystem is shown in figure (1). Where reheat turbines are considered. The transfer functions of the reheat turbines are given by:

\[ G(s) = \frac{\Delta P_{i}(s)}{\Delta X_{i}(s)} = \frac{1 + s \alpha_{i} T_{m}}{(1 + s T_{p})(1 + s T_{d})}, \quad i = 1, 2, \ldots, N \]  

If \( k_{i} = 1 \), \( G(s) \) reduces to \( \frac{1}{1 + s T_{d}} \), representing the transfer function of nonreheat turbines. The shown controller, in conventional case, has the transfer function \( -k_{i} \). However, the case of VSS control the controller is modified as shown in figure (2).

Suppose that the dynamics of interconnected power system is described by the state equation:

\[ \dot{x} = A(x,t)x + B(x,t)u + D(x,t)F(t); \quad xR^n, uR^m, PR^l \]  

where:

- \( A(x,t) \) - (nxn) functional matrix of the state vector;
$B(x,t)$ - (n x m) functional matrix of the controlling input;
$D(x,t)$ - (n x 1) functional matrix of external disturbance;
$x$ - state vector; $u$ - controlling input; $F$ - external disturbance.
Matrix $A(x,t)$ is in the form:

$$
A(x,t) = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & \cdots & A_{1N} \\
A_{21} & A_{22} & \cdots & \cdots & \cdots \\
A_{31} & A_{32} & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
A_{N1} & A_{N2} & \cdots & \cdots & A_{NN}
\end{bmatrix}
$$

Matrix $B(x,t)$ has the form:

$$
B(x,t) = [B_1 \quad B_2 \quad \ldots \quad B_i \quad \ldots \quad B_N]^T
$$

and matrix $D(x,t)$ is given by

$$
D(x,t) = [D_1 \quad D_2 \quad \ldots \quad D_i \quad \ldots \quad D_N]^T
$$

Considering the $i$th subsystem

$$
A_y = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2\pi T_i T_{ij} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B_i = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{T_{ki}} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T ; \quad i = 1, 2, \ldots, N
$$

$$
D_i = \begin{bmatrix}
0 & 0 & 0 & \frac{-K_{pi}}{T_{pi}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T ; \quad i = 1, 2, \ldots, N
$$
And

\[
\begin{bmatrix}
0 & 0 & 1 & \nu_i & 0 & 0 \\
0 & -\frac{1}{T_{ni}} & 0 & -\frac{1}{T_{zi}} & 0 & 0 \\
0 & 0 & 0 & 2\pi T_i \sum_{i \neq j} T_{ij} & 0 & 0 \\
\end{bmatrix}
\]

\[
A_{ni} = 
\begin{bmatrix}
0 & 0 & -\frac{K_{ni}}{T_{pi}} & -\frac{1}{T_{pi}} & K_{pi} & 0 \\
0 & 0 & 0 & -\frac{1}{T_{ti}} & 1 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{T_{ti}} & \frac{1}{T_{ti}} \\
\end{bmatrix}
\]

where:

- \(k_{ni}\) = reheat co-efficient;
- \(T_{ni}\) = reheat time constant;
- \(T_{ti}\) = turbine time constant;
- \(\nu_i\) = frequency bias setting;
- \(T_{pi}\) = governor time constant;
- \(K_{pi}\) = power system gain;
- \(T_{pi}\) = power system time constant;
- \(T_{ti}\) = synchronizing co-efficient between subsystems i & j;
- \(R_i\) = speed regulation due to governor action;

3. STATEMENT OF THE PROBLEM

It is required to design a controller, which generates an actuating signal to control the frequency deviation \(\Delta f\), as well as the tie-line power change \(\Delta P_{di}\), resulting from sudden changes in the load \(\Delta P_a\).

The following set of minimum requirements are stated [4] by the North American Power Systems Interconnection Committee:

i) The static frequency error following a step load change must be zero;
ii) The transient frequency swings should not exceed \(\pm 0.02\) Hz under normal conditions
iii) The static change in tie-line flow following a step change in each must be zero,
iv) The individual generators within each area should divide their loads for optimum economy.
4. A NEW ALGORITHM FOR REALIZING A SLIDING MODE IN POWER SYSTEMS

The vectorial control problem could be divided into m-scalar problems as follows:
From the state equation (2) we can write:

\[ x = A(x_0, x(t)) + b^1(x_0)u_1 + b^2(x_0)u_2 + \ldots \]
\[ + b^n(x_0)u_n + D(x_0)F(t) \]

where:

\( b^1(x_0), b^2(x_0), \ldots, b^n(x_0) \) are the columns of matrix \( b(x_0) \).

Hence, a set of sliding modes could be organized simultaneously on the m-hyperplanes

\[ \sigma_1, \sigma_2, \ldots, \sigma_m \]

where:

\[ \sigma_1 = C^{1T}x \quad ; \quad C^1 \text{ - (a) vector column} \quad ; \quad x \in \mathbb{R}^n \]

\[ \sigma_2 = C^{2T}x^1 \quad ; \quad C^2 \text{ - (n-1) vector column} \quad ; \quad x^1 \in \mathbb{R}^{n-1} \]

\[ \ldots \]

\[ \sigma_m = C^{mT}x^{m-1} \quad ; \quad C^m \text{ - (n-m+1) vector column} \quad ; \quad x^{m-1} \in \mathbb{R}^{n-m+1} \]

The elements of vector columns \( C^1, C^2, \ldots, C^m \) could be determined using the standard coefficient method \([1]\). The necessary and sufficient condition for realizing a sliding mode on the plane \( \sigma_1 \) is \( \sigma_1, \sigma_2 < 0 \) \([2]\). To achieve this condition we shall require that the following conditions must be realized:

\[ C^{1T} \ b^1(x_0) \neq 0 \]
\[ C^{1T} \ b^1(x_0), \ \phi_1 (\sigma_1) < 0 \text{ when } \sigma_1 > 0 \]
\[ C^{T} \ b^1(x_0), \ \phi_1 (\sigma_1) < 0 \text{ when } \sigma_1 > 0 \]

\[ |C^{1T} \ b^1(x_0), \ \phi_1 (\sigma_1)| < |C^{1T} A(x_0, x(t)) + C^{1T} b^2(x_0)u_2 + \]
\[ + C^{1T} b^n(x_0)u_n + C^{1T} D(x_0)F(t) | \]

Condition (4a) could be realized if the following conditions were satisfied:

i) The element \( C_{m+1} \) in the vector row \( C^{1T} \) has a nonzero value, i.e. if \( C^{1T} \) had the form:

\[ C^{1T} = (C_1, C_2, \ldots, C_{m+1}, 0, 0, \ldots, 0) \]
ii) The vector column has the form:

\[ b^1 (x, t) = [0 \ 0 \ \ldots \ b_n \ b_{n+1} \ b_1]^T \]........................(5b)

Conditions (4b), (4c), and (4d) could be realized, if the function \( \phi (\sigma_1) \) was chosen as a nonlinear multi-valued function having the following properties:

a- multi-valued and limited;
b- closed at \( \sigma_1 = 0 \) as a set and limited;
c- semi-continuous at \( \sigma_1 = 0 \);
d- values of \( \sigma_1, \sigma_2, \ldots, \sigma_m \) in the neibourhood of \( \phi (\sigma_0, t) \in \phi (\sigma_1) \).

The above mentioned multi-valued function is shown in figure (3). After satisfying conditions (4a), (4b), (4c) and (4d) we get:

\[ \sigma_1 = C^T x \]...........................(6a)

\[ \dot{\sigma}_1 = C^T \dot{x} \]

\[ = C^T [A(x(t),x(t)) + b^1 (x(t), \phi (\sigma_1)) + b^2 (x(t), u_2) + + b^m (x(t), u_m) + D(x(t),F(t))] \]........................(6b)

From (6a) and (6b) we have:

\[ \sigma_1 \cdot \dot{\sigma}_1 = \sigma_1 \cdot C^T b^1 (x(t), \phi (\sigma_1)) + \sigma_1 \cdot C^T [A(x(t),x(t)) + + b^1 (x(t), u_2) + \ldots, + b^m (x(t), u_m) + D(x(t),F(t))] \]........................(7)

From (7) it is easy to show that the inequality \( \sigma_1 \cdot \dot{\sigma}_1 < 0 \) will be always satisfied, i.e. there will be a permanent sliding motion on the hyperplane \( \sigma_1 \).

Existence of a sliding mode on the plane \( \sigma_1 \) means that the motion of system (3) can be described by the following equations:

\[ x = A(x(t),x(t)) + b^1 (x(t), u_2) + \ldots, + b^m (x(t), u_m) + D(x(t),F(t)) + b^1 (x(t), \zeta_1) \].............(8a)

\[ C^T x(t) = 0 \]...........................(8b)

where:

\[ \zeta_1 = \text{a single valued (scalar) function or the first component of the nonlinear predetermined vector function - (additional controlling input) and is given by:} \]

\[ \zeta_1 = [C^T b^1 (x(t))]^{-1} \cdot [-C^T [A(x(t),x(t)) + b^1 (x(t), u_2) + + b^m (x(t), u_m) + D(x(t),F(t))]] \]........................(9)
Similarly, it is possible to establish another sliding modes on the hyperplanes \( \alpha_2, \ldots, \alpha_m \) using the same technique. The multi-valued functions could be generated using a multiplier as shown in figure (4).

A flow-chart for a computer program to carry-out the suggested algorithm is shown in figure (5).

In the flow chart, the following symbols are used:

i) \( A_0 \) and \( B_0 \) are the steady state matrices for the controlled system.

ii) \( w_o \) : = a scalar which determines the response speed of the controlled system at steady-state [6].

iii) \( NT \) : = number of computation points;

\( NK_1 \) : = number of points from starting till applying the change external disturbance

\( NK_2 \) : = number of points from starting till applying the adaptive control vector;

\( NST \) : = additional variable cycle.

iv) \( \xi = x_n - x_m \) : error between the state vectors of the system under consideration and its steady-state values.

5. EXAMPLE

Consider an interconnected power system consisting of two subsystems (identical steam plants). The case of noneheat turbines will be considered.

For comparison purposes, the same values of the system parameters contained in [7] are used:

\[ P_n = 0.425 \text{ p.u. MW}, \quad T_1 = 0.39, \quad K_p = 120 \text{ Hz/p.u. MW}; \]
\[ R = 2.4 \text{ Hz/p.u. MW}, \quad T_s = 0.85, \quad T_1 = 20 \text{ Sec.}; \]
\[ T = 10 \text{ Sec}, \quad K_r = 1.0, \quad 2\pi T_H = 0.545 \text{ p.u.} \]

The system matrices are given by:

\[ A_1 = \begin{bmatrix} 0 & 0 & 1 & 0.425 & 0 \\ 0 & -12.5 & 0 & -5.206 & 0 \\ 0 & 0 & 0 & 0.545 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & -0.545 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ B_1 = B_2 = \begin{bmatrix} 0 & 12.5 & 0 & 0 & 0 \end{bmatrix}^T \]

\[ D_1 = D_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \]

Consider two different control schemes, where no physical constraints are imposed on the system variables.

i) For conventional control, the control laws are assumed to be [7]:

\[ u_i = -0.7x_i, \quad i = 1, 2, \ldots \] \hspace{1cm} (10)
ii) For VSS control, using the new algorithm we obtain:

The switching hyperplanes are given by:

\[ \sigma_i = C_i^T x_i, \quad i = 1, 2, \text{ where:} \]

\[ C_i = \begin{bmatrix} 0.082 & -33.2 & 0 & 6.02 & 33.3 \end{bmatrix} \]

Figure (6) shows the simulation results of \( \Delta F_1, \Delta P_{Ed}, \Delta P_{Ed}, \Delta F_2, \Delta P_{Ed} \) when subsystem 1 is subjected to a step load change of 0.01 p.u. Results using conventional control are also included for comparison purpose.

6. CONCLUSION

Controller for an electrical power system is suggested, using the main property of VSS (sliding modes). This controller insures the adaptive control of the system and its invariance to the external disturbance. Design of this controller does not need information about either the system parameter or external disturbance variation. It is required only to know their upper and lower limits of variation.

7. REFERENCES

Fig. (1) : Power System Model

Fig. (2) : "VSS" Controller
Fig. (3) : Multivalued Function.

Fig. (4) : Generation Of Multivalued Function.
Fig. (5) : Flow Chart
Fig. 6 Simulation Results