Natural Flow Convection In A Vertical Multilayered Porous Media With Varying Permeabilities

(Part 2: Heat Transfer)

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Abstract

This paper outlined numerically a solution for the phenomenon of heat flow by natural convection in a two dimensional multilayered vertical porous
medium heated from one side with vertical isothermal walls and insulated horizontal walls. The study is concentrated on the effect of changing both the permeability ratio \( K_r \) and width ratio \( W_r \) of an inner sublayer in a three layered porous medium on the different regimes of the heat flow. Numerical results are reported for the range of \( 0 < W_r < 1 \), \( 0.1 < K_r < 10 \) and \( 0 < Ra < 6000 \) and for an Aspect ratio \( A = 3 \). It is found that the heat transfer characteristics depend on both \( K_r \) and \( W_r \) besides the known dependence on the Ra and the aspect ratio. Three regimes of heat flow is obtained by increasing Rayleigh number, conduction, transient and boundary layer regime. A numerical correlation expresses the effect of both \( W_r \) and \( K_r \) on \( Nu \) is derived for the boundary layer regime for this particular case as follows:

\[
Nu = C \left( \pm 0.3836 W_r + 1 \right) \left( 0.0454 K_r + 0.9546 \right) Ra^{0.5}
\]

where the +ve is for the case of \( K_r > 1 \), the -ve is for the case of \( K_r < 1 \) and \( C \) is constant depends on the Aspect ratio.

1. INTRODUCTION

Recently considerable attention has been focused upon problems involving heat transfer in porous media. In many of these systems, such as building insulation, geothermal sources, and pebble bed thermal storage units, the mode of heat transfer is natural convection. The majority of existing studies were concerned with the natural convection in a homogeneous porous medium, which may not always be a good model for the physical configuration. A better model is to consider the non-homogeneous case. To handle the non-homogenous porous medium case, a model is assumed in the present work, in which the porous medium's composed of three vertical porous layers of different permeabilities, with vertical isothermal walls at different temperatures and adiabatic top and bottom walls. Considerable work has been performed on natural convection in an isotropic porous layer. An overview of this research is provided in the first part of this study [1]. The boundary layer regime for convection in the porous media was attractive for some researchers. One of the earlier theoretical studies for the boundary layer regime for convection in a vertical porous layer was done by Weber [2]. He showed that when the temperature difference between the walls, or equivalently, the Rayleigh number, is sufficiently increased, the basic flow exhibits boundary layer character. Tong and Subramanian [3] have solved the boundary layer equation derived from the Brinkman's extended model using the Weber's approach.
The purpose of this study is to analyse and understand the phenomenon of the convective heat transfer in a vertical layered porous media with vertical isothermal walls at different temperatures, under the effect of the inhomogeneity of the porous medium, and determine the different regimes of convective heat transfer, specially the boundary layer regime.

2. MATHEMATICAL MODEL AND SOLUTION PROCEDURE

Consider a two-dimensional rectangular vertical cavity (shown in Fig. 1) of width W and height H, filled with multilayered porous medium. Each layer is homogenous, isotropic and has constant permeability K. The porous medium is saturated with a single phase fluid of density \( \rho \) and viscosity \( \mu \). In Fig 1, \( T_H \) and \( T_C \) represent the hot and cold vertical walls of the cavity respectively while the horizontal top and bottom walls are insulated. The effect of both the drag and inertia are neglected and the flow will obey Darcy's law. The fluid properties are assumed constant except for the density change with temperature which gives rise to the buoyancy force. This is treated by invoking the Boussinesq approximation. While the permeability values \( K \) of the porous layers are different, the values of the thermal diffusivity in the layers is the same. This assumption is done to study the effects of the change of the permeability alone.

A nondimensional form of the mass, momentum and energy equations, using the Darcy's law and the Boussinesq approximation are used in a model solved by the finite difference technique. Nondimensional variables \( X, Y, \Psi, \theta, U \) and \( V \) are used for the distances \( x, y \), stream function \( \phi \), temperature \( T \) and horizontal and vertical velocities respectively. Their values are defined by:

\[
X = x/W, \quad Y = y/H, \quad \Psi = \phi H/\alpha W, \quad \theta = (T-T_C)/(T_H-T_C)
\]

The governing parameters for the present problem are the aspect ratio \( A \) and the Darcy-Rayleigh number based on the height of the cavity \( Ra \),

\[ A = W/H \quad \text{and} \quad Ra = K \beta \gamma (T_H - T_C) H/\alpha \nu \]

where \( \alpha, \beta, \gamma, \nu \) and \( K \) are the thermal diffusivity, coefficient of thermal expansion, acceleration due to gravity, dynamic viscosity and Permeability of the porous medium, respectively.

The boundary conditions for the non-dimensional stream function and temperature are as follows:

- \( \Psi = 0 \) on all walls, \( \partial \Psi/\partial Y = 0 \) for \( Y = 0,1 \) and \( 0 < X < 1 \),
- \( \theta = 1 \) for \( X = 0 \) and \( 0 < Y < 1 \), \( \theta = 0 \) for \( X = 1 \) and \( 0 < Y < 1 \)
The effect of the fluid motion on the heat transfer across the layers can be expressed by the Nusselt number, which is defined for the vertical hot wall in the nondimensional form as:

\[ Nu = \int_0^1 Nuy \bigg|_{x=0} \cdot dY \]  

and  \[ Nuy = - \frac{\partial \theta}{\partial x} \]  

where \( Nu \) is the average (mean) Nusselt number at the wall, \( Nuy \) is the local Nusselt number.

Because the vertical layers have different permeabilities \( K \), the Darcy-Rayleigh number \( Ra \) will be different for each layer. It will be:

\[ Ra = K_r \cdot Ra_H \]

where:
- \( Ra_H \) is the Darcy-Rayleigh number in the layer in contact with the hot wall,
- \( K_r \) is the permeability ratio of each sublayer = \( K/K_H \), and
- \( K_H \) is taken as the permeability value of the porous medium layer in contact with the hot wall.

After the numerical solution of the continuity, momentum and energy equations and obtaining the stream function and the temperature fields as well as the velocities in both the \( x \) and \( y \) directions, equation (1) was integrated numerically to get the \( Nu \). Detailed information for the numerical procedure is found in [1].

3. RESULTS AND DISCUSSION

The two dimensional natural convection in a multilayered with different permeability fluid saturated porous medium has been analyzed for vertical isothermal walls at different temperatures and with adiabatic top and bottom walls. The study was done for 3 layered porous medium in which the first and third layers have equal thicknesses and permeabilities. A wide range of the effective parameters are considered such as:

1. the permeability ratio of the core region varies from 0.1 to 10
2. the ratio of the width of the inner sublayers to the total width of the cavity varies form 0.0 to 1.0.

for a wide range of the Darcy Rayleigh number based on the height of the cavity up to 6000.
3.1 The validity of the model

A relation similar to equation (1) for the heat flow along the cold wall \( x=W, X=1 \) was obtained and integrated numerically. The obtained \( \text{Nu} \) was compared with this calculated from equation (1). The results were found to be nearly identical, the discrepancy between the results of the two approaches was less than 0.75%.

A comparison was done in the first part of this work [1] with the work of Lauriat and Prasad [4] for the values of the vertical velocity \( V/Ra \) and the temperature \( \theta \) at the midheight section where \( Y=0.5 \) and the horizontal velocity \( U/Ra \) at the middle vertical plane \( X=0.5 \) for the case of a single layer porous media with \( W_f = 0.0 \). The comparison shows good agreement.

Another comparison is done with the corresponding values of Lauriat and Prasad [4] for the local \( \text{Nu}_y \) at an aspect ratio \( A=5 \) and \( Ra=500 \) and for the average \( \text{Nu} \) at the hot wall for an aspect ratio \( A=5 \) and \( 0<Ra<6000 \) for the case of the single layer, i.e. \( W_f = 1 \) and \( K_f = 1 \). The comparison is shown in Figs. 2 and 3. The Darcy Rayleigh number \( Ra \) is based on the width of the cavity by Lauriat and Prasad [4], while it is based on the height of the cavity in the present research, so the values of \( Ra \) by Lauriat and Prasad [4] must be changed by the multiplication of the aspect ratio to suit the values of the present definition. The comparison shows also a good agreement and proves the validity of the model.

3.2 Effect of permeability ratio

To show the effect of the permeability ratio, a case of three layers porous media with equal widths are studied. In which the aspect ratio \( A=3 \), the permeability ratios for the inner to the outer sublayers \( K \), differ from 10 to 0.1 and \( Ra=250 \).

The behavior of the local rate of heat transfer \( \text{Nu}_y \) along the hot wall is shown in Fig. 4. It takes the typical trend as obtained in [4,5], in which the higher values of the rate of heat transfer exist at the bottom of the wall and then it drops to the lower values at the top of the wall. Fig. 4 shows the increase of the rate of local heat transfer with the increase of the inner sublayer permeability ratio.
Fig. 5 shows the effect of the permeability ratio of the inner sublayer on the average rate of heat transfer expressed by Nu for different Ra. Fig. 6 shows the variation of Nu/Nu₁ with Ra for different values of Kᵣ. Where, Nu₁ is the value of Nu for the case where the whole cavity is filled with the porous material of the layer adjacent to the hot wall, i.e., Kᵣ = 1 or Wᵣ = 0 for the inner sublayer. It is shown that Nu/Nu₁ is greater than 1 for Kᵣ > 1 and less than 1 for Kᵣ < 1, i.e., Nu is higher than its value for a homogenous filled cavity by Kᵣ > 1 and less than it by Kᵣ < 1.

Both the Nu/Nu₁-Ra and Nu-Ra curves take nearly the same form for the different permeability ratio Kᵣ. In all cases where Ra = 0 at the hot wall, Nu/Nu₁ and Nu take the unity value and the heat is transferred by pure conduction. By the increase of Ra, Nu/Nu₁ increases in a transient region until it takes the parallel straight line form by Ra > 1000, where Nu/Nu₁ depends only on Kᵣ.

The behavior of Nu/Nu₁ is shown in Fig. 7. It gives a linear relation as follows:

\[ \frac{Nu}{Nu_1} = 0.0454Kᵣ + 0.9546 \quad (2) \]

Kim and Vafia [6] studied the natural convection about a vertical plate embedded in a homogenous porous media. They concluded that the Nusselt number depends only on the Rayleigh number Ra in the thermal boundary layer, where the heating effect of the wall is felt. This conclusion is also confirmed by Weber [2] by studying the boundary layer regime for convection in a vertical homogenous porous layer. The mean value of Nu for the boundary layer heat flow in the homogenous porous media can be given according to [2,6] in our notations by the following relation:

\[ Nu = C Ra^{0.5} \quad (3) \]

Where C is a constant and is dependent on the aspect ratio A. By Weber [2] this constant is obtained as \( 1/(\sqrt{3}A) \) for the boundary layer flow. In this case, the heat conducted from the wall into the fluid is carried upwards by the convective movement of the fluid in the steady state, and the fluid is driven upwards by buoyancy and restricted by bulk friction. This means that outside this layer, where the fluid is isothermal and the buoyancy effect is absent the fluid is nearly motionless.
In our case, for the Nu/Nu₁ straight line zone, where Ra>1000, and Nu/Nu₁ is mainly a function of Kᵣ, it can be said that the heat is transferred in a boundary layer flow. This flow consists of upward boundary layer flow on the hot wall, downward boundary layer flow on the cold wall and motionless flow in the core, thus equation (3) can be considered. The constant C in equation (3) must be dependent on both the permeability ratio Kᵣ and the width ratio Wᵣ of the inner sublayer besides the aspect ratio A. Because the aspect ratio A and the width ratio Wᵣ are constants and equation gives that Nu/Nu₁ = f(Kᵣ), it can be said that the effect of both Kᵣ and Wᵣ on C can be separated, and C can be written as:

\[ C = f(Wᵣ) \cdot f(Kᵣ) \cdot f(A) \]  
(4)

3.3 Effect of sublayers width

To show the effect of the sublayers width ratio Wᵣ, constant values for the permeability ratios of the sublayers will be considered. Figs. 8-14 show the effect of the width ratio of the inner sublayer Wᵣ on the Local and mean Nu at the vertical hot wall. The width ratio Wᵣ takes the values 0.0, 0.2, 0.4, 0.6, 0.8, 1.0, for an aspect ratio A=3. Two cases are considered. In the first case, the permeability of the inner sublayer is taken as five times greater than the outer layers Kᵣ=5 and Ra=150. In the second case, the permeability of the inner sublayer is taken as five times less than it in the outer layers Kᵣ=0.2 and Ra=400.

Figs. 8 and 9 show the distribution of the local rate of heat transfer Nu along the hot wall. The Figures show that the rate of local heat transfer increases with the increase of the inner sublayer width ratio for Kᵣ>1, and decreases with the increase of the sublayer width for Kᵣ<1.

Figures 10 and 11 show the variation of the Nu for different values of the width ratio for the two cases of study. The behaviour of the Nu-Ra curve is nearly the same as that expressed in Fig. 5. It consists also from the three zones. The zero Ra at the hot wall where Nu=1 and the heat transferred by pure conduction, the transient zone and the boundary layer flow zone. It is shown that the Nu increases with the increase of the width ratio Wᵣ by the case of Kᵣ=5 and Nu decreases with the increase of the width ratio Wᵣ by the case of Kᵣ=0.2.

Figures 12 and 13 show the behavior of the function Nu/Nu₁ for the two expressed cases. It is shown that the value of Nu is higher than its value for a homogenous filled cavity by Kᵣ=5 and less than it by Kᵣ=0.2. It is also shown
that in the boundary layer flow zone, where $Ra > 1000$, the value of \( \frac{Nu}{Nu_1} \) is nearly constant and depends on the width ratio $W_r$ only. This relation is shown in Fig. 14. It gives a linear relation as follows:

\[
\frac{Nu}{Nu_1} = \pm 0.3836 \ W_r + 1 \quad (5)
\]

the +ve is for the case of $K_r > 1$ and the -ve for $K_r < 1$.

For the general case, equations 2 and 5 can be combined together to give the effect of both the permeability and width ratios in the boundary layer flow regime where $Ra$ is nearly equal to 1000. It can be expressed as follows:

\[
\frac{Nu}{Nu_1} = ( \pm 0.3836 \ W_r + 1) (0.0454 \ K_r + 0.9546) \quad (6)
\]

where the +ve is for the case of $K_r > 1$ and the -ve for $K_r < 1$.

Equation (3) which expresses the heat transfer in the boundary layer flow regime can be developed to take into consideration the effects of both the permeability ratio $K_r$ and width ratio $W_r$ for the three layered porous media and written as:

\[
Nu = C \cdot ( \pm 0.3836 \ W_r + 1) (0.0454 \ K_r + 0.9546) \ Ra^{0.5}
\]

where the +ve is for the case of $K_r > 1$, the -ve for the case of $K_r < 1$ and the constant $C$ depends on the aspect ratio $A$.

4. CONCLUSIONS

The significance of the effect of the non-uniform permeability of the sublayers on natural convection heat transfer in a two-dimensional vertical multilayered porous medium heated from one side has been thoroughly investigated numerically. Considerations were given to 3 layered porous medium in which the first and third layers have equal thicknesses and permeabilities. The variation was made for the permeability and thickness of the inner (second) sublayer.

For a constant width ratio, both the rate of local heat transfer and the mean heat transfer at the hot wall increase with the increase of the inner sublayer permeability ratio and are higher than its values for a homogeneous filled cavity by $K_r > 1$ and less than it by $K_r < 1$. 
For a constant permeability ratio both the rate of local heat transfer at the hot wall and the mean heat transfer increase with the increase of the inner sublayer width ratio for \( K_r > 1 \), and decreases with the increase of the sublayer width for \( K_r < 1 \).

The behavior of \( \text{Nu} \) at the hot wall with the increase of \( Ra_H \) takes 3 stages:
- \( \text{Nu} = 1 \) and the heat transferred by pure conduction when \( Ra_H = 0 \)
- transient heat flow by \( 0 < Ra < 1000 \)
- boundary layer flow by \( Ra > 1000 \), in which the mean rate of heat transfer depends on both the width ratio, the permeability ratio in addition to the dependence on both the Rayleigh number and the aspect ratio. A correlation is derived numerically for this relationship as follows:

\[
\text{Nu} = C \cdot (0.03836 \cdot W_r + 1) \cdot (0.0454 \cdot K_r + 0.9546) \cdot Ra^{0.5}
\]

where \( C \) is a constant depends of the aspect ratio \( A \), the +ve is for the case of \( K_r > 1 \) and the -ve for the case of \( K_r < 1 \).

5. NOMENCLATURE

- **A**: Aspect ratio = \( H/W \)
- **g**: Acceleration due to gravity, \( m^2/s \)
- **H**: Height of the porous material, \( m \)
- **K**: Permeability of the porous layer, \( m^2 \)
- **K_H**: Permeability of the porous layer adjacent to the hot wall, \( m^2 \)
- **K_r**: Ratio of the permeability of the porous layer to the permeability of the porous layer adjacent to the hot wall = \( K/K_H \)
- **Nu**: mean Nusselt number,
- **Nu_H**: Nu for the case where the whole cavity is filled with the porous material of the layer adjacent to the hot wall, i.e. \( K_r \) for the inner sublayer = 1 or \( W_r = 0 \).
- **Nu_H**: the local Nu at the vertical hot wall
- **p**: Pressure, Pa
- **Ra**: Darcy-Rayleigh number = \( g \cdot \beta \cdot K \cdot H \cdot (T_H - T_C)/\alpha \cdot u \)
- **Ra_H**: Darcy-Rayleigh number for the layer adjacent to the hot wall
- **T**: Temperature, K
- **T_H, T_C**: Temperature of the hot and cold isothermal surfaces, K
- **u, v**: Field velocities in the x and y directions, m/s
\( U, V \)  
Non-dimensional field velocities in the \( X \) and \( Y \) directions respectively

\( x, y \)  
Spatial coordinates

\( X, Y \)  
Dimensionless distances in the \( x \) and \( y \) axis respectively

\( W \)  
Width of the porous material, \( m \)

\( W_r \)  
Width ratio

\( \alpha \)  
Thermal diffusivity of the porous layers, \( m^2/s \)

\( \beta \)  
Coefficient of volumetric thermal expansion, \( 1/K \)

\( \mu \)  
Dynamic viscosity of the fluid

\( \nu \)  
Kinematic viscosity of the fluid, \( m^2/s \)

\( \rho \)  
Fluid density

\( \psi \)  
Stream function

\( \psi \)  
Dimensionless stream function

\( \theta \)  
Non-dimensional temperature = \((T-T_C)/(T_H-T_C)\)

6. REFERENCES


Fig. 1 Schematic diagram of the rectangular multilayered porous cavity

Fig. 2 The local $Nu$ along the hot wall
- Lauriat and Prasad [4]
- Present work

Fig. 3 The average $Nu$ with $Ra$ along the hot wall
- Lauriat and Prasad [4]
- Present work

Fig. 4 Effect of the permeability ratio of the inner sublayer $Kr$ on the local $Nu$ at the hot wall
- $Nu = 250$
- $Wr = 1/3$
- $A = 3$
Fig. 5 Effect of the permeability ratio of the inner sublayer $Kr$ on the average $Nu$ at the hot wall

Fig. 6 Effect of the permeability ratio of the inner sublayer on the behavior of $Nu/Nu_1$ at the hot wall

Fig. 7 The behavior of $Nu/Nu$ at the hot wall with the permeability ratio of the inner sublayer in the boundary layer regime
Fig. 8 The local Nu at the hot wall for Kr=5 and Ra=150

Fig. 9 The local Nu at the hot wall for Kr=0.2 and Ra=150

Fig. 10 Effect of the Width ratio of the inner sublayer Wr on the average Nu at the hot wall for Kr=5,
Fig. 11. Effect of the Width ratio of the inner sublayer $W_r$ on the average $N_u$ at the hot wall for $K_r=0.2$

Fig. 12. Effect of the Width ratio of the inner sublayer $W_r$ on $N_u/N_u$ at the hot wall for $K_r=5$

Fig. 13. Effect of the Width ratio of the inner sublayer $W_r$ on $N_u/N_u$ at the hot wall for $K_r=0.2$

Fig. 14. The behavior of $N_u/N_u$ at the hot wall with the Width ratio of the inner sublayer in the boundary layer regime