Position and Velocity Analysis of Free-Ended Two-Link Object for Whole Arm Manipulation


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Abstract—this paper presents position and velocity analysis of free-ended two-rigid-links object manipulated by two arms as an initial step simplification of holding and manipulating a human body by a humanoid robot. Rolling as well as sliding at the points of contact of the two-links object constrained by the holding two-arms is explained. Position and velocity analysis of the manipulated object, position and velocity constraints as well as expressions for sliding displacement and sliding velocity are presented. Simulation results for sliding displacement, sliding velocity, object links displacements and velocities as well as object configurations for different input rolling velocities are presented. The presented position and velocity analysis proves that: 1- through manipulating two-links object by two-arms, rolling at one point or both points of contact is associated with sliding at both points of contact. There is no rolling without sliding and there is no sliding without rolling. 2- It is possible to define the configuration of the object from contact points and hence contact angle measurements using tactile sensors. 3- It is possible to compute the amount of sliding at both contact points from contact angle measurements. 4- It is possible to define the object links velocity as well as sliding velocity from the contact rolling velocities estimations or measurements.

I. INTRODUCTION

Human interactive robotics has received a considerable attention by different research groups seeking their promising human-friendly assist and cooperation. Among these groups is RIKEN robotics research group who introduced RI-MAN for holding and transferring a patient, [1], [2]. RI-MAN is designed to grasp, hold and transfer a human applying its whole two-arms, Fig. 1. Whole arm manipulation is an approach to manipulation that employs all the available manipulation surfaces of the robot to act upon and sense the environment, [3]. Restraining large size objects, lifting heavy loads or assembling mating parts are examples of suitable tasks that require whole arm manipulation. Whole arm grasps are formed by wrapping the arms (or fingers) around the objects, [4]. Manipulating a human, (the task is a patient), as shown in Fig. 1, lies in the taxonomy of dynamic manipulation in which task dynamics is significant for analysis and planning. Kinematic, static and quasi-static manipulation analysis is important steps for dynamic manipulation. Dynamic manipulation has many benefits such as increasing the repertoire of actions to manipulators, increasing

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the speed of manipulation and saving the
complexity and the mass of the robot, [5]. With
dynamic manipulation, some of the complexity of
the robot system is transferred from the hardware
(joints and actuators) to planning and control, [6].
Many research works for different dynamic
manipulation tasks can be found in the literature.
Motion planning of a 1-DOF dynamic pitching
robot throwing a ball in a horizontal plane is
presented in [7]. Dynamic manipulation of an
object over a plate inspired by the handling of a
pizza peel is presented in [8]. Motion planning
and controllability for nonprehensile (without a form-
or force-closure) manipulation for throwing and
catching a disc using two planar manipulators are
presented in [9]. Carrying a polygonal shape
object applying two mobile robots with
nonprehensile manipulators has been presented in
[10]. So far the presented work is concerned with
one-link object to be manipulated by one or two
manipulators. However an application like holding
and/or transferring a human, (patient for instance),
is an example of multi-link object which is not
tackled in research literature, (up to the author’s
knowledge), in robotics manipulation. To define,
apply and assure stable holding and manipulating
as well as control, it is important to analyze and
understand kinematics and dynamics of the
manipulated multi-link object. As an initial step
towards analysis of holding and manipulating a
human body object, this paper introduces
two-dimensional position and velocity analysis of
free ended two-link rigid body object hold by two
arms, Fig. 3. Kinematic relations as well as
kinematic constraints are to be introduced.
Through analysis and simulation, it is aimed to
answer the following questions:

1- given the points of contact, (and hence the
angles of contact), for a two-link object held by
two-arms, at any instant:
   a) Is it possible to get information about the
      object configuration?
   b) Is it possible to get information about sliding
      amounts of the object links over the arms?
   c) How are rolling and sliding displacements
      associated with each other?

2- given the rate of change of angles of contact or
   rolling velocity:
   a) Is it possible to get information about
      object-links different velocities?
   b) Is it possible to get information about the
      sliding velocities of links over arms?
   c) How are links sliding velocities associated
      with rolling velocities?

This paper is organized as follows: rolling and
sliding of one link object over one arm in a plane is
presented in section II. Position analysis of
two-link object constrained by two-arms with
rolling and sliding motions, position constraints,
sliding displacement expressions and position
simulation results are introduced in section III.
Velocity analysis of two-link object constrained by
two-arms, velocity constraints, sliding velocity
expressions and the velocity simulation results for
different rolling velocities are introduced in section
IV. Conclusion and directions for future work are
to be presented in section V.

II. ROLLING AND SLIDING OF ONE LINK OBJECT

In this section, rolling and/or sliding of a stem
rigid-link object over a rigid-link arm system, Fig.
2 is explained. Contact type is assumed to be hard
contact. The coordinate system is introduced first
followed by explanation of the different possible
rolling and/or sliding motions.

A. Coordinate system

The coordinate system is defined as shown in
Fig. 2. A reference coordinate frame 0, O_0 − x_0 y_0
is defined at the center of the circular cross section of the arm; two frames \( S_{11} \) and \( S_{1c} \) both initially having the origin at the point of contact between the link and the arm, \( s_1 \) are also defined. Frame \( S_{11} \), \((O_{11} - x_{11}y_{11})\), is fixed to the link while frame \( S_{1c} \), \((O_{1c} - x_{1c}y_{1c})\), is at the point of contact between the link and the arm; \( x_{1c} \) is in the direction of line from \( O_0 \) to \( s_1 \) which is also normal to the plane including the point of contact as shown in Fig. 2. Frame \( e \), \((O_e - x_e y_e)\) is attached to the right end of the arm. The angle \( \theta_1 \), denoted as the angle of contact, is measured from \( x_0 \) to \( x_{1c} \) and being positive counter clockwise; the length \( l_1 \) expresses link segment length from the origin of frame \( S_{1c} \) to the origin of frame \( e \). The total length of the object link is \( a_1 \); Clearly \( l_1 \) is a variable while \( a_1 \) is a constant.

The rolling and/or sliding motion between the link object and the arm can be clarified in terms of the relative position of the two reference frames \( S_{11} \) and \( S_{1c} \) before and after the motion. After a motion of rolling and/or sliding, frames \( S_{11} \) and \( S_{1c} \) are denoted by \( S'_{11} \) and \( S'_{1c} \) respectively. Knowing the initial angle of contact \( \theta_1(0) \) and the initial length \( l_1(0) \) and defining the motion, (rolling and/or sliding), the change in angle of contact \( \Delta \theta_1 \) and the change in length \( \Delta l_1 \) can be defined. To explain the motions, three cases are considered: 1) sliding of the link over the arm (without rolling); 2) rolling of the link over the arm (without sliding); and 3) both sliding and rolling of the link over the arm.

**B. Sliding (without rolling)**

Assuming that the link has the possibility to slide (without rolling) over arm, then the contact point after sliding will remain at its location before sliding, \( S'_{1c} \) will be identical to \( S_{1c} \). Hence the angle of contact after sliding, \( \theta_1 \), will be equal to the initial angle of contact, \( \theta_1(0) \), then

\[
\Delta \theta_1 = \theta_1 - \theta_1(0) = 0
\]

Moreover the frame on the link side \( S_{11} \) as well as the whole link will move according to the amount of sliding displacement, \( d_1 \). \( S_{11} \) will become in a new position \( S'_{11} \). The length \( l_1 \) after sliding, will depend on its initial length, \( l_1(0) \), and amount of sliding \( d_1 \).

\[
\Delta l_1 = l_1 - l_1(0) = d_1
\]

**C. Rolling (without sliding)**

Assuming that the link has the possibility to roll without sliding, then both \( S_{1c} \) and \( S_{11} \) will be identical and move on the circle of radius \( r \), (the peripheral of the arm), with the amount of rolling distance, \( r \Delta \theta_1 \), to another position denoted by \( S'_{1c} \) and \( S'_{11} \). The change in the angle of contact \( \Delta \theta_1 \) is a
function of the angle of contact after rolling, \( \theta_1 \), and the initial angle of contact, \( \theta_1(0) \),
\[
\Delta \theta_1 = \theta_1 - \theta_1(0)
\]
(3)
The change in length because of rolling can be defined as:
\[
\Delta l_1 = l_1 - l_1(0) = -r \Delta \theta_1
\]
(4)

D. Sliding and rolling

Assuming that the link has the possibility to slide and to roll, then both \( S_{1e} \) and \( S_{1l} \) will move to different new positions according to rolling angle and sliding displacement. \( S_{1e} \) will become \( S'_{1e} \) according to amount of roll, \( r \Delta \theta_1 \), while \( S_{1l} \) become \( S'_{1l} \) according to the amount of sliding, \( d_1 \). The new length \( l_1 \) will depend on \( r \Delta \theta_1 \) as well as \( d_1 \) and the initial length \( l_1(0) \). Expressions for change in the angle of contact and the change in length will be the summation of both the sliding case and the rolling case;
\[
\Delta \theta_1 = \theta_1 - \theta_1(0)
\]
(5)
The change in length because of both sliding and rolling can be defined as:
\[
\Delta l_1 = l_1 - l_1(0) = d_1 - r \Delta \theta_1
\]
(6)

III. Position Analysis of Constrained Two-Link Object

Understanding the kinematics and dynamics of the human body as an object manipulated by the whole arms of a humanoid robot is important for stable manipulation and control. As an initial step simplification of this difficult problem, the author considers a two-dimensional system of two slender-rigid-link object constrained by two-arms as shown in Fig. 3. In this section position analysis is presented. Sliding and/or rolling motion of two-rigid-link object constrained by two arms is to be presented.

The object is a two free-ended rigid-links, (link 1 and link 2), with total lengths \( a_1 \) and \( a_2 \) connected with a passive revolute joint. The two-link object is constrained by two arms, (right and left arms). Link 1 has a point contact with the cylindrical right-arm at \( s_1 \) while link 2 has a point contact with the cylindrical left-arm at \( s_2 \). For the purpose of analysis, the base coordinate frame, (frame \( b \)), \( o_b - x_b y_b \), the right arm coordinate frame, (frame \( r \), \( o_r - x_r y_r \), left-arm coordinate frame, (frame \( l \), \( o_l - x_l y_l \)), contact points coordinate frames, (frame \( s_1 \), \( o_{s_1} - x_{s_1} y_{s_1} \) and (frame \( s_2 \), \( o_{s_2} - x_{s_2} y_{s_2} \) ), (frame \( 3 \), \( o_3 - x_3 y_3 \)) are defined as shown in Fig. 3. The angles of contact, \( \theta_1 \) is the angle from the \( x \)-axis of frame \( b \) to the \( x \)-axis of frame \( s_1 \) while the angle of contact \( \theta_2 \) is the angle from the \( x \)-axis of frame \( b \) to the \( x \)-axis of frame \( s_2 \), Fig. 3, both being positive in a counter clockwise sense. \( l_1 \) and \( l_2 \) define link segment lengths from origins of frames \( s_1 \) and \( s_2 \) at the points of contact to the passive joint, origin of frame \( 3 \), respectively. It is aimed to get for a two-link manipulator object, constrained as shown in Fig. 3, the constraining relation between \( l_1, l_2, \theta_1, \theta_2 \).

A. Position of passive joint 3

The position of the passive joint, frame \( 3 \), through the right-arm branch, can be described as:
\[
p^3 = p^2 + r^b_{r s_1} + R^b_{r (s_{13})}
\]
(7)
where \( p^b \) is the position vector from the origin of frame \( b \) to the origin of frame \( r \) expressed in frame \( b \); \( r^b_{r s_1} \) is the position vector from origin of frame \( r \) to origin of frame \( s_1 \) expressed in frame \( b \); \( R^b_{r (s_{13})} \) is the position vector from origin of frame \( s_1 \) to the origin of frame \( 3 \) expressed in frame \( s_1 \); and \( R^b_{r (s_{13})} \) is the rotation matrix from frame \( s_1 \) to frame \( b \).

![Fig. 3. Coordinate systems of free-ended two-link object by constrained by two-arms](image)

B. Constrained sliding and rolling:

In a similar way, the position of the passive joint, frame \( 3 \), through the left-arm branch, can be
described as:
\[ p^b_3 = p^b_1 + r^b_{1,3} + r^b_{2,3} \tag{8} \]
where \( p^b_1 \) is the position vector from the origin of frame \( b \) to the origin of frame \( l \) expressed in frame \( b \); \( r^b_{1,3} \) is the position vector from origin of frame \( l \) to the origin of frame \( s_2 \) expressed in frame \( b \); \( r^b_{2,3} \) is the position vector from the origin of frame \( s_2 \) to the origin of frame \( 3 \) expressed in frame \( s_2 \); \( R^b_2 \) is the rotation matrix from frame \( s_2 \) to frame \( b \). The relative position of the two arms can be expressed as:
\[ p^b_r = p^b_1 + R^b_r r^r_{1,3} \tag{9} \]
where \( r^r_{1,3} \) is the position vector from the origin of frame \( r \) to the origin of frame \( l \) expressed in frame \( r \); and \( R^b_r \) is the rotation matrix from frame \( r \) to frame \( b \). Substituting (9) into (8) and equalizing (8) and (7), then
\[ R^b_{1,3} r^b_{1,3} + R^b_{2,3} r^b_{2,3} = R^b_r r^r_{1,3} + l^b_{1,3} + R^b_r r^r_{2,3} \tag{10} \]
from which, the following constraint equation can be derived,
\[ \begin{bmatrix} s \theta_1 & s \theta_2 \\ -c \theta_1 & -c \theta_2 \end{bmatrix} \begin{bmatrix} l_{1,3} \\ l_{2,3} \end{bmatrix} + \begin{bmatrix} x_{r,3} - r_a (c \theta_1 - c \theta_2) \\ y_{r,3} - r_a (s \theta_1 - s \theta_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{11} \]
where \( s \theta_i = \sin(\theta_i) \); \( c \theta_i = \cos(\theta_i) \); \( x_{r,3} \) and \( y_{r,3} \) are the components of position vector \( r^r_{2,3} \) in \( x \) and \( y \) directions respectively; \( r_a \) is the radius of both right-arm and left-arm cross section.

**C. Configuration determination from tactile sensors measurements**

Determination of the configuration of the manipulated object is important in dynamic manipulation planning and control. Human do that applying his vision as well as his skin force/position sensors. According to the constraint equation (11), it is possible to define the configuration from tactile sensor measurements. Constraint equation (11) defines the lengths \( l_1, l_2 \) in terms of the contact angles \( \theta_1, \theta_2 \) and the relative position of the two-manipulating arms. For given arm size, \( r_a \), relative position, \( R^b_r \), of the two-arms; if it possible to measure (or estimate) the angles of contact \( \theta_1 \) and \( \theta_2 \), it is possible to determine the lengths \( l_1, l_2 \) which completely define the configuration of the constrained two-links object. Constraint equation (11) clarifies that if roll happens, the configuration will change. Also, in another way, if the configuration of the object is changed, the angles of contact will be changed. Measurements of the angles of contact can be obtained through the tactile sensors proposed by another research team in our institute for the humanoid robot RI-MAN, [11].

**D. Rolling-sliding relation**

It is important to define the sliding-rolling relation for future study of planning and control for the current problem of dynamic manipulation. As shown in section II, (equation 6), sliding, \( d_i \), is a function of the change in angle of contact, \( \Delta \theta_i \) and the change in length \( \Delta l_i \) for any two system states. The amount of sliding, \( d_i \) (referred to the frame \( s_j \)), can be expressed as:
\[ \{d_i = \Delta l_i + r \Delta \theta_i; \ i = 1 \} \{d_i = \Delta l_i - r \Delta \theta_i; \ i = 2 \} \tag{12} \]

Given the angles of contact \( \theta_i (i = 1, 2) \), and hence the lengths \( l_i (i = 1, 2) \), from equation (11), for any two different instants, the change in length \( \Delta l_i \), and the change in angle of contact \( \Delta \theta_i \) can be obtained, from which the amount of sliding \( d_i (i = 1, 2) \), can be estimated at both points of contact. Equation (12) assures that for manipulating two-links by two-arms both rolling and sliding are associated with each other. If roll happened over one or the two arms, sliding at the contact points exist. If sliding at the contact points happened, roll also exists at one or two of contact points. So, if the amount of roll is defined, (as it can be estimated through tactile sensor measurements), the sliding can be defined by (12).
E. Position Simulation Results

A program is implemented in MATLAB to simulate the governing equations of the system presented in this section. Three cases are considered: 1) rolling of link 1 around right arm only, (point of contact $s_1$ and so the angle of contact $\theta_1$ are changing); the rate of change of the angles of contact considered for simulation are $\dot{\theta}_1 = -30 \text{ deg/s}$ and $\dot{\theta}_2 = 0 \text{ deg/s}$; 2) rolling of link 2 around left arm only, (point of contact $s_2$ and so the angle of contact $\theta_2$ are changing); the rate of change of the angles of contact considered for simulation are $\dot{\theta}_1 = 0 \text{ deg/s}$ and $\dot{\theta}_2 = 40 \text{ deg/s}$; 3) rolling of link 1 around right arm and rolling of link 2 around left arm, (points of contact $s_1$ and $s_2$ and angles of contact $\theta_1$ and $\theta_2$ are all changing); the rate of change of the angles of contact considered for simulation are $\dot{\theta}_1 = -30 \text{ deg/s}$ and $\dot{\theta}_2 = 40 \text{ deg/s}$; Simulation parameters are shown in Table I. Simulation results of case 1, case 2 and case 3 are shown in Figs. 4, 5 and 6 respectively.

Shown in Figures 4a, 5a and 6a, the amount of roll, length and sliding displacements of link 1 around right-arm, for the three different cases, (different rates of change of angles of contact). Also shown in Figures 4b, 5b and 6b, the amount of roll, length and sliding displacements of link 2 around left arm, for the three different cases, (different rates of change of angles of contact as above). These figures assure the association of sliding to roll. Even roll may happen around one arm, the associated sliding exist at both points of contact. The roll is associated with sliding and sliding is associated with rolling.

Shown in Figures 4c, 5c and 6c the position of the passive joint between the two-link object. The positions of passive joint in addition to the contact points completely define the two-link object in the plane. Shown also in Figures 4d, 5d and 6d the configuration changes for the three cases of different contact angles. These figures show that if the angles of contact are measured or estimated by a tactile sensor, the configuration of the object, (which is important for future study of stable dynamic manipulation of two-link object by two-whole-arms), is completely defined.

IV. VELOCITY ANALYSIS OF CONSTRAINED TWO-LINK OBJECT

In this section, the analysis of two-link object constrained by two-arms is extended to include object links’ velocities. Velocity of the passive joint is expressed, the velocity constraint equation, the sliding velocity expressions and the simulation results are introduced.

A. Velocity of passive joint 3

Differentiating the position equation (7), the velocity of the passive joint can be obtained,

$$v^p_3 = v^p_2 + r^b_{r1} + R^b_{s1} s^s_{s1} + R^b_{s1} s^s_{s1}$$  

where $r^b_{r1} = r_a \dot{\theta}_1 [-s\theta_1 \quad c\theta_1 \quad 0]'^T$;

$$R^b_{s1} (\theta_1) = \begin{bmatrix} -s\theta_1 & -c\theta_1 & 0 \\ c\theta_1 & -s\theta_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$r^s_{s1} = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}'$$ ;  

and

$$R^b_{s1} (\theta_1) = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (14) expresses the velocity of the passive joint 3, $v^p_3$, as a function of the angle of contact, $\theta_1$, rolling velocity, $\dot{\theta}_1$, the length from the point of contact to the passive joint, $l_1$, and its rate of change, (the links velocity), $l_1$, as well as the velocity of the right arm, frame $r$, expressed in base frame $b$, $v^b_R$.

B. Velocity constraint equation

Now we would like to get the constraint equation for velocity. In a similar way to above, differentiation is executed for the position of the passive joint, (through the left-arm contact point), of equation (8) from which it is possible to obtain the following velocity equation:

$$\begin{bmatrix} p^b_{3x} \\ p^b_{3y} \end{bmatrix} = \begin{bmatrix} p^b_{1x} \\ p^b_{1y} \end{bmatrix} + \begin{bmatrix} -r_a s\theta_2 + l_2 c\theta_2 \\ r_a c\theta_2 + l_2 s\theta_2 \end{bmatrix} \dot{\theta}_2$$

(15)
Fig. 4. Simulation results of free-ended two-link object constrained by two-arms for rolling around right-arm only, at $s_1$, a) roll, length and sliding displacements of link 1 around right-arm, b) roll, length and sliding displacements of link 2 around left-arm, c) position of joint 3, d) configuration changes.

Fig. 5. Simulation results of free-ended two-link object constrained by two-arms for rolling around left-arm only, at $s_2$, a) roll, length and sliding displacements of link 1 around right-arm, b) roll, length and sliding displacements of link 2 around left-arm, c) position of joint 3, d) configuration changes.
Differentiating the relative position of the two arms, equation (9), the following rate equation is obtained:
\[ \dot{p}_{r}^a = \dot{p}_{r}^b + \dot{R}_{r}^b r_{r} + R_{r}^b \dot{r}_{r} \] (assuming \( R_{r}^b = l_3 \)),
then
\[
\begin{bmatrix}
    \dot{p}_{r}^a_x \\
    \dot{p}_{r}^a_y
\end{bmatrix}
= \begin{bmatrix}
    \dot{p}_{r}^b_x \\
    \dot{p}_{r}^b_y
\end{bmatrix} + \begin{bmatrix}
    \dot{x}_{r,l} \\
    \dot{y}_{r,l}
\end{bmatrix}
\] (16)
Substituting (16) into (15) and equalizing the right hand side of the result to that of (14), then
\[
\begin{bmatrix}
    s\theta_1 & s\theta_2 \\
    -c\theta_1 & -c\theta_2
\end{bmatrix}
\begin{bmatrix}
    l_1 \\
    l_2
\end{bmatrix}
+
\begin{bmatrix}
    r_0 s\theta_1 + l_1 c\theta_1 & -r_0 s\theta_2 + l_2 c\theta_2 & \theta_1 \\
    -r_0 c\theta_1 + l_1 s\theta_1 & r_0 c\theta_2 + l_2 s\theta_2 & \theta_2
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_{r,l} \\
    \dot{y}_{r,l}
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    0
\end{bmatrix}
\] (17)
Equation (17) expresses the constraint equation for velocity; for given velocities of rolling, \( \dot{\theta}_1 \) and \( \dot{\theta}_2 \), the links velocity, \( l_1 \) and \( l_2 \) are constrained by equation (17) because of the kinematic structure. \( \dot{x}_{r,l} \) and \( \dot{y}_{r,l} \) are the components of the relative velocity of the left arm with respect to the right arm in \( x \) and \( y \) directions. The same result can be obtained by differentiating the position constraint equation (11).

C. Sliding velocity estimation
Differentiating equation (12), sliding velocity \( \dot{d}_i \) can be obtained as a function of velocity of \( i^{th} \) link, \( l_i \) and velocity of \( i^{th} \) rolling velocity, \( \dot{\theta}_i \) as follows:
\[
\begin{align*}
\dot{d}_i &= l_i + r \dot{\theta}_i; \quad i = 1 \\
\dot{d}_i &= l_i - r \dot{\theta}_i; \quad i = 2
\end{align*}
\] (18)
Estimating the velocities of rolling \( \dot{\theta}_1, \dot{\theta}_2 \), the velocities of links, \( l_1, l_2 \), and sliding velocities \( d_1, d_2 \), can be estimated.

D. Velocity Simulation Results
The above equations (17) and (18), (in addition to the position and position constraint equations of section III), are simulated through an implemented MATLAB program. The independent input variables are the contact angle velocities, (\( \dot{\theta}_1 \) and \( \dot{\theta}_2 \)), and positions, (\( \theta_1 \) and \( \theta_2 \)). The rolling velocities are assumed as a linear function of time...
with different coefficients; \( \dot{\theta}_1 = -1.5 \ t \) [rad/s]; \( \dot{\theta}_2 = t \) [rad/sec]; the angles \( \theta_1 \) and \( \theta_2 \) are obtained through integration \( \theta_i(t) = \theta_i(0) - \Delta t + \dot{\theta}_i \Delta t \), \( i = 1, 2 \). \( \Delta t \) is the sampling time, assumed to be 0.01 s. In real time implementation, the angles of contact can be obtained from tactile sensor measurements and the rates of change can be estimated. The parameters for simulation are as in Table I. the relative velocity of the arms is assumed zero. The simulation results are shown in Fig. 7. The links' velocities and sliding velocities as well as the linear rolling velocity of link 1 over right-arm and the linear rolling velocity of link 2 over left-arm are shown in Figures 7a and 7b respectively. These results assure that if it is possible to estimate the velocity of rolling, it is possible to estimate the object links' velocities as well as the sliding velocities. Shown in Figures 7c and 7d, the position of the passive joint 3 and some configurations of the object respectively. Figures show that not only object configuration is detectable but also the object links' velocities.

V. CONCLUSION

Position and velocity analysis of free-ended two-link object held by two arms is presented. The main conclusions derived from the presented research work can be summarized as: 1- for dynamic manipulation of two-link object by two arms, rolling of a link about an arm is associated with sliding of the link over the arm at the point of contact. There is no rolling without sliding and there is no sliding without rolling; 2- through the measurements of angles of contact applying a tactile sensor, the configuration of the manipulated two-link object can be determined; 3- through estimating the rolling velocities, links' velocities and the sliding velocities at both points of contact can be estimated. Object configuration determination, sliding displacements and sliding velocities, would facilitate planning and control for dynamic manipulation of the current task.

FUTURE WORK

For dynamic analysis and control, it is important to analyze forces including gravity forces and friction forces at the points of contact. It is also important
to define stable manipulation and to control object sliding. So, force analysis as well as sliding control for stable manipulation will be of future interest.

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