Mansoura Engineering Journal, Vol. 19, No.2, June 1994.

SPECIFICATION LIMITS AND QUALITY OF CONFORMANCE OF RANDOMLY ASSEMBLED PRODUCTS

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حدود المواصفات (المقاسات) التصميمية وجودة المطابقة للمنتجات المجمعة عشوائيا

خلاصة :

تعرف جودة مطابقة المنتج بمدى (درجة) تطابق مواصفاته (مقاساته) الحقيقية مع مواصفاته المحددة له • ويتعلق موضوع هذا البحث في مدى مطابقة المنتجات المصنعة بالتجميع العشوائي لاجزائها • وقدتم تعريف مطابقة المنتج المصنع باحتمالية توافق هذا المنتج مع المواصفات (التفاوتات) التصميمية التي حددت له •كما ويقدم هذا البحث نماذجا (متباينات) لتمثيل العلاقة بين التفاوتات الاجمالية النهائية للمنتج المجمع من اكثر من جزء ، والتفاوتات الفردية للاجزاء المكونة له ، ومدى مطابقته •كما ويستعرض البحث كيفية الاستفادة من هذه النماذج في التصنيع بهدف تحديد المواصفات التصميمية للمنتج المصنع بالتجميع كوحدة كاملة ولاجزائه كوحدات منفصلة لضمان درجة معينة من المطابقة في مواصفاته الحقيقية الناتجة بعد التصنيع •

ABSTRACT

Quality of conformance is defined as the degree to which a product or service conforms to its design specifications. In this paper, random assembly products are considered, and the degree of conformity of a random assembly is defined as the probability that the assembly will meet its specified tolerance. Furthermore, Models of inequalities are formulated to show the relationships between assembly total tolerances, individual part tolerances, and assembly degree of conformity for a random assembly consisting of two or more parts (components). These inequalities can serve as a tool to set up parts dimensional tolerances and/or assembly dimensional tolerances so that a predetermined assembly degree of conformity is met, and vise versa.

INTRODUCTION AND BACKGROUND

The purpose of a manufacturing system is to produce saleable goo saleable the goods must function satisfactorily, and must fur customers demands. Manufacturing is based on production specific costs. Production specifications usually incorporate dimensional materials to be used, surface finishes, any heat treatments, etc.

Interchangeable manufacturing systems for quantity production have recently been widely used because they have certain economic advantages: (1) parts can be produced in quantity with less demand on labor skill and effort, (2) parts can be assembled instead of fitted, (3) assemblies so made can be serviced by a simple system of replacement parts drawn from stock. This is more convenient for the user and cheaper than product reconditioning which may involve the manufacture of new parts [6].

Dimensional tolerance has been defined as the permissible or acceptable variation in the dimensions such as height, width, length, depth, angles, and diameter of a part or an assembly of parts [5]. Tolerances are unavoidable in production. This is because it is virtually impossible to manufacture two parts precisely of the same dimension. It is well accepted that in most cases, the smaller the tolerance, the better will be the quality of the product. However, smaller tolerances require the use of high precision machine tools in manufacturing the parts therefore increase production cost. Figure 1 indicates the relationship between the tolerance and the production costs.

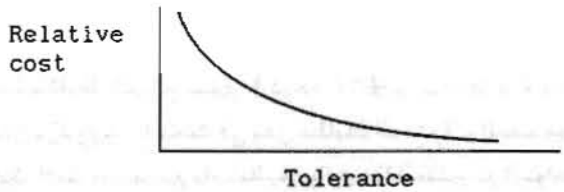

Figure 1: The relationship between tolerance and production costs.

As can be seen, very small tolerances result in very high production cost. Therefore, small tolerances should not be specified when designing components unless they serve a certain purpose in that design [11].

There are three different attitudes involving tolerance specification. The first concerns the designer, whose goal is to ensure proper function. The second are those who must see the part is manufactured; their goal is to produce the part as economically as possible. Finally, those responsible for assembling individual parts into components and units; their major concern is to complete the assembly without problems [7].

The quality of design of a product, whether a part or an assembly, is determined by the product technical specifications, while the quality of conformance of a product depends solely upon the manufacturing process ability to meet the product specifications requirements [1]. It is management's responsibility, with respect to the setting of specifications, to ensure that compatibility exists among those who design products, those who manufacture them, and those who will use them. Whether this compatibility is achieved or not, the suitability of a particular process or equipment will be assessed on the basis of existing product specifications. Quality, on the other hand, has been defined in terms of "quality of conformance" [2]. The use of the term and the context within which the word "quality" is used is defined as the degree to which a product or service conforms to its design specifications (tolerances). Thus, the degree of conformity of a product may be expressed as the probability that it will satisfy its specified tolerance for its quality characteristic. When dealing with a quality characteristic that can be expressed as measurement, it is customary to exercise control over both the average (mean) value of the quality characteristic and its variability (variance). Such control is made by the statistical control charts for means and variance.

Statistically, a process is said to be under statistical control if its mean value (the mean value of the quality characteristic being controlled) is centered on the desired value and its variability is totally attributed to chance causes. When this is the case, the process can be said to be functioning at the predetermined level of accuracy. For more details refer to statistical quality control books such as [1], [2], and [9].

A main purpose of interchangeable manufacturing systems is to produce parts in large quantities. If the manufacturing process is under statistical control, then the variations of this process is assumed to be normally distributed. This leads to the assumption that dimensional variation in parts produced is normal. Furthermore, the sampling distribution of the process variables are considered to be normally distributed based on the "Central Limit Theorem" of statistics. Hence, individual part dimension and part mean dimension may be considered as independent, normally distributed random variables. Therefore, most quality control practitioners set production process natural tolerance limits at ±3 standard deviations from the mean to cover 99.78% of the area under the curve. Figure 2 shows the $\pm\sigma$, $\pm2\sigma$, and $\pm3\sigma$ intervals from the mean of the normal distribution, with their corresponding %age areas under the curve, where $\bar{X}$, σ are the mean and standard deviation of the distribution. For more details see [1], [2], [3], [6].

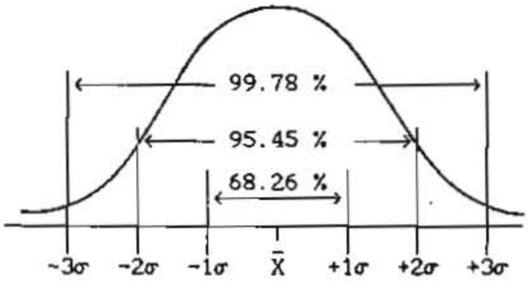

Figure 2: The normal probability distribution.

One way of determining a dimensional tolerance of an assembly is by algebraic sum of the dimensional tolerances of its constituent parts or components. But due to the random variation in the variables of the manufacturing process, the dimensional tolerances of the parts produced may vary from part to part. Hence, when extreme values of part dimensional tolerances occur within an assembly, the resulting assembly dimensional tolerance will fall outside the tolerance range prescribed for the assembly. This occurs because of the tolerance "build up" effect in assemblies due to the algebraic accumulation of individual part tolerances [6], [2]. Therefore, if parts and assembly dimensional tolerances are based on algebraic sum relationship, replacement parts cannot then be supplied separately. To avoid tolerance build up in random assemblies, statistical tolerance is used to assign assembly and constituent parts dimensional tolerances.

In the following section, models of inequalities are formulated to show the relationships between assembly total tolerances, individual part tolerances, and assembly degree of conformity for a random assembly product consisting of two or more parts (components). These inequalities can serve as a tool to set up parts and/or assembly dimensional tolerances, so that a prescribed degree of conformity for the assembly is achieved, and vise versa.

FORMULATION OF MODELS

The following assumptions are considered in the formulation of the models:

(1) The variation of sizes for each dimension considered is normally distributed.
(2) The individual part dimensions are completely independent of each other.
(3) Individual parts to be assembled are randomly selected from large quantities.
(4) The basic (nominal) size of each part is equal to the mean of all such parts, and the tolerance range is equal to 6 standard deviations. That is, if a bilateral tolerance is given as ±0.003, then the standard deviation is equal to 0.001.

Consider two randomly selected parts A, and B, as shown in Figure 3 , whose basic sizes (mean dimensions) are $\bar{X}_a$ and $\bar{X}_b$ respectively, with upper and lower tolerance limits of (U_a, L_a) and (U_b, L_b) respectively, are to be stacked together in an assembly. Let the resulting dimension of the assembly be Y with upper and lower tolerance limits of U_y and L_y, respectively.

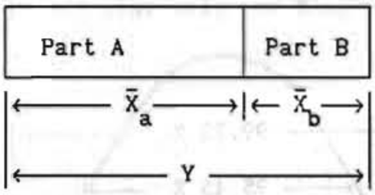

Figure 3: An assembly consisting of two parts A and B.

Based on the randomness of the selection and the "Central Limit Theorem" $\bar{X}_a$, $\bar{X}_b$ are independent, normally distributed random variables, while Y is the dependent random variable. The dimension of the assembly Y can be found by the following equation:

$$Y = \bar{X}_a + \bar{X}_b \qquad (1)$$

since $\bar{X}_a$ and $\bar{X}_b$ are independent, normally distributed random variables, then by the reproductive property of the normal distribution [3], the dimension of the assembly Y is normally distributed with a mean of $\bar{Y}$ and a standard deviation σ_y given by

$$\sigma_y^2 = \sigma_a^2 + \sigma_b^2 \qquad (2)$$

where σ_a and σ_b are the standard deviations of $\bar{X}_a$ and $\bar{X}_b$ respectively.

Assuming a tolerance range of ±3 standard deviations for the assembly tolerance limits, the assembly standard deviation can be found in terms of the assembly tolerance limits as follows:

$$\sigma_y = \frac{U_y - L_y}{6} \qquad \text{..................................(3)}$$

and,

$$\sigma_a = \frac{U_a - L_a}{6} \qquad \text{..................................(4)}$$

$$\sigma_b = \frac{U_b - L_b}{6} \qquad \text{..................................(5)}$$

An important inequality in statistics is the Camp and Meiddle inequality [2]. It is an adaptation of Tchebycheff's inequality [3]. Camp and Meiddle state that under certain circumstances more than $1-(1/2.25t^2)$ of any distribution will fall within the closed range $\bar{X}\pm t\sigma$, where $\bar{X}$, and σ are the mean and standard deviation of the distribution, and $t>1$. These circumstances are that the distribution must have only one mode, and the mode must be the same as the arithmetic mean, and that the frequencies must decline continuously on both sides of the mode.

As can be seen, the Camp-Meiddle inequality can be applied to the distribution of Y (the dimension of the assembly) since it is normally distributed and satisfies the conditions stated for the inequality. Hence, the degree of conformity of the assembly, that is the probability that the basic size of the assembly falls within its upper and lower tolerance limits can be denoted by DC and expressed as,

$$DC \geq 1-(1/\ 2.25t^2) \qquad \text{..........................(6)}$$

Now, consider the following two cases in which the basic size of the assembly ($\bar{Y}$) is centered, and is not centered, between the lower (L_y) and upper (U_y) specification limits as shown in Figure 4.

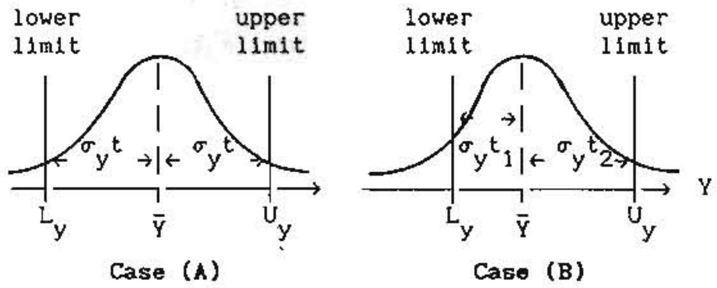

Figure 4: Upper and lower specification limits of $\bar{Y}$

Case (A): If the upper and lower tolerance limits of the assembly are at equal distances from the basic (nominal) size (dimension) $\bar{Y}$.

Since the range specified by Camp-Meidlle inequality is $\pm\sigma t$, then t can be found in terms of σ_y, U_y, and L_y as follows:

$$t = 0.5\left[\frac{U_y - L_y}{\sigma_y}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

by substitution for t in Inequality (6) it follows:

$$DC \geq 1 - \frac{4.0}{2.25}\left[\frac{\sigma_y^2}{(U_y - L_y)^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

then, substituting for σ_y^2 from Equation (2) in Inequality (8), we get

$$DC \geq 1 - \frac{4.0}{2.25}\left[\frac{\sigma_a^2 + \sigma_b^2}{(U_y - L_y)^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

and substituting for σ_a and σ_b from Equations (4) and (5) in Inequality (9), it results that,

$$DC \geq 1 - \frac{4.0}{81.0}\left[\frac{(U_a - L_a)^2 + (U_b - L_b)^2}{(U_y - L_y)^2}\right] \quad \ldots\ldots\ldots\ldots\ldots(10)$$

Case (B): If the upper and lower tolerance limits of the assembly are not at **equal distances from the basic (nominaL) size (dimension) $\bar{Y}$.**

In this case, instead of having $(\sigma_y t)$ at both sides of $\bar{Y}$, the tolerenc range of $\bar{Y}$ is decomposed into $(\sigma_y t_1)$ and $(\sigma_y t_2)$ on the left and the right sides of $\bar{Y}$ respectively, as shown in Figure 4 (note, $t_1 + t_2 = 2t$). Hence, the degree of conformity of the assembly becomes:

$$DC \geq 0.5\left[1 - \frac{1}{2.25\ t_1^2}\right] + 0.5\left[1 - \frac{1}{2.25\ t_2^2}\right] \quad \ldots\ldots\ldots\ldots(11)$$

substituting for $t_1 = (\bar{Y} - L_y)/\sigma_y$ and $t_2 = (U_y - \bar{Y})/\sigma_y$ in Inequality (11), we get

$$DC \geq 1 - (\sigma_y^2 / 4.5)\left[\frac{1}{(\bar{Y} - L_y)^2} + \frac{1}{(U_y - \bar{Y})^2}\right] \ldots\ldots\ldots(12)$$

but from Equations (2), (4), and (5) we have,

$$\sigma_y^2 = \left[(U_a - L_a)^2 + (U_b - L_b)^2\right] / 36.0$$

and by substitution for σ_y^2 in Inequality (12), the following resultant inequality is obtained.

$$DC \geq 1 - \frac{1}{162.0}\left[(U_a - L_a)^2 + (U_b - L_b)^2\right]\left[\frac{1}{(\bar{Y} - L_y)^2} + \frac{1}{(U_y - \bar{Y})^2}\right] \quad \ldots.(13)$$

Generalization of Models

Consider an assembly of n parts. Part i has a mean (nominal) dimension (of a quality characteristic variable) $\bar{X}_i$ with upper and lower tolerance (specification) limits U_i and L_i respectively, for i=1,2,...,n. Let the total assembly resulting dimension be T, then the mean (nominal) dimension of the assembly will be $\bar{T}$, with upper and lower tolerance (specification) limits of U_T, and L_T respectively.

The general form of Inequality (10) becomes as follows:

$$DC \geq 1 - \frac{4.0}{81.0}\left[\frac{\sum_{i=1}^{i=n} (U_i - L_i)^2}{(U_T - L_T)^2}\right]$$

or,

$$DC \geq 1 - \left[\frac{4.0 \sum_{i=1}^{i=n} (U_i - L_i)^2}{81.0\,(U_T - L_T)^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(14)$$

and, the general form of Inequality (13) becomes as follows:

$$DC \geq 1 - \frac{1}{162.0}\left[\sum_{i=1}^{i=n} (U_i - L_i)^2\right]\left[\frac{1}{(\bar{T} - L_T)^2} + \frac{1}{(U_T - \bar{T})^2}\right]$$

or,

$$DC \geq 1 - \left[\frac{\sum_{i=1}^{i=n} (U_i - L_i)^2}{162.0}\right]\left[\frac{1}{(\bar{T} - L_T)^2} + \frac{1}{(U_T - \bar{T})^2}\right] \quad \ldots\ldots\ldots.(15)$$

It can be observed from Inequalities (14) and (15) that the maximum possible degree of conformity of the assembly can be better achieved when the upper and lower specification limits of the assembly dimension are both specified (i.e. bilateral tolerance), and placed at equal distances from the mean (basic) dimension (size) of the assembly. It can also be verified that Inequality (15) yields to Inequality (14) if tolerance limits L_T and U_T are placed at equal distances from the mean $\bar{T}$. This can be done by substituting $(U_T-L_T)/2$ for both $(\bar{T}-L_T)$ and $(U_T-\bar{T})$, since each term will count for half of the tolerance range in this case.

However, unilateral specification (tolerance) limits may also be used. This still can be facilitated by Inequality (15), by excluding the term which is related to the unspecified limit from the inequality. This exclusion can be made by assigning a value of infinity to the unspecified limit. For example, if only the upper specification limit for the assembly (U_T) is specified, then the lower specification limit for the assembly (L_T) is assigned a value of infinity, and therefore the term $1/(\bar{T}-L_T)$ will equate to zero and be eliminated from the inequality.

Other cases may arise when using Inequalities (14) and (15). Such cases and the accompanying manipulations in the inequalities may be identified as follows:

1. A lower bound on the degree of conformity (DC) of the assembled product may be found by substituting the inequality sign by an equality sign in Inequalities (14) and (15).

2. Suppose the degree of conformity of the assembly is specified, and the specification limits of each constituent part is known, and it is required to find the specification limits of the assembly. In this case, all is required is simple mathematical manipulations on the models so that the terms containing U_T and L_T are expressed in terms of DC, U_i and L_i.

3. On the other hand, when the assembly degree of conformity and the assembly specification limits are known, and it is required to determine the specification limits of each component part, i.e. U_i, and L_i. In this case, the summation term of the U_i and L_i needs to be expressed in terms of DC, U_T and L_T. Furthermore, the specification limits U_i, and L_i could be the same for all constituent parts, if not, then they can be expressed as ratios of each other.

In all cases a viable and rational tool for solving these models is a computer program that incorporates some search methods of engineering optimization [10].

CONCLUSIONS

Quality and reliability are important attributes of products and systems [4]. Interchangeable manufacturing facilitates quantity production of parts, assembly and replacement of parts, at lower costs and higher quality of conformance. Quality of conformance of parts and assemblies may be expressed in terms of the degree to which manufactured products, consisting of one part or assemblies of parts. adhere to their prescribed specification (tolerance) limits. Assembly product specification limits should be related statistically, not algebraically, to the specification limits of its constituent parts or components.

The quality of conformance, or the degree of conformity of a random assembly product in terms of the specification (tolerance) limits of its quality characteristic was defined as the probability that the product will meet such specifications. As noted before, this conformance is essential to facilitate parts assembly and replacement. The models of inequalities presented in this paper can serve as a tool for relating specification limits incorporated in product design and manufacturing. They are applicable for both random assembly products and their associated components and/or parts.

REFERENCES

1. DelMar, D., and Sheldon. G., *Introduction to Quality Control*, West Pub. Co., New York, 1988.

2. Grant, E. L., and Leavenworth, R. S., *Statistical Quality Control*, 6th ed. McGraw-Hill Int., New York, 1988.

3. Hines, W. W., and Montgomery, D. C., *Probability and Statistics In Engineering and Management Science*, John Wiley & Sons Inc., New York, 1972.

4. Kapur, K. C.,and Lamberson, L. R., *Reliability in Engineering Design*, John Wiley & Sons Inc., New York, 1977.

5. Kalpakjian, S., *Manufacturing Processes for Engineering Materials*, 2d ed., Addison Wealey Inc., New York, 1991.

6. Lissaman, A. J., and Martin, S. J., *Principles of Engineering Production*, 2d ed., ELBS, Sevenoaks, Kent, 1982.

7. Ludema, K. C., and Cddell, R. M., and Atkins, A. G., *Manufacturing Engineering: Economics And Processes*, Printice-Hall Int., Englewood Cliffs, New Jersey, 1987.

8. Miller, I., and Fruend, J. E., and Johnson, R. A., *Probability and Statistics for Engineers*, 4th ed., Printice-Hall Inc., Englewood Cliffs, New Jersey, 1990.

9. Ott, E. R., *Process Quality Control: Troubleshooting and Interpretation of Data*, McGraw-Hill Int., New York, 1975.

10. Reklaitis, G. V., and Ravindran, A., and Ragsdell, K. M., *Engineering Optimization: Methods and Applications*, John Wiley & Sons. New York, 1983.

11. Wakil, S. D., *Processes and Design for Manufacturing*, , Printice-Hall Inc., Englewood Cliffs, New Jersey, 1989.