NATURAL CONVECTION HEAT TRANSFER IN PARTIALLY DIVIDED VERTICAL ENCLOSURES WITH ADIABATIC END WALLS

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ABSTRACT - A numerical study has been carried out on natural convection, which occurs in building enclosures, with perfectly adiabatic horizontal end walls and finitely adiabatic baffles extending from the ceiling of the enclosures. The results have been obtained for the air-filled (Pr = 0.71) and for water-filled (Pr = 3.7) enclosures with isothenral hot and cold walls. The flow is considered to be two dimensional, laminar and steady. During the course of this work Rayleigh number has a value varies between $10^4$ to $10^7$ and the aspect ratio of the enclosure has two values (A = 1 and 1/2), and the aperture ratio has three value (A_p = 3/4, 1/2 and 1/4). The effect of the aspect ratio and the effect of the aperture ratio on the heat transfer and the natural convection flow are discussed. At low Rayleigh separation bubble increases while the strength of the main flow decreases with the decrease of the aperture ratio. The average Nusselt number for the enclosure is significantly smaller in the presence of the baffles.

INTRODUCTION

Natural convection in enclosure is of great interest in the areas of electronic cooling, design of energy efficient buildings, solar collector design, etc. In the light of this, the subject has received a fair amount attention in the literature. Ostrach [1] and Catton [2] have reviewed the heat transfer literature pertaining to enclosure convection.

Recently, heat transfer in partially divided enclosures has received limited attention due to its application in the design of energy efficient buildings. Proben and Word [3] studied experimentally the heat transfer behaviour in a partitioned enclosure with aspect ratios of 18.2 and 26.4. Janikowski and co-workers [4] extended the work in ref. [3] to investigate the thermal resistance of an air filled enclosure with an aspect ratio of 5 and fitted with vertical baffles extending from the floor and ceiling. Changes et. al [5,6] have presented detailed numerical computations of natural convection in a square cavity fitted with adiabatic partitions extending from the floor and ceiling. Their results are obtained for Rayleigh number in the range of $10^6$ to $10^7$.
and the radiation effect is also considered. Bajorek and Lloyd [7] carried out an experimental study of heat transfer for the same configuration as it used in refs. [5,6]. A typical comparison of the results in refs. [5,7] shows that the experimentally determined Nusselt numbers are significantly higher than the predicted values. To explain this disagreement, it should be noted that in the numerical calculations [5,6], the partitions and end walls are assumed to be adiabatic. Bajorek and Lloyd [7] used plexiglass side walls and baffles which have a thermal conductivity nearly 25 times that of air. However, Zimerman and Acharya [8] studied numerically the free convection heat transfer in an enclosure with perfectly conducting horizontal end walls and finitely conducting baffles. Results obtained using the Boussinesq model for density variation, agree reasonably well with measurements of refs [5,6]. Except at low Rayleigh numbers, a separation bubble is observed behind the baffle. The strength of the separation bubble increases while the strength of the main flow decreases with increasing baffle conductivity. They concluded that, the average Nusselt number for the enclosure is significantly smaller in the presence of the baffles. Except at low Rayleigh numbers, where baffle conductivity has little influence, the Nusselt number values decrease with increasing baffle conductivity. Olsen and co-workers [9] experimentally studied and steady-state natural convection, which occurs in building enclosures for Rayleigh number of order 10^5 in a full-scale room and in a small-scale physical model containing R = 114. For isothermal opposing end walls at different temperatures, a quite good agreement is found between the full-scale room and the scale model in flow patterns, velocity levels, temperature distributions, and heat transfer, even though the radiation heat transfer is not scaled between the two models. Their configurations are tested with the enclosure empty, with a vertical partition extending from the floor to midheight, and with the vertical partition raised slightly off the floor. A numerical finite difference study has been carried out, by Chang et al. [10], for the two-dimensional radiation-natural convection interaction phenomena in square enclosures with equal vertical finite-thickness partitions located at the centers of the ceiling and floor. In their work both participating gases (CO_2 and NH_3) and nonparticipating gas (air) are considered. Results of a companion pure natural-convection study dealing with the effects of Grashof numbers and size and location of the partitions are given in [11]. They adopted a nonuniform grid and the upwind difference scheme is used. The difference equations are formulated on the bases of the control volume approach. It is found that, the partitions provide an effective means to increase the internal resistance. Nansteel and Groff [12] experimentally investigated the enclosure fitted with partial vertical divisions located at the center of ceilings. The study was carried out with water (Pr = 3.5) for Rayleigh number in the range of 2.3 x 10^6 - 1.1 x 10^7 and for aspect ratio A = 1/2 with different values of aperture ratio A_p = 1, 1/3, 1/2 and 1/4. In their work, the laminar flow pattern is observed for A_p < 1. The results obtained in ref. [12] have been correlated, for the case of adiabatic partial divisions as follows:

\[ Nu = 0.762 \frac{0.473}{A_p} + 0.226 \]  

Consequently the above literature, one may conclude that, the partial divisions decrease the overall heat transfer coefficient. And the adiabatic partitions increase the thermal resistance. Correlation (1) shows the dependence of the Nusselt number on the aperture ratio (A_p).
The objective of the present study is to study the heat transfer by natural convection in a two-dimensional rectangular enclosure fitted with partial vertical divisions extending from the centers of the ceiling. The horizontal walls of the enclosure are adiabatic, while the vertical walls are maintained at different temperatures. The partial vertical divisions are assumed to be adiabatic and give the aperture ratio \( \lambda_p = 3/4, 1/2 \) and 1/4. The study is carried out with water (Pr = 3.7) and with air (Pr = 0.7) and for Rayleigh number in the range of \( 10^4 \) - \( 10^7 \). The investigated enclosures have aspect ratios \( A = 1 \) and 1/2. The governing equations of the problem have been solved by using the semi-implicit method for pressure linked equation [13-15]. The power law difference scheme has been used.

**MATHEMATICAL FORMULATION**

Consider the two-dimensional rectangular enclosure shown in Fig. (1), and assume that the enclosed fluid (air) is Newtonian and of constant transport properties. Further assume that the fluid is Boussinesq - incompressible, then the following statements for the conservation of mass, momentum, and energy can be written:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

(2)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}
\]

(3)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + Gr \cdot \theta
\]

(4)

\[
\frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

(5)

These dimensionless equations are based on the following set of definitions, in which the lower-case symbols represent the physical (dimensional) quantities:

\[
(X, Y) = \frac{(x, y)}{H}
\]

(6)

\[
(U, V) = \frac{(u, v)}{(\psi / H)}
\]

(7)

\[
P = \frac{p}{(\rho \nu^2 / H)}
\]

(8)

\[
\theta = \frac{(T - T_c)}{(T_b - T_c)}
\]

(9)
\[ Gr = \frac{g \beta H^3 \left( T_h - T_c \right)}{U^2}. \]  \hspace{1cm} (10)

The conditions imposed along the rectangular perimeter of the enclosure cross section state that the top and bottom walls are impermeable, no-slip, and adiabatic, while the side walls are impermeable, no-slip, and at different temperatures:

\begin{align*}
U &= 0, \ V = 0, \ \theta = 0.5 \quad \text{at} \quad X = 0 \\
U &= 0, \ V = 0, \ \theta = -0.5 \quad \text{at} \quad X = A \\
U &= 0, \ V = 0, \ \frac{\partial \theta}{\partial Y} = 0 \quad \text{at} \quad Y = 0 \text{ and } 1
\end{align*} \hspace{1cm} (11, 12, 13)

Along the impermeable vertical partition extending from the centre of the ceiling of the enclosure ( X = A/2), the condition of no slip is:

\[ U = 0 \text{ and } V = 0, \] \hspace{1cm} (14)

in which the + and - subscripts indicate the left and right sides, respectively, of the partition. Another condition is the rectangular enclosure fitted with adiabatic partitions extending from the centre of the ceiling Fig. (1):

\[ \frac{\partial \theta}{\partial X} = 0 \text{ and } \frac{\partial \theta}{\partial Y} = 0 \] \hspace{1cm} (15)

in which the e subscripts indicate the end of the partition.

The domain of the enclosure is divided into 34 x 34 grids and the thickness of the baffle is 3\( \Delta X \), and the baffle length varies according to the aperture ratio value \( A_p = 1/4, 1/2 \text{ and } 1 \).

The dimensionless governing equations in the primitive variable form are solved numerically using SIMPLE method [15]. Finite-difference equations are derived by integration of the differential equations over a small rectangular control volume. The power-law differencing scheme is used in coefficient computations the details of solution is reported in ref. [16].

RESULTS AND DISCUSSION

The main results of the present calculations for a rectangular air-filled enclosures and a water-filled enclosures are presented in graphical and tabular form. Solutions were computed for \( Pr = 0.71 \) (air) and \( Pr = 3.7 \) (water), \( Ra \) varied between \( 10^3 \) and \( 10^6 \) and the cavity aspect ratios \( A = 1, 0.5 \text{ and } 0.3 \) at different aperture ratios. In case of air-filled enclosures the aperture ratios \( A_p = 3/4, 1/2 \text{ and } 1/4 \), while \( A_p = 3/4 \text{ and } 1/2 \) in the case of water-filled enclosures. Table (1) shows the average Nusselt number values for air-filled enclosures and \( Ra = 10^4, 10^5 \text{ and } 10^6 \). In the case of square cavity (A = 1), the results have been reported at \( A_p = 3/4, 1/2 \text{ and } 1/4 \). One may observe that the Nusselt number results have the highest values at the aperture ratio \( A_p = 1/4 \) while the results of the Nusselt number obtained at \( A_p = 1/2 \) have
the lowest values. And as the Rayleigh number value increases the results obtained at $A_p = \frac{1}{4}$ and $\frac{1}{2}$, on Fig. (2), show two parallel lines. On the other hand, table (1) and Fig. (2) show that the results, reported at $A_p = \frac{3}{4}$, located in between the values displayed at $A_p = \frac{1}{4}$ and $\frac{1}{2}$.

<table>
<thead>
<tr>
<th>$A_A$</th>
<th>$A_P$</th>
<th>$Ra_4$</th>
<th>$Ra_5$</th>
<th>$Ra_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$\frac{3}{4}$</td>
<td>2.01</td>
<td>4.21</td>
<td>8.6</td>
</tr>
<tr>
<td>1.0</td>
<td>$\frac{1}{2}$</td>
<td>1.93</td>
<td>3.86</td>
<td>7.66</td>
</tr>
<tr>
<td>1.0</td>
<td>$\frac{1}{4}$</td>
<td>2.3</td>
<td>4.46</td>
<td>8.56</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>1.4</td>
<td>2.21</td>
<td>4.32</td>
</tr>
</tbody>
</table>

Table (1) The average Nusselt number for air-filled enclosure

Table (1) and Fig. (2) illustrate the Nusselt number values obtained at aspect ratio $(A = 1/2)$ and aperture ratio $(A_p = 1/2)$. Obviously, the results obtained at $A = 1/2$ are much over than the whole set of data obtained in the case of square cavity.

Table (2) shows the average Nusselt number values for water-filled enclosure and $Ra = 10^4$, $10^5$ and $10^6$. In the case of square cavity $(A = 1)$, the results presented are for $A_p = 1/2$ and $3/4$. It is seen that the data obtained at $A_p = 1/2$ are higher than the same obtained at $A_p = 3/4$. However, the results obtained for the water-filled enclosure with $A = 1/2$ and $A_p = 1/2$ show a remarkable increase in the Nu values than the same obtained in the case of enclosure with $A = 1.0$.

<table>
<thead>
<tr>
<th>$A_A$</th>
<th>$A_P$</th>
<th>$Ra_4$</th>
<th>$Ra_5$</th>
<th>$Ra_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$\frac{3}{4}$</td>
<td>2.91</td>
<td>8.12</td>
<td>13.61</td>
</tr>
<tr>
<td>1.0</td>
<td>$\frac{1}{2}$</td>
<td>3.57</td>
<td>11.16</td>
<td>18.35</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>5.26</td>
<td>13.75</td>
<td>21.81</td>
</tr>
</tbody>
</table>

Fig. (3) also shows a comparison between the obtained average Nusselt number values for the air-filled enclosures with the aspect ratio $(A)$ equal to $1/2$ without partition and with aperture ratio $(A_p)$ equal to $1/2$. It is seen that the addition of the partition extending from the ceiling decrease the Nusselt number values by about 56%.
FLOW REGIMES

Streamlines are shown in Figs. (4 and 5) with the streamlines plotted in uniform increments of $\Delta \psi$ (from a lower bound of $\psi_L$ to an upper bound of $\psi_U$). The values of $\Delta \psi$ as well as the stream function value $\psi_m$ are indicated on the top right corner of the figure (in case of air filled enclosures Fig. (4)) and at the top of the fig. (5) in case of water filled enclosures.

At a Rayleigh number of $10^4$, the isotherm plot (not shown) is nearly conduction like and the natural convection flow is rather weak ($\psi_m \approx 3.6 \times 10^{-5}$ in case of air and $\psi_m \approx 2.6 \times 10^{-5}$ in case of water). For the above two cases ($A = 1$ and $A_p = 3/4$) the flow does not separate behind the baffle. As the Rayleigh number is increased to $10^5$, the flow is completely separated for the case of air filled enclosure and the maximum stream function value increases about one hundred time as the Rayleigh number value increases from $10^4$ to $10^5$ (Fig. 4). In the case of water filled enclosures the flow does not separate completely and as the Rayleigh number value increases from $10^5$ to $10^6$ the maximum stream function ($\psi_m$) value increases about 150 times as indicated on Fig. (5). One may also observe that, as the aperture ratio ($A_p$) decreases the flow separation behind the baffle takes place in the case of air filled enclosures as well as in the case of water filled enclosures. However, as the aspect ratio ($A$) of the cavity decreases to be equal to 1/2 the flow separation exists even with the aperture ratio equal to 3/4 (see Figs. (4 and 5)). As the Rayleigh number is increased up to $10^7$, the strength of flow field increases and the separation bubbles grow in size as shown in Fig. (5), for the case of $A = 1/2$ and $A_p = 1/2$. One may observe that, the flow in the enclosure shown in Fig. (5), is divided into two main vortices adjusted near the vertical walls of the enclosure.

Out of this discussion one may conclude that, the strength of the separation bubble increases while the strength of the main flow in which moving up the hot wall and down the cold one decreases with the increase of the baffle height.

CONCLUSIONS

In this work an attempt has been made to show that the natural convection that occurs in a building enclosure due to differentially heated and cooled vertical walls can be simulated in models containing either air or water. The used mathematical models are enclosures with vertical partitions extending from the ceiling. During this work Rayleigh number varies between $10^4$ and $10^7$, while Prandtl number equals 0.71 in case of air and equals 3.7 in case of water used as a fluid fills the enclosure. In this investigation two aspect ratios are used ($A = 1$ and 1/2) and almost three aperture ratios also used ($A_p = 3/4$, 1/2 and 1/4).

The foregoing paper concluded that:

1. The average Nusselt number value increases with the decrease of the aperture value.
2. The average Nusselt number value decreases with the decrease of the aspect ratio.
3. As the aperture ratio decreases, i.e. as the partition length increases the flow separation takes place and two eddies appears in the domain of the fluid flow.
4. The aperture ratio has remarkable effect on the rate of heat transfer through the enclosure filled with water than the same filled with air, specially in the case of square cavity.
In general one may conclude that the partition represents a high thermal resistance to the heat transfer across the fluid filled enclosures, with a high sensitivity to the aperture ratio.

**NOMENCLATURE**

- \( A \): aspect ratio, \((H/L)\)
- \( A_p \): aperture ratio, \((h/H)\)
- \( P \): specific heat, \((\text{KJ/Kg, K})\)
- \( g \): acceleration of gravity, \((\text{m/s}^2)\)
- \( \text{Gr} \): Grashof number, \((g \beta \Delta T H^3 / \nu^2)\)
- \( H \): cavity height
- \( \theta \): heat transfer coefficient \((\text{W/m}^2 \text{ K})\)
- \( k \): thermal conductivity, \((\text{W/m} \text{ K})\)
- \( L \): cavity width
- \( Nu \): average Nusselt number, \((h H/k)\)
- \( Ra \): Rayleigh number, \((Gr \ Pr)\)
- \( P \): dimensionless pressure
- \( p \): dimensional pressure
- \( Pr \): Prandtl number, \((\mu cp/k)\)
- \( T \): Temperature, \(\text{K}\)
- \( U,V \): component of dimensionless velocity in X and Y direction respectively
- \( u,v \): component of velocity in the x and y direction, respectively
- \( X,Y \): dimensionless distances in x and y cartesian coordinates
- \( x,y \): rectangular cartesian coordinates
- \( \Delta X, \Delta Y \): dimensionless mesh sizes in x and y cartesian coordinates

**Greek Letters**

- \( \beta \): volumetric expansion, \((1/\text{K})\)
- \( \Theta \): dimensionless temperature
- \( \rho \): fluid density, \((\text{Kg/m}^3)\)
- \( \mu \): dynamic viscosity, \((\text{Kg/m.s})\)
- \( \nu \): kinematic viscosity, \((\text{m}^2/\text{s})\)
- \( \psi \): stream function

**SUBSCRIPTS**

- \( c \): cold side
- \( h \): hot side
- \( l \): lower value
- \( m \): maximum value
- \( u \): upper value
REFERENCES

Fig. (1) The Schematic diagram

Fig. (2) Nusselt Number Values Versus Rayleigh Number for air

Fig. (3) Comparison between results (air) of cavity at $A = \frac{h}{2}$ without partition and with $A_p = \frac{h}{2}$
\[ \Delta \Psi = 3.0 \times 10^{-3} \]
\[ \Psi_m = 3.3 \times 10^{-2} \]
\[ \Delta \Psi = 4.0 \times 10^{-4} \]
\[ \Psi_m = 6.8 \times 10^{-3} \]
\[ \Delta \Psi = 4.0 \times 10^{-5} \]
\[ \Psi_m = 6.8 \times 10^{-3} \]
\[ \Delta \Psi = 5 \times 10^{-3} \]
\[ \Psi_m = 7.7 \times 10^{-2} \]
\[ \Delta \Psi = 3.0 \times 10^{-7} \]
\[ \Psi_m = 4.5 \times 10^{-6} \]

Fig. (4) Streamlines at \( Ra = 10^4 \) and \( 10^6 \) for air at \( \Lambda = 1 \) and \( \Lambda_p = 3/4 \), \( 1/2 \) and at \( \Lambda = 1/2 \) and \( \Lambda_p = 1/2 \)
Fig. (5) Streamlines at $Ra = 10^4, 10^6$ and $10^7$ for water
at $\Lambda = 1$ and $A_p = 3/4$ and $1/2$ and
at $\Lambda = 1/2$ and $A_p = 1/2$