TRANSIENT STABILITY ANALYSIS BASED ON THE POST-DISTURBANCE TWO-MACHINE EQUIVALENT


Abstract

This paper presents a method of power system transient stability analysis, based on two-machine equivalent. The method first computes the actual system trajectory using time domain simulation of fault-on system, then at clearing time, evaluates the kinetic energy and acceleration of each machine in order to identify the severely disturbed machine(s). Each of the severely disturbed machines and all remaining machines are grouped together in one equivalent machine, forming the two-machine equivalent.

Based on the relative kinetic energy of the two-machine equivalent, a method for detecting the system stability or instability and computing the critical clearing time is developed.

The proposed method is applied successfully to a 9-machine 9-bus power system and a 4-machine 19-bus power system, which represents the 220 KV Mid-Delta region network of Egypt. The simulation results show that the method can provide reliable and quick information about the transient stability analysis of a power system and it may become effective for on-line applications.

1. INTRODUCTION

Two of the main aims of the transient stability analysis (TSA) are to detect system stability or instability and to compute the critical clearing time (CCT) for a given fault conditions.
major topics in power system analysis, planning and operation. Three methods: the numerical integration method, the extended equal area criterion (EEAC) method and the transient energy function (TEF) method are widely used for detection of system instability and determination of CCT.

Numerical integration method has unlimited modeling capability and produces time responses of all quantities, some of which may be needed for detailed examination of the stability phenomenon or for simulating special protection schemes. However, it has two shortcomings. Firstly, it is inherently slow due to the integration process involved in solving the differential equations, especially for a large power system. Secondly, it only yields a yes-or-no type answer on the stability problem, without indication on the degree of stability. The trial-and-error approach, for deriving stability limits, requires much CPU time, hence not suitable for on-line applications [1,2].

The EEAC method is based on the well-known equal area criterion [3,4], which clusters a multimachine into two subsystems: one containing the critical machine, and the other consisting of the remaining machines. These two subsystems are then transformed to a one-machine to infinite-bus (OMIB) system for which the equal area criterion is used for a direct solution. A significant advantage of EEAC method is the algebraic expressions it provides for the calculation of CCT and transient stability margin (TSM). This makes the conventional TSA particularly easy, and gives the possibilities for on-line TSA. However, the equivalent OMIB system does not always represent the dynamics of the original system accurately, except for the case where both the critical machine group and the remaining machine group are coherent.

The transient energy function (TEF) method is investigated in the context of on-line TSA [5,6]. It is based on determining whether the TEF value \( V \) at clearing time is less than a threshold value \( V_{cr} \), the TEF at the controlling unstable equilibrium point (UEP) [7,8,9]. However, the controlling UEP has to be computed through iterative procedures and faces serious convergence problems in calculation. Furthermore the TEF method has some modeling limitations and for a large power system the computation time is considerable [10].

This paper begins with decomposition of a multimachine system into two groups, based on the machine states (kinetic energy and acceleration) at fault clearing. The group of severely disturbed machines are aggregated into a single equivalent machine and the rest of the machines in the system are similarly combined in a group and replaced by another equivalent machine. The severely disturbed machines are considered to be that having the highest values of kinetic energy and acceleration.

It is shown by the developed method that the minimum value of relative kinetic energy of the two-machine equivalent, which is easy to be determined, can be used for detection of system stability or instability and computation of an accurate CCT.

The proposed method is evaluated in a 3-machine 9-bus system and 4-machine 19-bus system, which represents 220 KV Mid-Delta network of Egypt, to detect system stability or instability and to
calculate the critical clearing time for different fault locations. Results obtained, using the proposed method and time domain method for the original systems are compared.

2. ENERGY ANALYSIS OF A ONE-MACHINE POWER SYSTEM

The objective of this Section is to review the kinetic energy \( V_{KE} \) and potential energy \( V_{PE} \) of a one-machine power system. An extension of this idea, for a multimachine system is described in Section 3. Consider a lossless one-machine system as shown in Fig.1. Assume that a 3-phase fault (see Fig.1) appears in the system at \( t=0 \) and it is cleared by opening one of the lines. The time simulation of system angle \( \delta \), \( V_{KE} \) and \( V_{PE} \) are shown in Fig.2. Two cases are illustrated in Fig.2. They are as follows:

- Case 1: clearing time \( tc \) slightly less than CCT.
- Case 2: \( tc \) is such that the system barely become unstable.

In Fig.2, during the fault, the potential energy of the generator reduces drastically (almost to zero) but the kinetic energy of the generator increases rapidly. Thus, the generator accelerates and its angle \( \delta \) increases. When the fault is cleared at time \( tc \), the potential energy of the generator increases, due to absorption of kinetic energy in the system. The system is considered to be stable if the maximum potential energy is greater than the kinetic energy gained during the fault period.

In this case, the fault is sufficiently large to make the machine critically unstable, its potential energy goes through a maximum before instability occurs. This maximum value of the potential energy along the post-fault trajectory, which is independent of the duration of the fault and the mode of instability is the critical value of total energy \( V_{KE}+V_{PE} \). Therefore, at \( V_{KE}>0 \), the system is stable if \( V_{KE} \) is less than the critical value of total energy \( V_{KE} \), as shown in Fig.2.

3. MATHEMATICAL MODEL

3.1 Multimachine System Equations

Given an \( N \)-machine system represented by classical model and constant impedance loads, the dynamics of \( i \)th machine can be described by the following two 1st-order differential equations [9]:

\[
\dot{\delta}_i = \omega_i \quad (1)
\]

\[
\dot{\omega}_i = P_{m_i} - P_{e_i} = f_i(\delta) \quad (2)
\]

where

\[
P_{e_i} = E_i^2 Y_{ii} \cos \theta_i + \sum_{j=1}^{N} E_j E_i Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \quad (3)
\]

and

\[
\delta_i, \omega_i = \text{rotor angle and speed of machine } i \text{ in the centre of inertia (COI) frame}
\]

\[
M_i = \text{inertia constant of machine } i
\]
3.2 The Two-Machine Equivalent

Suppose during a disturbance the group A consists of those machines, which are severely disturbed and the group B consists of the rest of the power system. After a disturbance is cleared the severely disturbed machines, which belong to group A, being to separate from the remaining machines. The rest of the machines in the system are less disturbed and the variations of their rotor angles during the transient period are not significant compared with the severely disturbed machines [12]. For a given disturbance, the procedure of the equivalent of the power system consists of

(a) dividing the system machines into two groups A and B.
(b) replacing the two sets by two equivalent machines (\(a\) and \(b\)).

The inertial centers for the group A and group B are defined by

\[
\delta_a = \sum_{i \in A} \delta_i M_i / \sum_{i \in A} M_i, \quad \delta_b = \sum_{j \in B} \delta_j M_j / \sum_{j \in B} M_j \tag{4}
\]

The two swing equations of the equivalent critical machine \(a\) and equivalent machine \(b\) are

\[
M_a \dot{\delta}_a = \sum_{i \in A} (P_{m_i} - P_{E_i}) \tag{5}
\]

\[
M_b \dot{\delta}_b = \sum_{j \in B} (P_{m_j} - P_{E_j}) \tag{6}
\]

where

\[
M_a = \sum_{i \in A} M_i, \quad \quad M_b = \sum_{j \in B} M_j \tag{7}
\]

\[
P_{E_i} = E_i^2 Y_i \cos \theta_i + \sum_{k \neq i} E_i E_k Y_{ik} \cos(\phi_{ik} - \theta_i) + \sum_{j \neq i} E_i E_j Y_{ij} \cos(\phi_{ij} - \theta_i) \forall i \in A \tag{8}
\]

\[
P_{E_j} = E_j^2 Y_{jj} \cos \theta_j + \sum_{q \neq j} E_j E_q Y_{jq} \cos(\phi_{jq} - \theta_j) + \sum_{i \neq j} E_j E_i Y_{ji} \cos(\phi_{ji} - \theta_j) \forall j \in B \tag{9}
\]

and

\[
\phi_i = \delta_i - \delta_a \forall i \in A, \quad \phi_j = \delta_j - \delta_b \forall j \in B \tag{10}
\]

From eqn(5) and eqn(6), the swing equation of the two-machine
The rotor angles in the above equations are defined with respect to centre of inertia (COI) of the corresponding group (eqn(10)). In Reference [3], the rotor angles of the individual machines are supposed to be represented by the corresponding partial centre of angles (PCAII), i.e., \( \delta = \delta_a \) and \( \delta = \delta_b \), \( \delta_a \in \beta \).

3.3 SELECTION OF SEVERELY DISTURBED MACHINES

The selection of severely disturbed machines to a large extent depends on how the disturbance affects the system [1, 8]. Hence, the informations available at the clearing time are used in machine grouping procedure as follows:

(a) classify the machines in a decreasing order of their kinetic energy at tc, i.e., based on the quantities \( \omega_i M_i (\omega) \) where \( \omega_i = i, 2, ..., N \) is the speed with respect to the COI at tc. These machines are grouped in set Aa.

(b) classify the machines in a decreasing order of their acceleration at tc, i.e., based on the quantities \( \gamma_i f_i (\delta) M_i \) with respect to COI. These machines are grouped in set Ab.

(c) select set A as Aa U Ab and then set B consists of the remaining machines.

The above procedure is very efficient, since the machine on the top
of the set $A$ will be most affected and the one at the bottom of the list, least affected. Furthermore, both $\omega$ and $\gamma (i=1,2,...,N)$ are available in the swing curve simulation of the fault-on system. Result of analysis (Section 3), shows that choice a few machines in set $A$ results in a satisfactory equivalent system.

4. TRANSIENT STABILITY ANALYSIS

4.1 System Stability or Instability Detection

The total energy of the two-machine equivalent at any instant can be represented as follows

$$ V = V_{KZ} + V_{PE} $$

The $V_{KZ}$ is the kinetic energy, which contributes to the system separation can be defined as

$$ V_{KZ}(t) = \frac{1}{2} M_{eq} (\omega_1) $$

where

$$ M_{eq} = \frac{M_a + M_b}{M_a + M_b} $$

$$ \omega = \omega_0 - \omega $$

Since the total energy $V$ is constant, it is clear from eqn(17) that $\frac{\partial V_{KZ}}{\partial t} = 0$. And its critical value $(V_{KZ})_{max}$ of the critically stable machine. Thus, the system under consideration is

stable if $V_{KZ} < V_{KZ}^{crit}$

unstable if $V_{KZ} > V_{KZ}^{crit}$

When the potential energy reaches its maximum value, $\frac{\partial V_{KZ}}{\partial t}$ becomes zero. At this point, $V_{KZ}$ reaches its minimum. Moreover, all of the injected kinetic energy must be converted into potential energy. Thus, if the minimum value of $V_{KZ}$ is defined as

$$ V_{KZ}^{min} = \min V_{KZ}(t) $$

where $t_u$ is the time of maximum potential energy. Then $V_{KZ}^{min}$ should equal zero for all $t > t_u$ and should be positive value for $t < t_u$. Thus, the system stability or instability can be detected as

stable if $V_{KZ}^{min} < 0$

unstable if $V_{KZ}^{min} > 0$

4.2 Computation of Critical Clearing Time

The minimum value of kinetic energy $(V_{KZ}^{min})$ is a function of the two-machine equivalent parameters and states $(\delta, \omega)$. Using the time-domain simulations, the $V_{KZ}^{min}$ is computed. The time at which $V_{KZ}^{min}$ changes from zero (or negative value) to positive value is the critical clearing time. The algorithm used for computation of the clearance time is given below.

5. PROGRAM ALGORITHM

The algorithm used for detecting the transient stability and
computing the critical clearing time based on the two-machine equivalent may be presented as follows:

Step 1: using the prefault load flow data, compute the initial state.

Step 2: produce the reduced internal Y-matrix for both faulted and post-fault systems.

Step 3: using the transient stability program, calculate the kinetic energy and acceleration for each machine at fault clearing. Identify the machine grouping: group A consists of critical machines and group B consists of the all remaining machines.

Step 4: for the post-fault system, calculate the parameters of the two-machine equivalent.

Step 5: continue the simulation of the two-machine equivalent system trajectory. At each time step, determine the kinetic energy \( V_{K\text{c}} \). Detect the \( V_{K\text{c}} \), which can be achieved as follows:

Compute the \( V_{K\text{c}} \) at each time step, i.e. \( V_{K\text{c}}(t-2\Delta t) \), \( V_{K\text{c}}(t-\Delta t) \), and \( V_{K\text{c}}(t) \). A local \( V_{K\text{c}} \) point occurs at time \( t-\Delta t \) when \( V_{K\text{c}}(t-2\Delta t) \) is less than \( V_{K\text{c}}(t-\Delta t) \) and \( V_{K\text{c}}(t) \).

Step 6: if \( V_{K\text{c}}>0 \), the system is stable. The system is unstable if \( V_{K\text{c}}<0 \).

Step 7: to compute the CCT, advance time by one integration step and go back to step 4 and continue. Since the location and type of fault does not change, a new machine grouping is not required. CCT lies between \( t_1 \)(where \( V_{K\text{c}}=0 \)) and \( t_2 \)(where \( V_{K\text{c}}>0 \))

If the step is small enough, \( t_1 \) may be taken as the CCT. If the step is large, however, an approximate region is found. Reduce step size and repeat Steps 4-7 for a more precise determination of the CCT.

8. RESULTS

The proposed method for detecting the transient stability and computing the critical clearing time is tested on two power systems. The 3-machine 9-bus system and 4-machine 19-bus system representing the high voltage network of Mid-Delta region of Egypt. Results are computed for the equivalent system and compared with the values obtained for the original system by time simulations of system differential equations. The integration method used for solving system dynamic differential equations is the Runge-Kutta fourth-order method.

8.1. 3-Machine 9-Bus System

The single-line diagram of the system is shown in Fig. 3, which is obtained from [13]. A three-phase fault is applied at bus #7 near machine #2, cleared by tripping one of the two lines between buses #7 and #8.
Figure 4 shows the kinetic energy and acceleration of each machine for tc=0.14s. Examining Fig. 4, the machines can be classified in a decreasing order of kinetic energies at tc (set A) as G2, G4 and G1. Classification of the machines in a decreasing order of their accelerations at tc (set A) is G2, G4, and G1. Therefore, the critical machine is G2 and the remaining machines are G4 and G1.

The simulation results for two-machine equivalent system are shown in Fig. 5 and Fig. 6. The time response of angle difference (\(\delta_{ab}\)) and kinetic energy for tc=0.14s is shown in Fig. 9. The system is stable in this case and \(V_{\text{EM}} = 0\).

Figure 6 shows the phase portrait (\(\delta_{ab}-\omega_{d}\)) and the time response of machine rotor angle, speed, kinetic energy, equivalent parameters of mechanical power and electrical power for clearing time of 0.18s (curve a) and 0.19s (curve b). For tc=0.18s, \(V_{\text{EM}} = 0\) and the system is critically stable. The system is unstable for tc=0.19s and \(V_{\text{EM}} > 0\). Therefore, the critical clearing time, which corresponds to the above fault location is 0.18s. This value coincide with the value obtained from the time simulation of the original system.

8.2. 4-Machine 19-Bus Typical System

This system represents the reduced power network of Mid-Delta region of the unified power system of Egypt. The system single line diagram is shown in Fig. 7. Its data and initial operating conditions are given in [14]. The initial operating conditions are calculated based on the recorded data on 19/6/1983 by Egypt load Dispatch Center [15]. Two cases are studied. They are as follows:

Case 1:

A 3-phase fault of tc=0.20s at bus # 23 is cleared by opening one of the two parallel lines between bus # 22 and bus # 23. Fig. 8 shows the kinetic energy and acceleration of each machine for faulted system. At clearing time, machine G4 has the largest values of kinetic energy and acceleration. Thus, G4 represents the severely disturbed group (group A) and all remaining machines (G2, G3 and G4) are grouped in group B. This grouping coincides with the results obtained in Fig. 9. In Fig. 9, the time response of rotor angles of G2, G3 and G4 are coherent.

Figure 10 shows the kinetic energy obtained for the two-machine equivalent. It is found that \(V_{\text{EM}} = 0\) and the system is stable. If the clearing time is increased (tc=0.27s) the system is unstable and G4 losses its synchronism and the corresponding \(V_{\text{EM}} > 0\), as shown in Fig. 10. For computing the CCT corresponding to the above fault location, \(V_{\text{EM}}\) is computed according to the above algorithm and illustrated in Fig. 11. The CCT is 0.26s at \(V_{\text{EM}} = 0\).

Case 2:

A 3-phase fault of tc=0.34s at bus # 13 is cleared by opening one of the two parallel lines between bus # 13 and bus # 14. Based on kinetic energies and accelerations of system machines at clearing time, which are shown in Fig. 12, the machines are grouped as: group
A contains machines $G_b$, which represents the severely disturbed machine and group B contains the remaining machines $G_a, G_c$ and $G_d$.

Figures 13 and 14 show the results obtained for the original system and the two-machine equivalent respectively when the system is critically stable and critically unstable. The comparison shows that the behaviours of the two systems from point of view of stability or instability and CCT value are similar.

7. CONCLUSIONS

A method to analyze the transient stability of multimachine power system is developed. The development of the analytical method is based on the two-machine equivalent, in which one machine is the equivalent of the severely disturbed machine (or machines) and the other one is the equivalent of all remaining machines in the original system. It is found that, for a particular disturbance, the combination of the machine's kinetic energy and acceleration, which are available at clearing time can be used effectively in selection of the severely disturbed machines.

It is shown by the developed method that the minimum value of relative kinetic energy of the two-machine equivalent, which is easy to be determined, can be used for detecting stability or instability of the power system. Moreover, it is used for determining an accurate critical clearing time.

Results of transient stability detection and assessment, using the proposed method are computed and compared with the values obtained by time-domain simulations of the original systems, which are 3-machine 9-bus power system and 4-machine 10-bus power system representing practical network.

Results demonstrate that the developed analytical method can provide reliable and quick information about the transient stability analysis of a power system and it may become effective for on-line applications.

8. REFERENCES


transient stability assessment of a multi-machine power system" 
IEEE Trans. Power Apparatus and System, Vol.PAS- 103, No.8, 
August 1984, p.2199-2205.

transient energy function method for on-line dynamic "security 
analysis", IEEE Trans. on Power Systems, Vol.8, No.2, May 1993, 
p.497-507.

power system. Part 1: Investigation of system trajectory",IEEE 

[8] Vittal V., Rajagopal S., et al.,"Transient stability analysis of 
stressed power systems using the energy function method", IEEE 

disturbance: energy associated with system separation", IEEE 
Trans. on Power Apparatus and Systems, Vol. PAS-102, No.11, 


stability program output analysis", IEEE Trans. on Power 

indexes for on-line analysis of 'worst-case'dynamic 
contingencies", IEEE Trans. on Power Systems, Vol.PWRS-2, No.3, 
August 1987, pp.660-668.


stability assessment of power systems", Mansoura Engineering 

Unified Electrical Network, A.R.E., 30-500 KV directory.
Fig. 1 One-machine infinite bus system

Fig. 2 Time response of $\delta$, $V_{ke}$, $V_{pe}$ for case 1: $tc=0.18s$, case 2: $tc=0.19s$
Fig. 3 3-Machine 9-bus system

Fig. 4 Response of \( V_{ke\alpha} \), \( V_{ke\beta} \) for \( tc=0.14s \)

Fig. 5 Response of \( \delta \), \( Vke_{eq} \) of equivalent system for \( tc=0.14s \)
Fig. 8  $\delta_{ab}$-eq, $V_{eq}$, $P_{meq}$, $P_{req}$ response for
(a): $t_c=0.18s$  (b): $t_c=0.19s$

Fig. 7  4-Machine 19-bus system

SYSTEM POWER STATIONS:
01- ABU-SULTAN
02- TALEFA
03- DAMMINUR
04- DAMITA
Fig. 8: Vke acceleration for original system for $t_c=0.26s$.

Fig. 9: Response of $\delta_i$ for $t_c=0.26s$.

Fig. 10: System instability detection for $t_c=0.26s$ and $t_c=0.27s$.

Fig. 11: Response of $(Vke)_{min}$ with different clearing times, $t_c$. 

CCT = 0.26s for 3-ph fault at bus #23.
Fig. 12  \( V_{ke} \), acceleration for \( t_c = 0.34 \) s

Fig. 13 \( \delta_i \) for original system

Fig. 14 \( \delta_{ab}, (V_{ke})_{eq} \) of equivalent system