A STUDY OF THE DIAMETER AND
CONTRACTION OF RING SPUN-YARNS

By

I. Rakha* and R. El-Bealy*

* Professor ** Assoc. Prof. in Textile Dept.
Faculty of Engineering, Mansoura University

1. Abstract:

This paper presents some predicting formulas of yarn diameter and contraction of ring spun yarns. The study concerned the influence of twist multiplier, yarn count and bulk density. Experiments carried out on different cotton yarns to justify the results of the theoretical analysis. The results obtained with these empirical formulas agree satisfactorily with the actual data.

1. Yarn Diameter:

Several expressions for the calculation of yarn diameter have been put forward. The practical formulas suggested to express the yarn diameter can be devided into three groups:

1) The first group indicate that the variation of the yarn diameter with yarn count only\(^{(1,2,3)}\). Also, Pierce\(^{(4)}\) and Onion et al.\(^{(5)}\) made their measurements under compression arising from the fabric. Pierce\(^{(4)}\) has stated that for cotton yarn, \(d \ (\text{yarn diameter}) = \frac{36}{\sqrt{N_e}}\).
The constant represents an assumed specific volume of 1.1 c.c/g for all cotton yarns and the effect of tension and twist are ignored. Ashenhurst\(^{(6)}\) used the equation

\[ d = K \sqrt{\text{yards per lb} + 840} \]
where \( K \) is a constant which takes into account bulk

density and compression of the yarn during weaving \( K = 0.95 
for cotton yarn\). Law\(^7\) proposed a similar relationship namely

\[
d = \sqrt{800 \times \text{counts for cotton yarns}}
\]

where \( d \) is the diameter of reciprocal (i.e. the number of

treads that can be laid side by side in unit distance "Inch"

without over lapping).

ii) In the second of these account is taken of the yarn twist\(^{3,15,16}\). Van Issum et al.\(^8\) pointed out that tension

and twist affect yarn diameter. They suggested the form which

can be translated to:

\[
d = K' / \sqrt{Ne} + (\psi - mT) \quad \text{(mills)}
\]

Where \( K' \) is a constant = 46.38

\( Ne \) is the yarn count

\( \psi \) is intercept of the regression line on the axis

of ordinates.

\( m \) is slope of the regression line.

\( T \) is twist per inch.

Also, the effect of twist on yarn diameter statistically
determined and the regression equation take the form\(^9\):

\[
d = 0.1765 + 0.004558T - 0.00114\frac{\alpha}{13} \quad \text{(mm)}
\]

Where \( \alpha \) is the twist factor and \( T \) is the yarn linear density

(tex). Another form\(^10\) for yarn count ranged between 17 tex

and 50 tex with different twist multiplier indicate the yarn

diameter,

\[
d = 4.496 / (\alpha_{2/3})^{0.2117} \cdot \text{Nm}^{0.3915}
\]

iii) The equations of the third group were set up with account taken

of the density of the yarn\(^11,12\)

\[
d = 1.13 \sqrt{\text{Nm}} \cdot \gamma
\]

Where \( \text{Nm} \) is metric yarn count and \( \gamma \) is the bulk density of

the yarn viz 0.3 - 0.9 kg/m\(^3\).
Barella\(^{(12)}\) discuss the influence and application of the
diameter factors and explore the effect of twist upon yarn
density and postulated the law of critical diameter.

\[
d = 2 \frac{\sqrt{\frac{N_m}{10^3 \pi \gamma}}}{\kappa^n} \quad \text{or} \quad d = \frac{a}{N_m} + b_o
\]

Where \(N_m\) : is metric yarn count (m/g) or (Km/kg),
\(\gamma\) : is the yarn density.
\(d_o\) : Experimental yarn diameter in mm and
\(K^n\) : is the coefficient equal to

\[
2 \left( \sqrt{\frac{1}{\pi \gamma}} - \sqrt{\frac{1}{\pi \gamma^3}} \right)
\]

\[
\sqrt{10^3} \cdot BL
\]

Where \(\gamma^3\) the fiber density, \(F_n\) : tenacity (g/den) and \(BL\) : Breaking length in Km and equal to \(F_n \cdot N_m/10^3\).

Monsikova\(^{(14)}\) suggested that the relation between the yarn
diameter and its count is hyperbolic and described by the
following equations:

\[
d = \frac{a_o}{N_m} + b_o \quad \text{or} \quad d = \frac{N_m}{C + F} \quad \text{or} \quad 10^3
\]

Also, the bulk density and count of yarn are linked by

\[
\text{hyberbolic relation} = \frac{N_m}{C + F} \quad \text{or} \quad 10^3
\]

When \(N_m\) is the metric yarn count; \(T\) is the tex of the yarn
and \(\gamma\) is the density of yarn. It was found to vary between
0.53 and 0.78 for \(N_m\) 12 to \(N_m\) 85 while the literature gives a
constant density of 0.8-0.9 Kg/m\(^3\) for cotton yarn. \(a_o, b_0, c, F, A\) and \(F\) are

constant coefficients were calculated by the method of least
squares Barella\(^{(13)}\) has suggested the following formula for
determining the yarn density as a function of twist multiplier

\[
\gamma = a_1 + b_1 \cdot \alpha_T^2 \cdot 10^{-4}
\]
where $a_i$ is the twist multiple in tex system, 
a and $b_i$ are coefficients that vary according to the type of fiber. Also, Karetzky$^{(10)}$ has found the type of fiber. Also, Karetzky$^{(10)}$ has found the relation between turns/m and yarn density as follows

$$\gamma = K_o \left( \frac{\tau}{\eta} \right)^{0.5} \text{ where } K_o = 0.0285$$

In the present work, the first consideration is the prediction of a formula for the calculation of yarn diameter. In these calculations the yarn is usually assumed to have a cylindrical shape with small circular cross-section$^{(21)}$.

The mass in gram of a length of yarn "L meter"

$$M = V \cdot \frac{1}{T} \gamma = \frac{\pi d^2}{4} L \gamma$$

Where $S$ : area of yarn cross-section; $V$ : the volume of length "L" of idealised yarn, $Y_{\gamma}$ : is the specific volume of the yarn expressed in Cm$^3$/g., $\gamma$ is the bulk density of yarn and $T$ is the yarn linear density "tex" by equalizing Equations (1.1) and (1.2) and solving

for diameter

$$d = 0.0357 \left( \frac{T}{\gamma} \right)^{0.5}$$

The formula can be considered as original form for determination yarn diameter but the quantity of bulk density varies as obtained from the earlier literaure$^{(13,14)}$.

From the experimental results shown in Table (1), the relationship between $Y = \frac{1}{\delta}$ and $\gamma$ is plotted graphically as shown in Fig.(1) and represented by the formula

$$\gamma = 0.0294 \frac{T^{0.4887}}{\delta}$$

where $\delta = a / \sqrt{T}$

By combining equations (1.4), (1.5) and (1.3) we get yarn diameter

$$d = 0.2082 \frac{T^{0.622}}{a^{0.844}}$$

Gregory$^{(10)}$ found that the twist multiplier for maximum strength could be represented as a function of "$\beta$" which equal to "$L_f \mu S$" where $L_f$ : is biased mean fiber length, $\mu$ is the fiber friction coefficient and $S$ is mean.
Specific surface of fibers (as a measure of fineness) and this coefficient "β" is derived by Sullivan's theory^{19}. Thus, the twist multiplier "α" calculated from the following formula^{21} taken into account the coefficient "β"

\[ α_f = τ \cdot β^{0.21} \cdot T^{0.58} \]  \hspace{1cm} (1.7)

where \( β = \frac{L_f}{υ_f} \cdot \mu_f \cdot \eta_m \cdot 10^{-2} \)  \hspace{1cm} (1.8)

The values of twist multiplier applied for determining the bulk density "γ" and the following formula obtained and represented graphically in Fig. (2).

\[ γ = 0.19 \left( α_f^{0.64} \cdot 10^{-2} \right) / T^{0.18} \]  \hspace{1cm} (1.9)

By substituting from equation (1.9) in equation (1.3), the yarn diameter can be expressed as follows:

\[ d = 0.0819 \left[ \frac{T^{1.18}}{(α_f \cdot 10^{-2})} \right]^{0.5} \]  \hspace{1cm} (1.10)

In Fig. (3), the equation (1.10) has been represented. In such a graph the values of yarn diameter deduced from equation (1.10) are closely corresponding to those obtained experimentally.

2. Yarn Contraction:

The theoretical calculation of contraction has been dealt with several research workers^{20,21,22}. The treatment is based on the consideration of an idealized twist geometry and the occurrence of fiber migration in the yarn.

Yarn contraction can be defined in terms of the length of zero twist yarn and the length of twisted yarn in two ways:

i) Contraction Factor "Cy" = \( \frac{\text{Length of zero twist yarn}}{\text{Length of twist yarn}} \)

Contraction factor is commonly used with staple yarn and \( 1 \leq Cy < m \), and

ii) Retraction Factor "Ry" :

\[ \frac{\text{Length of zero twist yarn} - \text{length of twisted yarn}}{\text{Length of zero twist yarn}} \]
where \( 0 \leq \text{Ry} < 1 \) and the retraction is very useful when dealing with continuous filament yarn.

Among the theoretical relationship suggested to express yarn contraction factor in terms of twist multiplier we shall retain here one of the best known as following:

\[
C_y = \frac{100}{\sqrt{1 + E \frac{\alpha_t^2}{\gamma}}}
\]

where:

- \( E \) is a constant takes several values as shown in Table (2).

| Table 2 |
|------------------|------------------|
| Besset Barello\(^{(23)}\) | \( E = 5.7 \times 10^{-5} \) |
| Obukh\(^{(24)}\) | \( = 4.9 \times 10^{-5} \) |
| Budnekov\(^{(25)}\) | \( = 5.97 \times 10^{-5} \) |

Another form suggested by Braschler\(^{(26)}\) to express yarn contraction as follow:

\[
C_y = \frac{2}{1 + \sqrt{1 + \frac{125.7}{10^3} \left( \frac{\alpha_t}{100} \right)^2}} \times 100
\]

Generally, from the idealized yarn geometry developed by Hearl\(^{(26)}\), the relationship between the twist angle \( \alpha \), turns per unit length \( \tau \) and diameter of average spiral \( ds \) is:

\[
\tan \alpha = 2 \pi R / h = \pi ds \tau \tag{2.1}
\]

Schwartz\(^{(27)}\) has reported that \( ds = \delta \cdot d \) \( \tag{2.2} \)

where \( \delta \) is known as schwartz’s constant and approaches unity for large numbers of fibers. Also, he has pointed out its usefulness in twist analysis. On the other hand, several research workers has reported the value of \( \delta \) as shown in the following Table (3)

| Table (3) |
|------------------|------------------|
| Barella\(^{(13)}\) | \( \delta = 0.5 \) |
| Sakalov\(^{(28)}\) | \( = 0.71 \) |
| Karetsky\(^{(17)}\) | \( = 0.67 \) |
| Calculated value* | \( = 0.80 \) |

\(*\) From the present investigation
In Basset's report\(^{(23)}\) the influence of twist on the length of cotton yarn is discussed and by analytical means arrives at the equation for yarn contraction

\[
Cy = \left( 1 - \frac{1}{\sqrt{1 + \tan \alpha}} \right) 100 \quad (2.3)
\]

From the theories that explain the phenomenon of yarn contraction, it would be possible in the present investigation to represent the relationship between contraction of ring spun yarns and twist multiplier, bulk density and yarn count.

If the values of yarn diameters Eq. (1.6) and (2.2) substituted in Equation (2.1) we get

\[
\tan \alpha = 0.523 \alpha_t^{0.75} T^{0.132} \quad (2.4)
\]

by combining this with the formula (2.3), a simplified expression is found for the contraction factor.

\[
Cy = \left( 1 - \frac{1}{\sqrt{1 + 0.2735 T^{0.244} \alpha_t^{1.51}}} \right) 100 \quad (2.5)
\]

When the values of \(\alpha_t\) and \(T\) are substituted in equation (2.5), the calculated contraction are not closer to the experimental values. Thus, we consider the formula of yarn diameter shown by equation (1.10).

\[
d = 0.0819 \left( \frac{T^{1.10}}{\alpha_t^{0.54}} \right)^{0.5} \quad (1.10)
\]

and combining this with equation (2.2) in equation (2.1) we get

\[
\tan \alpha = 0.2057 \alpha_t^{0.75} T^{0.09} \quad (2.6)
\]

When the above formula is substituted in equation (2.3) the contractions caused by twist referred to twist coefficient and yarn linear density can be expressed as follows:

\[
Cy = \left( 1 - \frac{1}{\sqrt{1 + 0.0423 \alpha_t^{1.36} T^{0.16}}} \right) 100 \quad (2.7)
\]

Also, a modified form of equation (2.7) in terms of twist multiplier \(\alpha\) and bulk density \(\varphi\) can obtained as follows

\[
Cy = \left( 1 - \frac{1}{\sqrt{1 + 8.05 (\alpha_t^2 / \varphi) 10^{-5}}} \right) 100 \quad (2.8)
\]
The results obtained by use of this equation are presented in Fig. (4). The curves in Fig. (4) indicate the relation between the values of twist multiplier and the percent contraction. The calculated contractions are in almost exact agreement with the experimental results carried-out in the present study and the results obtained according to carminata's tables(29) and those of the Shirley Institute(32) and reproduced here in Fig. (4). On the same figure, it can be seen Brachler's, Bessett-Barella, and Obukh-formulas give the lowest values.

3. Conclusions:

The study of yarn diameter and contraction of ring spun yarns permits establishing the following conclusions:

i) Starting from the established theories for calculating yarn diameter and contraction, the study affords a empirical formula's to predict the probable.

Yarn diameter by means:

\[ d = 0.0819 \left( T^{1.10} / \sigma_{T}^{0.64} \right)^{0.5} \]

Yarn contraction by means:

\[ Cy = \frac{1}{\sqrt{1 + 8.05 (\sigma_{T}^{2}/\gamma) 10^{-5}}} \]

Where \( \sigma \) is the twist multiple, \( \gamma \) is the yarn density and yarn count in tex.

ii) The predicted values of yarn diameter and contraction is clearly dependent on twist multiplier, bulk density and fiber parameters.

iii) The calculated results deduced from the suggested practical formulas agree satisfactorily with the experimental data.

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Fig. (1) Specific volume $v$ (cm$^3$/gm)

Fig. (2) Specific volume $v$ (cm$^3$/gm)
Fig. (3)

Fig. (4)