Yarn Crimp Ratio in Woven Fabrics
Theory and Practice

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I. Abstract:
Because of the importance of accuracy in measuring and estimating yarn crimp ratio in woven fabrics, non classical methods were searched for. In this paper mathematical formulae were derived to express yarn crimp ratio in terms of weave structure, yarn spacing, and yarn diameter. It is discriminated between floated weaves and extended weaves. In floated weaves yarn spacing is constant and equal to yarn spacing at the point of intersection. In extended weaves, yarns are separated only at points of intersection. To increase accuracy and save effort and time consumed in measuring yarn crimp ratio, a new experimental method is introduced. In this method yarn crimp ratio is determined by weighing crimped yarns instead of descrimping them.

II. Introduction:
There have been many simple empirical models to relate fabric parameters. These models need more modifications by using the electronic computer systems [1]. Simple mathematical models had been improved and modified in the form of dimensionless relations [2]. The weave structure can be expressed in terms of average float or float ratio. It was found that the mean value of warp tension decreases by increasing the float ratio [3].

Fabric abrasion resistance is affected by yarn crimp ratio because crowns formed as yarn bends round a transverse thread will protrude from the fabric surface and meet the destructive abrasive agent first. The largest amount of fabric shrinkage is that represented by increase of crimp. Control of crimp percentage is necessary when a fabric is designed to give a desired degree of extensibility and when crimp balance between warp and weft is required after finishing processes. Knowledge of crimp value is useful in calculating fabric cost and yarn requirement [4].

It is clear that yarn crimp ratio plays important parts in fabric analysis, fabric research and design, process control, and economics of fabric production.
III. Nomenclature:

- \( A_i \) The number of weft yarns over which the warp end passes in the \( i^{th} \) fraction of warp weave repeat.
- \( B_i \) The number of weft yarns under which the warp end passes in the \( i^{th} \) fraction of warp weave repeat.
- \( C_1 \) Warp crimp ratio in the woven fabric.
- \( C_2 \) Weft crimp ratio in the woven fabric.
- \( (C_1)_p \) Warp crimp ratio in plain-woven fabric.
- \( (C_2)_w \) Weft crimp ratio in plain-woven fabric.
- \( d_1 \) Diameter of warp end (cm).
- \( d_2 \) Diameter of weft yarn (cm).
- \( F_1 \) Average float of warp end.
- \( F_2 \) Average float of weft yarn.
- \( G_i \) The number of warp ends over which the weft yarn passes in the \( i^{th} \) fraction of weft weave repeat.
- \( H_i \) The number of warp ends under which the weft yarn passes in the \( i^{th} \) fraction of weft weave repeat.
- \( K_1 \) Warp cover ratio.
- \( K_2 \) Weft cover ratio.
- \( L_1 \) Warp modular length (cm).
- \( L_2 \) Weft modular length (cm).
- \( L \) Length of weave repeat.
- \( m \) Number of fractions in repeat of weft interlacing.
- \( n \) Number of fractions in repeat of warp interlacing.
- \( n_1 \) Average number of ends / cm.
- \( n_2 \) Average number of picks / cm.
- \( N \) Number of yarns.
- \( P_1 \) Distance between two neighbouring warp ends at a weft intersection (cm).
- \( P_2 \) Distance between two neighbouring weft yarns at a warp intersection (cm).
- \( W \) Width of weave repeat.
- \( \theta_1 \) Warp weave angle.
- \( \theta_2 \) Weft weave angle.

IV. Mathematical Derivation:

Figures 1 and 2 show an example of floated weaves and extended weaves (3/1) respectively.
IV.1. Fabrics of Floated Weaves:

Let warp interlacing repeat be \( A_1, A_2, A_3, \ldots, A_k \) and weft interlacing repeat be \( G_1, G_2, G_3, \ldots, G_k \).

Then

Number of warp interlacings / repeat = \( 2n \)

Number of picks / repeat = \( \sum_{i=1}^{k} A_i + \sum_{i=1}^{k} B_i \)

Warp length / repeat = \( 2nL + P_1 \left( \frac{1}{\sum_{i=1}^{k} A_i + \sum_{i=1}^{k} B_i} - 2n \right) \)

Length of repeat = \( P_1 \left( \frac{1}{\sum_{i=1}^{k} A_i + \sum_{i=1}^{k} B_i} \right) \)

Warp Crimp Ratio

\[
C_1 = \frac{2n(L - P_1)}{P_1 \left( \sum_{i=1}^{k} A_i + \sum_{i=1}^{k} B_i \right)} = \frac{2n}{\sum_{i=1}^{k} A_i + \sum_{i=1}^{k} B_i} \left( \frac{L - P_1}{P_1} \right)
\]

\[
(C_1)_{pl} = \frac{2n}{\sum_{i=1}^{k} A_i + \sum_{i=1}^{k} B_i} \cdot (C_1)_{pl}
\]

\( (C_1)_{pl} \) is warp crimp ratio in plain woven fabric. Similarly

\[
C_2 = \frac{2m}{\sum_{i=1}^{k} G_i + \sum_{i=1}^{k} H_i} \cdot (C_1)_{pl}
\]

\( (C_2)_{pl} \) is weft crimp ratio in plain woven fabric.

\[
C_3 = \frac{(C_2)_{pl}}{F_2}
\]

and

\[
C_4 = \frac{(C_2)_{pl}}{F_2}
\]

\( F_1 \) and \( F_2 \) are average float of warp and weft, respectively.

From a previous study [5],

\[
(C_1)_{pl} = \sec \theta_1 - \frac{2d}{F_2} \tan \theta_1 + \frac{2d}{F_2 \theta_1 - 1}
\]

when \( \theta_1 < \frac{\pi}{2} \).
\[
(C_2)_{\text{unit}} = \text{Sec} \theta_2 \frac{2d}{P_1} \tan \theta_2 + \frac{2d}{P_1} \theta_2
\]  
(6)

when $\theta_2 < \frac{\pi}{2}$

When $\theta_2 = \frac{\pi}{2}$, $\theta_1 = 0$, $C_1 = \frac{\pi d}{P_1} - 1$, and $C_2 = 0$

When $\theta_2 = \frac{\pi}{2}$, $\theta_1 = 0$, $C_2 = \frac{\pi d}{P_1} - 1$, and $C_1 = 0$

\[
\sin \theta_1 = \frac{2dP_1 - d_1 \sqrt{P_1^2 - d_2 (2d + d_1)}}{P_1^2 + d_1^2}
\]  
(7)

\[
\sin \theta_2 = \frac{2dP_1 - d_2 \sqrt{P_1^2 - d_1 (2d + d_1)}}{P_1^2 + d_2^2}
\]  
(8)

\[
d_1 + d_2 = 2d
\]  
(9)

d_1 and d_2 are warp yarn and weft yarn diameters, and P_1 and P_2 are yarn spacing for warp and weft, respectively.

Yarn crimp ratio can also be expressed as follows:

\[
C_1 = \frac{\text{Warp interlacings / repeat}}{\text{Picks / repeat}}
\]  
(10)

\[
C_2 = \frac{\text{Weft interlacings / repeat}}{\text{Ends / repeat}}
\]  
(11)

\[(C_1)_{\text{unit}}\] and \[(C_2)_{\text{unit}}\] are functions of:
- warp diameter d_1
- weft diameter d_2
- warp spacing P_1
- weft spacing P_2

The contribution of weave structure in the value of $C_1$ and $C_2$ is represented in warp interlacings / repeat as a ratio of picks / repeat and weft interlacings / repeat as a ratio of ends / repeat (equations 10 and 11). In other words the contribution of weave structure is represented in warp average float and weft average float (equations 3 and 4).

Example:
Let d_1 = 0.02 cm \hspace{1cm} d_2 = 0.03 cm
P_1 = 0.05 cm \hspace{1cm} P_2 = 0.07 cm

\[\theta_1 = 27.43^\circ \hspace{1cm} \theta_2 = 28.07^\circ\]

Crimp ratios can be calculated from equations 5 and 6.
\[(C_1)_{\text{unit}} = 0.0979 \hspace{1cm} (C_2)_{\text{unit}} = 0.08995\]

Table (1) shows the effect of weave structure (average float) on yarn crimp ratios:
IV.2. Fabrics of Extended Weaves:

In the above-mentioned analysis yarn spacing \( P \) is taken constant i.e. yarns are uniformly spread along and across the woven fabric. In extended weaves every group of yarns interface alike and behave as one yarn i.e. they run side by side without gaps except at points of intersection where yarn set is divided into groups. Basket weave, warp rib and weft rib are popular examples of such weaves. In this case yarn spacing is not constant.

Length of weave repeat \( L = 2n \bar{P} + \left( \sum A_1 + \sum B_1 - 2n \right) d_2 \)  \(  \tag{12}  \)

Warp length \( L \) / repeat = \( \frac{2n \left( L_1 - \bar{P} \right) d_2}{2n \bar{P}_1 + \left( \sum A_1 + \sum B_1 - 2n \right) d_2} \)

: Warp Crimp Ratio \( C_i = \frac{2n \left( L_1 - \bar{P} \right) d_2}{2n \bar{P}_1 + \left( \sum A_1 + \sum B_1 - 2n \right) d_2} \)  \(  \tag{13}  \)

\[
C_i = \frac{\left( L_1 - \bar{P}_1 \right)}{\bar{P}_1} \cdot \frac{1 + \frac{\left( \sum A_1 + \sum B_1 - 2n \right) d_2}{2n \bar{P}_1}}
\]  \(  \tag{14}  \)

From (12)

\[ \bar{P}_1 = \frac{L - \left( \sum A_1 + \sum B_1 - 2n \right) d_2}{2n} \]  \(  \tag{15}  \)

\[ \bar{P}_2 = \frac{L}{2n} - \left( F - \frac{1}{n_2} \right) d_2 \]

\[ F = \frac{L_2 - \bar{P}_1}{n_2} \]  \(  \tag{16}  \)

\( n_2 \) is overall weft denisty (picks/cm).

Similarly

Weft Crimp Ratio

\[
C_i = \frac{\left( L_2 - \bar{P}_1 \right)}{\bar{P}_1} \cdot \frac{1 + \frac{\left( \sum G_1 + \sum H_1 - 2m \right) d_2}{2m \bar{P}_1}}
\]  \(  \tag{17}  \)

If width of repeat is \( w \)

\[ \bar{P}_1 = \frac{w - \left( \sum G_1 + \sum H_1 - 2m \right) d_2}{2m} \]
\[ P_i = \frac{w}{2m} - (F_2 - 1) d_i \]
\[ P_i = \frac{F_1}{n_1} - (F_2 - 1) d_i \]  

\( n_1 \) is overall warp density (ends/cm).

From (14) and (17)
\[ C_1 = \frac{2nF_1}{L} (C_i)_{w1} \]  
and
\[ C_2 = \frac{2mF_1}{w} (C_i)_{w1} \]

It may be more appropriate to count the number of weft threads/unit length of the fabric than to measure the length of weave repeat. The unit length must be as great as possible in order to obtain as many complete weave repeats as possible. After counting the number of packs/unit length, the overall number of packs/cm (\( n_2 \)) can be computed
\[ L = \sum A_i + \sum B_i \]

Similarly
\[ W = \sum C_i + \sum D_i \]

It can be derived that
\[ C_1 = \left[ 1 - \frac{n_2 d_2 \left( \sum A_i + \sum B_i - 2n_2 \right)}{\sum A_i + \sum B_i} \right] (C_i)_{w1} \]
\[ C_2 = \left[ 1 - \frac{K_2 (F_2 - 1)}{F_1} \right] (C_i)_{w1} \]  
and Similarly
\[ C_1 = \left[ 1 - \frac{K_1 (F_2 - 1)}{F_1} \right] (C_i)_{w1} \]
\[ K_1 = n_1 d_1 \]
\[ K_2 = n_2 d_2 \]

V. Procedure of Calculations:

V.1. Fabrics of Float Weaves:

Given: Weave structure, yarn diameters (warp and weft), and yarn spacings (warp and weft)

Calculations:
- From weave structure, yarn average float is estimated (warp and weft).
- From yarn diameters and yarn spacings, weave angles and yarn crimp ratios of plain weave can be calculated.
- From yarn crimp ratio of plain weave and yarn average float, yarn crimp ratio of the considered weave structure is estimated.
V.2. Fabrics of Extended Weaves:
Given: Weave structure, yarn diameters (warp and weft) and average yarn densities (warp and weft).
Calculations:
- From weave structure, yarn average float is estimated (warp and weft).
- From yarn diameter and yarn density, yarn cover ratio can be estimated (warp and weft).
- From yarn average float, yarn diameters, and yarn densities, yarn spacing for plain weave (yarn spacing at points of intersection) can be estimated.
- From yarn spacing at points of intersection and yarn diameters, yarn crimp ratio for plain weave can be estimated.
- From yarn cover ratios, yarn average floats, and yarn crimp ratios for plain weave, yarn crimp ratios for the considered weave structure can be estimated.

Figures 3 and 4 show a discrimination between floated and extended weaves with respect to crimp ratio.

VI. A New Method for Measuring Yarn Crimp Ratio:
The usual procedure followed to estimate yarn crimp ratio [5] is to measure:
1. The distance between two points on the yarn in the plane of the fabric (L_c).
2. The distance between the previously considered two points when the yarn is straightened i.e. decrimped (L_s).
Then yarn crimp ratio is estimated as follows:
\[
C = \frac{L_s - L_c}{L_s} \tag{27}
\]
Determining the length L_s needs straightening the crimped yarn by applying a certain tensile load (decrimping load). The decrimping load is that sufficient to straighten the yarn without causing any elongation. This decrimping load is a function of [4] yarn material, yarn structure, and yarn count i.e. yarn properties. It is clear that the value of the decrimping load is not only difficult to be specified but also a critical value. This means that a load less than the proper decrimping load results in a value of crimp ratio less than the real value and a load more than the proper decrimping load results in a value of crimp ratio more than the real value. There is no doubt that the previous formula (equation 27) of calculating yarn crimp ratio is right but the problem lies in determining the value of L_s. This problem can be overcome by the following procedure:

Multiplying the numerator and denominator in (27) by yarn cross sectional area and yarn density (weight/unit volume), then
\[
C = \frac{W_s - W_c}{W_s} \tag{28}
\]
\(W_s\) is the weight of the unravelled yarn, \(W_c\) is the weight of a portion of the yarn of length equal to the distance between the ends of the unravelled yarn in the plane of the fabric. This formula is based on the imporvement of yarn regularity as it is suitable for only regular yarns. \(W_c\) can be calculated from yarn count and \(L_c\). If \(L_c\) is in meters and yarn count is in \(N_m\) (metric count), then
\[
W_c = \frac{L_c}{N_m} \tag{29}
\]
\(W_c\) is in gm.
Fig. (3): Effect of Average Float on Warp Crimp Ratio

- Floated Weave - Extended Weave

Fig. (4): Effect of Average Float on Weft Crimp Ratio

- Floated Weave - Extended Weave
\[ C = \frac{W_i - L_i}{N_m} \]

\[ C = \frac{W_s}{L_s} \cdot \frac{N_m}{N_m} - 1 \]  \hspace{1cm} (30)

If \( W_s \) is the weight of one unravelled yarn, \( C \) will be the corresponding crimp ratio. If \( W_s \) is the mean weight of a number of unravelled yarns of the same \( L_s \), then \( C \) will be the average yarn crimp ratio. It can be concluded that if \( N \) yarns have a weight \( W \), the \( W_s \) will be equal to the total weight divided by the number of yarns i.e.

\[ \frac{W}{N} = \frac{W_s}{N_s} \cdot \frac{N_m}{N_m} \]  \hspace{1cm} (31)

It is worth noting that this relation is another form of the formula used to calculate weight of \( N \) crimped yarns of the same crimped length \( L_s \) yarn crimp ratio \( C \), and yarn count \( N_m \). The last formula can be rewritten as follows:

\[ C = \frac{N_m}{(N_m)_c} - 1 \]  \hspace{1cm} (32)

\((N_m)_c\) is the metric count of the crimped yarn.

VII. Procedure of Crimp Ratio Testing:

1. Prepare a rectangular or square sample of fabric of adequate dimensions.
2. Measure length and width of the sample to obtain the yarn crimped length \( L_s \) of each of warp and weft.
3. Unravel a number \( N \) of yarns (e.g. warp) and weigh them to obtain the weight \( W \) of \( N \) yarns.
4. Estimate the yarn count \( N_m \) of each of warp and weft if it is unknown.
5. Substitute in formula (31) to give the average crimp ratio of \( N \) yarns.

VIII. Verification:

Yarn crimp ratio in different samples of woven fabric is estimated by 4 different methods: 3 experimental methods besides the theoretical method considered in this paper. The 3 experimental methods are:

1. Decrimping method.
2. Weighing method and using nominal counts.
3. Weighing method and using estimated values of counts (counts are estimated by decrimping and weighing).

Table (2) shows the results of these 4 methods. It is clear that although styles 1 and 4 have the same constructional details, they differ in values of crimp ratios estimated by the 3 experimental methods. It is also clear that the first and third methods give the same results. In square fabrics where warp and weft specifications are the same (styles 1, 2, 4, 5 and 8) the experimental values of crimp ratio are not the same for warp and weft.

To determine the most suitable method for crimp ratio estimation, fabric weight (g/m²) is measured and also estimated using crimp ratios obtained from the different 4 methods. This is shown in Table (3). It is clear from the table that the decrimping
method and the weighing method (estimated counts) of crimp estimation give nearly the same value of estimated fabric weight. It is also clear that the weighing method (nominal counts) give fabric weight which is the nearest to the measured value in 4 styles and the theoretical method is the nearest in 3 styles whereas the other 2 methods are the nearest in 1 style.

Table (3): Fabric Weight (g/m²) Measured and Estimated Using Crimp Ratios Obtained by the 4 Different Methods of Crimp Ratio Estimation

<table>
<thead>
<tr>
<th>Style</th>
<th>Measured Value</th>
<th>Decrimping Method</th>
<th>Weighing Method (nominal counts)</th>
<th>Weighing Method (estimated counts)</th>
<th>Theoretical Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>166.67</td>
<td>151.71</td>
<td>161.70</td>
<td>151.71</td>
<td>159.80</td>
</tr>
<tr>
<td>2</td>
<td>218.18</td>
<td>205.40</td>
<td>211.27</td>
<td>205.40</td>
<td>209.90</td>
</tr>
<tr>
<td>3</td>
<td>200.00</td>
<td>177.49</td>
<td>186.40</td>
<td>177.78</td>
<td>172.85</td>
</tr>
<tr>
<td>4</td>
<td>168.75</td>
<td>156.95</td>
<td>157.30</td>
<td>156.95</td>
<td>159.80</td>
</tr>
<tr>
<td>5</td>
<td>206.00</td>
<td>195.10</td>
<td>181.99</td>
<td>195.00</td>
<td>207.90</td>
</tr>
<tr>
<td>6</td>
<td>168.57</td>
<td>161.07</td>
<td>166.69</td>
<td>161.10</td>
<td>146.68</td>
</tr>
<tr>
<td>7</td>
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<td>168.70</td>
<td>163.60</td>
<td>169.29</td>
<td>161.00</td>
</tr>
<tr>
<td>8</td>
<td>240.00</td>
<td>227.20</td>
<td>246.30</td>
<td>227.00</td>
<td>243.56</td>
</tr>
</tbody>
</table>

IX. Conclusion:
Both the weighing method (nominal counts) and the theoretical method can be preferably used in estimating yarn crimp ratio in woven fabrics. Weighing methods comprises unravelling yarns from fabric, weighing them and then estimating yarn nominal count (as in specifications of fabric), number of unravelled yarns, and yarn crimped length (the new method of crimp estimation). The theoretical method needs measuring yarn densities (or yarn spacing), and yarn diameter (from yarn count), and determining yarn average float (from weave structure repeat). Then substituting in mathematical formulae derived in this paper gives the values of yarn crimp ratios.

References:
<table>
<thead>
<tr>
<th>Style</th>
<th>Structure</th>
<th>Weave</th>
<th>Average Float</th>
<th>Specifications</th>
<th>Crimp Ratio</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Decrimping Method</td>
<td>Weighing Method</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( C_1 )</td>
<td>( C_2 )</td>
</tr>
<tr>
<td>1</td>
<td>Plain 1/1</td>
<td>1</td>
<td>20/1 x 20/1</td>
<td>0.252</td>
<td>0.089</td>
</tr>
<tr>
<td>2</td>
<td>Dobby 1/2</td>
<td>1.25</td>
<td>24 x 24</td>
<td>0.098</td>
<td>0.115</td>
</tr>
<tr>
<td>3</td>
<td>Twill 2/1</td>
<td>2</td>
<td>16 x 12</td>
<td>0.134</td>
<td>0.033</td>
</tr>
<tr>
<td>4</td>
<td>Plain 1/1</td>
<td>1</td>
<td>24/1 x 20/1</td>
<td>0.105</td>
<td>0.111</td>
</tr>
<tr>
<td>5</td>
<td>Plain 1/1</td>
<td>1</td>
<td>16/1 x 16/1</td>
<td>0.123</td>
<td>0.088</td>
</tr>
<tr>
<td>6</td>
<td>Plain 1/1</td>
<td>1</td>
<td>12/1 x 6.5/1</td>
<td>0.1475</td>
<td>0.152</td>
</tr>
<tr>
<td>7</td>
<td>Warp Rib 2/2</td>
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<td>16 x 14</td>
<td>0.063</td>
<td>0.079</td>
</tr>
<tr>
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<td>Plain 1/1</td>
<td>1</td>
<td>24 x 24</td>
<td>0.119</td>
<td>0.125</td>
</tr>
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</table>