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A NEW METHOD FOR DESIGNING THE SHOULDER OF PACKAGING MACHINES

طريقة جديدة لتصميم أكتاف التشكيل لماكينات التعبئة

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الخلاصة:

كتف التشكيل في ماكينة التعبئة هو الجزء الذي يقوم بتوجيه رقائق البلاستيك التي تصنع منها الأكياس ثم ثنيها على شكل أنبوب يتم لحامه وتعبئته ثم تقطيعه إلى أكياس. ويتم تصنيع الكتف عن طريق ثني رقيقة معدنية بمحاذاة منحنى محدد يسمى "منحنى الثني" الذي يتحكم في خصائص شكل الكتف كنصف قطره وارتفاعه وزاويته وغيرها. ومن المعروف أن الاختيار الجيد لهذه الخصائص هو الذي يتحكم في التوجيه الصحيح لرقائق البلاستيك وانسيابها على الكتف في نعومة دون تجعد. وبفحص الطرق الموجودة في المراجع والخاصة بتصميم منحنى الثني، وجد أن لها عدة عيوب منها أنها تعجز عن تحديد خصائص شكل كتف التشكيل في بعض الحالات أو تعطي حلولا محدودة لبعض الخصائص بينما تدع الخصائص الأخرى دون تحديد في البعض الأخر. إضافة الى انها لا تتسم بالمرونة بالنسبة للتغيير في خصائص شكل الكتف.

والبحث الحالي يناقش هذه الطرق كما يقدم طريقة جديدة من أهم سماتها أنها مرنة لأي عدد اختياري من خصائص شكل الكتف، كما أنها تتميز بالدقة والكفاءة. وهذه الطريقة تستخدم أسلوب B-Splines لنمذجة منحنى الثني، وأساليب المفاضلة للحصول على الحل الأمثل. وقد تم تطبيق هذه الطريقة على عدد من الأكتاف الحقيقية فأعطت نتائج مرضية مقارنة بالطرق الأخرى.

ABSTRACT

The shoulder of a packaging machine is the part which guides the packing material, plastic sheet for example, and folds it into the shape of a tube. It is manufactured through bending a piece of sheet metal along certain curve called bending curve. All geometry features of the shoulder are determined through the bending curve. The methods found in the literature to design the bending curve have some drawbacks. For many practical specifications of the shoulder geometry, these methods give no solution or give a solution for some specifications and let the other specifications uncontrolled. They are not flexible for new geometrical or mechanical specifications. The present work discusses these methods and introduces a new method, which is flexible for arbitrary number of specifications and gets the closest solution to the specified one. The method uses B-splines to model the bending curve and optimization techniques to get the

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optimal solution for certain specifications. The application of the proposed method to real shoulders gives satisfactory results in comparison with the other methods.

KEYWORDS

Computer Aided Design, B-splines, Optimization methods, Differential geometry, Isometric mapping.

1. INTRODUCTION

One of the most critical parts of packaging machines is the shoulder (see Fig. 1). The speed and the reliability of the machine depend to a great extent on the design and manufacture of the shoulder. Fig. 2 shows the working principal of the shoulder. The plastic sheet (the film) is drawn from the rollers and guided over and then through the shoulder to fold it into the shape of a tube. The sheet is sealed vertically and horizontally at the bottom to form a bag. The bag is then filled from above by the product to be packed. Then it is sealed at the top and cut off. The technological question is: which shape of the shoulder that ensures a correct undisturbed guidance of the film and how to manufacture it in a smooth way. The shoulder can be manufactured through marking the bending curve by a groove on the back of a piece of sheet metal (see Fig. 3). The lower part of the sheet is then bent progressively into smaller cylinders until the final cylinder is reached, whilst at the same time letting the upper part turn downward lower and lower to form the shoulder.

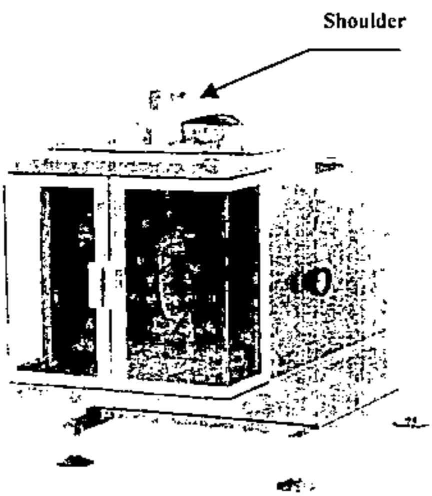

Fig.1. Packaging machine

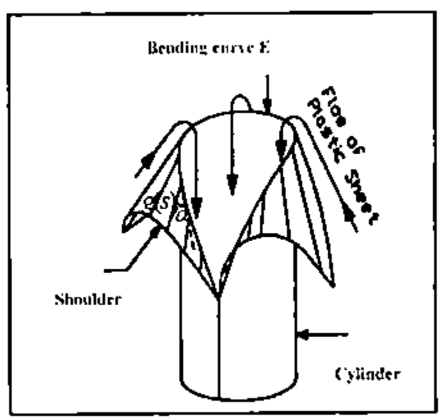

Fig. 2. The shoulder

Upon thorough search in the literature, only three works concerning the shoulder design are found. It is proved in these works [1 - 3] that all geometrical specifications of the shoulder are determined by the bending curve. In [1] the shoulder is proposed to be consisting of two truncated cones connected by a planar triangle. The geometrical parameters of the shoulder are the radius of the cylinder, R, the height, H, the angle between the shoulder and the cylinder at the heighest point, θ_0, and the open angle of the triangle, β, (see Fig. 3). An equation is proposed which determines the bending curve according to specified values of the above geometrical parameters. This equation fails to give a bending curve for some values of the geometrical parameters. Moreover, this equation does not tolerate for additional geometrical parameters. In [2], the idea of isometric correspondence between the shoulder and the plane is introduced. Using differential geometry an expression for the angle θ_0 is deduced. An equation is also proposed which considers the geometrical parameters R, H and θ_0. The plane triangle with its open angle β is not considered. The method has the same drawbacks as the previous one. In [3] a more general formulation of the problem is proposed. As in [2] the classical differential geometry with the idea of developable surface is utilized. The conditions of a shoulder surface free of singularities and containing a planar triangle are deduced. An equation for calculating the bending curve is proposed. The constants of the equation are determined to satisfy the following geometrical parameters: the ratio H/R, the angles θ_0, β, as defined above, and the angle θ_1 which is the angle between the shoulder and the cylinder at the lowest point. The equation is not flexible for additional geometrical parameters. One must add new terms to the equation and deduce from the beginning the relations of the constants. Moreover, this equation fails to provide solutions for many real shoulders which are presented in this work (see section 5). To overcome these drawbacks, two procedures are introduced. The first one is to model the bending curve by means of B-splines which possess many advantages. They offer one common mathematical form for free form shapes, provide the flexibility to design a large variety of shapes and can be evaluated reasonably fast by numerically stable and accurate algorithms [4, 5]. So, if one B-spline fails to get reasonable results, one can switch to another B-spline with other characteristics in the same program only by changing certain parameters. The second procedure is using an optimization technique to determine the optimal control points of a B-spline curve, which give the closest solution to the required one. This technique overcomes the problem of getting no solution for certain requirements.

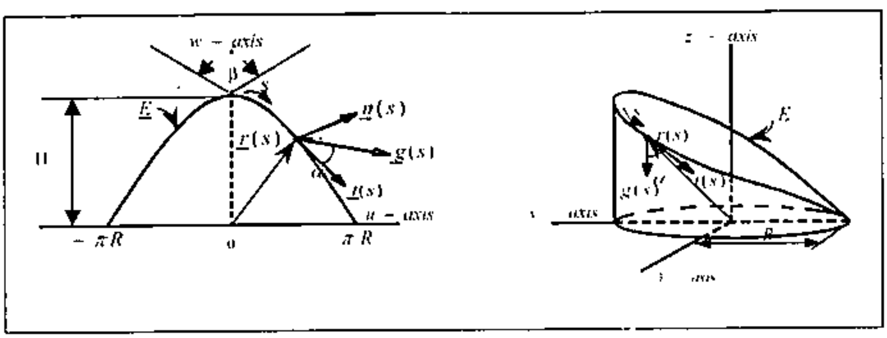

Fig. 3. Modeling of the bending curve with the shoulder

2. THE SHOULDER PROBLEM

The geometrical parameters of the shoulder are important because they determine the smoothness of the guidance of the plastic film over the shoulder into the cylinder. The angle θ_0 determines the amount of film bending. The planar triangle with angle β determines how perfectly the plastic film fits the shoulder. The angle θ_t determines the smoothness of the transition of the film from the shoulder to the cylinder at the lowest point. The ratio H/R is the most important parameter which determines the undisturbed guidance of the film. Although no research works are found which deal with the mechanics of the transport of the film over the shoulder into the cylinder, some industrial experience can give reasonable values of these parameters. The problem in industry is: for some desired values of these parameters, what is the matched bending curve required to manufacture that shoulder without stretching or tearing of the flexible plate. The problem has an interesting extension if the industry deals with developing new shoulder or manufacturing an existing one through reverse engineering. The relation between the bending curve and the geometrical parameters can be obtained by considering the shoulder surface as developable surface. The bending curve $\underline{E}$ (=w (u)) deforms into the bending curve E (=z (x, y)) on the cylinder (see Figs 2, 3), while the plane above $\underline{E}$ deforms into the shoulder and the plane under $\underline{E}$ deforms into the cylinder. All deformations take place without stretching or tearing. Using differential geometry [6], the shoulder is completely determined by a specification of its generators g. So, the parametric representations of the shoulder surface and its isometric correspondence (the plane above $\underline{E}$) are given by:

$$\mathbf{F} = \mathbf{F}(s,v) = \mathbf{r}(s) + v\mathbf{g}(s) \qquad (1)$$

$$\underline{\mathbf{F}} = \underline{\mathbf{F}}(s,v) = \underline{\mathbf{r}}(s) + v\underline{\mathbf{g}}(s) \qquad (2)$$

where s stands for arc length along the bending curves and v stands for arc length along the generators (g, $\underline{g}$). Equations (1, 2) describe also the cylinder and its isometric correspondence (the plane under $\underline{E}$). The generators g, $\underline{g}$ are given by:

$$\mathbf{g}(s) = \cos\alpha(s)\mathbf{t}(s) + \sin\alpha(s)[\cos\phi(s)\mathbf{n}(s) + \sin\phi(s)\mathbf{b}(s)] \qquad (3)$$

$$\underline{\mathbf{g}}(s) = \cos\alpha(s)\underline{\mathbf{t}}(s) + \sin\alpha(s)\underline{\mathbf{n}}(s) \qquad (4)$$

where **t**, **n**, **b** are the tangent, normal and binormal vectors to E while $\underline{\mathbf{t}}$, $\underline{\mathbf{n}}$ are tangent and normal vectors to $\underline{E}$. Angles α and ϕ can be found by equating the first fundamental coefficients of surface **F** to those of surface $\underline{\mathbf{F}}$ (properties of isometric surfaces [6]). One gets:

$$\cos\phi(s) = \frac{\underline{\kappa}(s)}{\kappa(s)} \qquad (5)$$

$$\tan\alpha(s) = \frac{-\kappa(s)\sin\phi(s)}{\varphi_s(s) + \tau(s)}, \qquad 0 < \alpha < \pi \qquad (6)$$

Subscript s denotes differentiation w.r.t. s. κ and $\underline{\kappa}$ are the curvatures of E and $\underline{E}$, respectively, while τ is the torsion of E. Equation (5) has two solutions, first one is for the shoulder and the second one is for the cylinder. For a cylinder of radius R, the curvature κ and torsion τ can be expressed as:

$$\kappa = \frac{(R^2 w_{uu}^2 + w_u^2 + 1)^{1/2}}{R(1 + w_u^2)^{3/2}} \qquad (7)$$

$$\tau = \frac{R^2 w_{uuu} + w_u}{R(R^2 w_{uu}^2 + w_u^2 + 1)} \qquad (8)$$

The curvature $\underline{\kappa}$ can be expressed as:

$$\underline{\kappa} = \frac{-w_{uu}}{(1 + w_u^2)^{3/2}} \qquad (9)$$

From equations (1-9) one can conclude that the shoulder surface F is completely defined by defining the bending curve $\underline{E}$ (w (u)). The angle θ between the tangent planes of the shoulder surface and the cylinder is consequently a function of the bending curve $\underline{E}$:

$$\cos\theta = -\frac{R^2 w_{uu}^2 - w_u^2 - 1}{R^2 w_{uu}^2 + w_u^2 + 1} \qquad (10)$$

In [3] the condition of a shoulder surface free of singularities (along which two different sheets are tangent to each other) is found to be:

$$R^3 w_{uuuu} + R w_{uu} < 0 \qquad (11)$$

Also it is proved in [3] that in order to have a plane triangle whose vertex lies on the highest point of the shoulder, there must be a discontinuity in the third derivative at that point on the bending curve $\underline{E}$. The opening angle β can be calculated as:

$$\tan(\beta/2) = -\frac{2R^2 w_{uuu}(0+)}{R^2 w_{uu}^2(0) + 1} \qquad (12)$$

3. MODELING OF BENDING CURVE BY B-SPLINES

The bending curve must satisfy the following conditions:

1- It must be an even function (w (u) = w (-u)) for symmetry consideration.
2- It must be continuous differentiable at least up to the third derivative except at the heighest point if a planar triangle is required.
3- It must be concave (w_{uu}<0)
4- It must satisfy equation (11) in order to have a shoulder surface free of singularities.

5- It must satisfy the boundary conditions w (πR) = w (-πR) = 0
6- It must contain a number of constants equals at least to the number of geometrical specifications.

In [3], the following equation is suggested:

$$w(u) = R\left[c_0 + c_2\lambda^2 + c_3|\lambda|^3 + c_4(\cos\lambda - 1 + \lambda^2/2) + c_5|\sin\lambda - \lambda + \lambda^3/6|\right] \quad (13)$$

where $\lambda = u/R, \qquad -\pi \le \lambda \le \pi$

This equation satisfies conditions (1, 2, and 6): the conditions (3, 4) must be checked after calculating the constants according to geometrical specifications and condition (5). In many practical cases, equation (13) gives no satisfactory solutions. So additional terms are needed in order to get better results. This adding process is not a flexible one. Firstly, one needs to keep conditions (1, 2) valid. Secondly, one must derive analytically the relations to be satisfied by the constants according to other conditions and geometrical specifications. So, the adding process is not a systematic one. One must write a computer program for each proposed equation. B-splines offer an efficient way to solve those problems. The process of changing the equation of the bending curve will now be a simple and systematic one in a single computer program and provides a more efficient way to get more accurate results. B-splines together with NURBS form the bases of computer aided geometrical design CAGD [5] which has applications in many areas, one of them is the reverse engineering [7]. B-spline curve is a piecewise polynomial curve. The bending curve w (u) can now be written as:

$$w(u) = \sum_{i=0}^{m} d_i N_i^n(u) \qquad (14)$$

The coefficients d_i are called "control points" or "de Boor points". They form the de Boor polygon (see Fig. 4). The normalized B-splines $N_i^n(u)$ are piecewise polynomials of degree n that are defined over the knots $\{u_0, u_1, \ldots, u_k\}$ They can be determined recursively as:

$$N_i^q(u) = (u - u_i)\frac{N_i^{q-1}(u)}{u_{i+q} - u_i} + (u_{i+q+1} - u)\frac{N_{i+1}^{q-1}(u)}{u_{q+i+1} - u_{i+1}} \qquad (15)$$

where

$$N_i^0 = \begin{cases} 1, & \text{if } u \in [u_i, u_{i+1}), \\ 0, & \phantom{\text{if }} u \notin [u_i, u_{i+1}). \end{cases}$$

Whereas zero is taken for the case 0/0. A B-spline curve is defined completely by its degree n, its control points and its knot sequence. The relation between the number of knots (k+1), the degree n and the number of control points (m+1) is:

$$k = m + n + 1 \qquad (16)$$

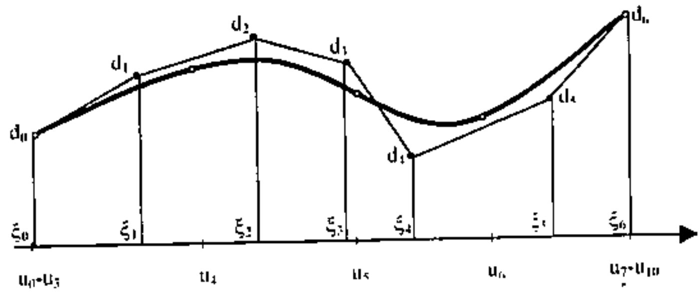

Fig. 4. B-spline control polygon

The first derivative of a B-spline curve of degree n over a partition $\{u_k\}$ is a B-spline curve of degree (n-1).

$$w_u(u) = \sum_{i=1}^{m} d_i^{(1)} N_i^{n-1}(u) \tag{17}$$

defined over the same partition, where

$$d_i^{(1)} = n\frac{d_i - d_{i-1}}{u_{i+n} - d_i} \tag{18}$$

For higher derivatives this scheme is to be repeated. The B-splines possess the following characteristics which are important in designing the bending curve:

a- If p knots $u_j = \ldots\ldots = u_{j+p-1}$ coincide (i.e. the knot j is of multiplicity p), the B-spline curve may become only C^{n-p} continuous at u_j.
b- The multiplicity n+1 is assigned to the first and last knots if the end control points required to be located on the curve.
c- B-splines possess the convex hull property. That is: any point of the curve lies in the convex hull of those control points that contribute to its evaluation.
d- If $x \in (u_l, u_{l+1})$, all $N_i^n(u)$ vanish at x except those with $i \in \{l-n, \ldots, l\}$.

The construction of bending curve using B-splines can now be done according to the following procedure:

1- To verify condition (1) of the bending curve, the number of control points (m+1) must be an odd number. The control points and the knot sequence must satisfy the following relations:

$$d_i = d_{m-i}, \qquad i = 0,\ldots\ldots,m$$
$$u_i = -u_{k-i}, \qquad i = 0,\ldots\ldots,k \tag{19}$$

II- To satisfy condition (5), one lets $u_0=\ldots\ldots=u_n=-\pi R$, consequently $u_{k-n}=\ldots\ldots=u_k=\pi R$ and $d_0=d_m=0$ (characteristic (b)).

III- To satisfy condition (2), one must let $n>3$ and $u_{(k+3-n)/2+i}=0$, $i=1,\ldots,n-2$ (characteristic (a)).

IV- To satisfy condition (3), the de Boor polygon must be concave relative to u-axis (characteristic (c)). This means:

$$A_i = \begin{vmatrix} \xi_{i-1} & \xi_i & \xi_{i+1} \\ d_{i-1} & d_i & d_{i+1} \\ 1 & 1 & 1 \end{vmatrix} < 0. \qquad i = 1,\ldots\ldots,m-1 \tag{20}$$

where ξ_i are the Greville abscissae (the u-coordinate of d_i, see Fig. 4) [4] which can be calculated from:

$$\xi_i = \frac{1}{n}(u_{i+1}+\ldots\ldots\ldots\ldots+u_{i-n}) \tag{21}$$

V- The other knots are made equal distance apart, unless the continuity is required to be less than (n-1) but not less than 3 between curve segments. Then the knots are repeated at the desired location.

VI- The number of control points (m+1) must be larger than or equal to (2G+1) where G is the number of geometrical specifications in order to satisfy condition (6). However, when optimization method is used (see next section) this condition can be relaxed.

VII- The control points (d_i, $i=1,\ldots,m/2$) are determined to get the closest values to the geometrical specifications and to satisfy conditions (3, 4) (see next section).

One can see that most of the bending curve characteristics are applied directly in a simple and systematic way. One can choose various B-spline degrees, various number of control points or various knot sequence (keeping the above-mentioned restrictions valid) to alter the equation of the bending curve (increasing its flexibility) without altering the process. That is to say, all possibilities of the bending curve are handled in the same program. Note that the derivatives of B-splines needed in the calculations are done systematically independent of the characteristics of the chosen B-spline. One can also change part of the B-spline curve without affecting the other parts through varying suitable control points (characteristic (d)). This has great advantages. So, one can adopt suitable values for θ_0 and θ_1 independently as an example. This flexibility increases the chance to get more accurate solution.

4. OPTIMIZATION TECHNIQUE

The geometrical specifications considered in this work are: R^*, H^* (or $(H/R)^*$), θ_0^*, θ_1^* and β^*. Note that R is satisfied directly (see procedure II above). To overcome the problem of getting no solution and also to determine systematically the control points of B-spline, an optimization problem is formulated which minimizes the squares of the errors (e^2) between the specified values and the calculated ones. The optimization problem can be defined as:

$$\min . f(\mathbf{D}) = W_1\left[(H/R)^* - (H/R)\right]^2 + W_2\left[\beta^* - \beta\right]^2 + W_3\left[\theta_0^* - \theta_0\right]^2 + W_4\left[\theta_1^* - \theta_1\right]^2$$

subjected to concave constraints defined by equations (20) and singularity constraints defined by equation (11). The singularity constraints are determined at some selected points along the bending curve. W_i are weighted factors. The design variables (**D**) are (d_i, i=1,...,m/2). H is calculated through equations (14, 15) by putting u=0. θ_0 and θ_1 are determined through equation (10) by putting u=0,πR respectively. β is determined through equation (12). All the derivatives in these equations and in the singularity constraints are determined through equations (17, 18) and their repetitions (see last section). Sequential linearization program [8] is used to solve this nonlinear optimization problem. A computer program is written to design the bending curve according to arbitrary B-spline.

5. RESULTS AND DISCUSSION

In Tables 1, 2 measured geometrical specifications of some real shoulders are shown. The application of equation (13) surprisingly gets no solutions for all cases. The cause of that is the complex roots in some cases and the unsatisfaction of singularity condition in the other cases. The results of the application of the written program to one case for various B-spline curves are shown in Table. 1. W_i are set to 1. These results indicate how one can increase the accuracy of the solution by changing the characteristics of the B-Spline curve. Fig. 5 shows the best bending curve found for that case. Table 2. shows the results for other cases of real shoulders which give satisfactory solutions. The characteristics of all B-spline curves in Tables 1,2 are indicated in Table 3., where L is the number of curve segments. Fig. 6 shows a manufactured shoulder from sheet metal according to the method described here. The measured geometry parameters of that shoulder fit accurately the calculated ones. These results indicate the validity of the proposed procedures which can get acceptable solutions in many practical cases.

Table 1. Results of various B-spline curves for one real shoulder.

	R (mm)	H/R	β (degree)	θ_0 (degree)	θ_1 (degree)	$(\sum e^2)^{1/2}$
measured	45.0	3.78	115.0	31.5	3.5	
cal. curve 1	45.0	3.823	97.035	46.055	18.354	27.477
cal. curve 2	45.0	3.92	104.71	36.0	24.18	23.54
cal. curve 3	45.0	3.496	101.638	32.52	23.586	24.14
cal. curve 4	45.0	3.75	113.458	30.098	2.489	2.32
cal. curve 5	45.0	4.2578	115.438	33.58	4.45	2.38
cal. curve 6	45.0	4.11	115.65	29.08	3.156	2.546
cal. curve 7	45.0	3.73	113.82	31.68	4.778	1.75

Table 2. Results of acceptable B-spline curves for some real shoulders.

	R (mm)	H/R	β (degree)	θ_0 (degree)	θ_1 (degree)	$(\sum e^2)^{1/2}$
measured	39.0	4.23	105.0	40.0	7.0	
cal. curve 8	39.0	4.337	104.25	41.555	6.63	1.77
measured	42.5	3.96	105.0	30.0	31.5	
cal. curve 9	42.5	4.08	104.33	30.55	32.6	1.41
measured	55.0	3.43	102.0	27.0	3.0	
cal. curve 10	55.0	3.35	101.076	25.59	2.73	1.71

Table 3. Characteristics of the calculated B-spline curves.

	n	m+1	L	d_1 (mm)	d_2 (mm)	d_3 (mm)	d_4 (mm)	d_5 (mm)	d_6 (mm)
curve 1	4	7	2	86.0	156.3	187.8			
curve 2	4	9	4	47.56	131.3	170.2	179.2		
curve 3	5	9	2	67.17	122.4	150.8	163.8		
curve 4	5	11	4	32.18	95.94	147.5	166.0	170.5	
curve 5	5	13	4	37.1	73.6	139.1	178.2	189.9	193.3
curve 6	6	11	2	59.88	118.6	163.6	181.2	188.9	
curve 7	6	13	4	26.6	79.08	125.3	154.6	165.7	168.8
curve 8	5	11	4	32.56	96.1	147.1	165.5	171.0	
curve 9	5	13	6	26.29	74.62	134.3	163.1	172.2	174.1
curve 10	5	13	6	23.85	71.2	135.5	172.9	183.1	185.2

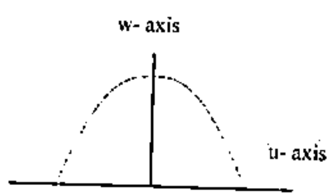

Fig. 5. Calculated bending curve

Fig. 6. Manufactured shoulder

CONCLUSION

A method is introduced to design the bending curve for specified geometry of the shoulder of a packaging machine. The method uses the B-spline to model the bending curve and optimization technique to obtain the closest solution to the required one. The calculation of the bending curve for real shoulders according to the proposed method gives satisfactory results compared to previous methods in the literature. The geometrical parameters of the manufactured shoulder fit accurately with the calculated ones. The results indicate the flexibility and accuracy of the proposed method.

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