Mansoura Engineering Journal (MEJ), Vol. 17, No.1, Maech 1992

ESTIMATION OF MOTION PARAMETERS USING A SIMPLIFIED INDIRECT METHOD

تقدير معاملات حركة الاجسام بتبسيط الطريقة
"غير المباشرة"

BY
Prof. Dr. Said H. El-Konyaly, Dr. Yehia M. Enab,
Dr. Sabry F. Saraya, Eng. Hesham I. Soltan.
Dep. of Automatic Control & Computer,
Faculty of Engineering, El-Mansoura Univ., EGYPT.

الملخص

يعالج هذا البحث مسالة تقدير معاملات حركة جسم متحرك كالموقع و السرعة عن طريق تحليل مجموعة من الصور الضوئية ماخوذة على التتابع. وقد استخدمت الطريقة "غير المباشرة" لاتمام ذلك. وتعتمد هذه الطريقة على حساب مايعرف بكمية "السريان الضوئى" ومنها يتم حساب معاملات الحركة المطلوبة. و البحث يقترح طريقة بسيطة نسبيا فى حساب السريان الضوئى بالمقارنة مع الطرق الموجودة حاليا. و الطريقة المقترحة تقلل كثيرا من تاثيرمشكلة "الخروج عن حدود النعومة" فى الصور و الاجسام. و اعتمد البحث على استخدام دالة "اقل قيمة لمتوسط مربع الخطا" استنتاج معاملات الحركة. بالاضافة الى ما سبق عالج البحث العلاقة بين احداثيات "بؤرة التمدد" ومعاملات الحركة وتم عمل المواءمة اللازمةلتقليل الخطا فى عملية التقدير. وقد تم اجراء مجموعة من التجارب المعملية عن طريق استخدام الحاسب فى تحليل مجموعتين من الصور الضوئية. الاولى تخليلية بواسطة برامج خاصة، و الثانية حقيقية عن طريق استخدام كاميرا رقمية ذات تحليلية عالية. و اثبتت النتائج جدوى الطريقة المقترحة فى تقدير معاملات الحركة بدقة معقولة.

KEYWORDS :
Artificial Intelligence, Machine Vision, Optical Flow, Motion Parameters, Focus of Expansion.

ABSTRACT :

In this paper an iterative scheme is used to compute the optical flow for a sequence of images. The problem of violating the smoothness constraint is discsussed and minimization of its effect is done. Also a least-squares algorithm is used to recover the motion parameters of moving camera in a fixed environment. In the case of pure translational motion, it is proved that the motion can be determined (up to a constant factor) by considering the direction of the optical flow at two points. In the same case, the relationship between the coordinates of the focus of expansion (FOE) and the motion parameters is also argued. A proposed norm based on that relationship is minimized to recover the motion parameters. Real and synthetic images are used to test the performance of the proposed technique.

1. INTRODUCTION

The optical flow can be defined as the apparent motion of the brightness pattern in a sequence of images. It carries valuable information about the nature and the depth of surfaces. Also, the relative motion between observer and objects may be detected. This means that, the optical flow can play an important role, as an indirect method, to determine the relative velocity components known as motion parameters. The analysis of dynamic images sequence finds a wide range of applications, including robot navigation, target tracking, traffic monitoring, motion of biological cells, cloud and weather systems tracking, image coding, etc.

A simple iterative algorithm for estimating the optical flow was developed by Horn & Schunck[1]. Also, Nagel[2,3,4], Hildreth[5], and others gave relevant topics. Several schemes for recovering the observer's motion indirectly have been suggested. These approaches can be classified into three categories: the discrete approach, the differential approach, and the least-squares approach[6]. In the least-squares approach, the whole optical flow field is used. A major shortcoming of both the discrete and the differential approaches is that neither allows for errors in the optical flow. This is why the least-squares approach is chosen to be used in this paper. Many papers tackled this subject[7,...,11] in this paper, the least-squares algorithm is used extensively for the cases of pure translational and pure rotational motion. In the case of pure translational motion the definition of Focus of Expansion (FOE) is used to develop a new technique to estimate the motion parameters. The proposed technique is relatively simple and gives a fast response. The complexity of computations in comparison with Horn method is discussed. A series of computer experiments are used to study the performance of this technique using both real and synthetic images.

2. SYSTEM ANALYSIS

2.1 The optical flow problem:

An iterative scheme presented in[6] was used to estimate the optical flow as follows:

$$u_{kl}^{n+1} = \bar{u}_{kl}^{n} - \frac{E_x \bar{u}_{kl}^{n} + E_y \bar{v}_{kl}^{n} + E_t}{1 + \lambda (E_x^2 + E_y^2)} E_x ,$$

$$v_{kl}^{n+1} = \bar{v}_{kl}^{n} - \frac{E_x \bar{u}_{kl}^{n} + E_y \bar{v}_{kl}^{n} + E_t}{1 + \lambda (E_x^2 + E_y^2)} E_y .$$

Where $\bar{u}$, and $\bar{v}$ are local averages of the x and y components respectively of the optical flow, u_{kl}^{n+1}, v_{kl}^{n+1} are the optical flow components at a point with coordinates (k, l) after the (n+1)th iteration, and E_x, E_y, and E_t are the three estimates of the first partial derivatives of brightness with respect to x, y, and t respectively at the point (k, l)[6]. There will be discontinuities in the optical flow on the silhouettes. This was overcome, to some extent, in the implementation by making the first two iterations be performed freely. Next, for each iteration, a threshold, T_1, has its value as a ratio of the maximum amplitude of the optical flow vectors among the image points is obtained. This maximum amplitude is obtained through the previous iteration. If the amplitude of the gradient of the optical flow at any point less than T_1, then the local averages, $\bar{u}$ and $\bar{v}$, must be evaluated separately. Namely, the flow amplitude at the neighbors which differ greatly from that in the current point are not included in the computation of $\bar{u}$ and $\bar{v}$. The difference between the current point flow amplitude and each neighbor point flow amplitude is compared with another threshold T_2. The value of T_2 is chosen empirically as a ratio of T_1. If there is not at least one point has a difference less than this threshold, we increase T_2 smoothly by a constant scale factor and attempt again for the four points. T_1 is decreased smoothly from an iteration to the next by a constant scale factor.

2.2 The recovery of motion parameters

Through the following analysis it will be assumed that the camera is moving with respect to a fixed environment. If there is more than one object with independent motion, then it well be assumed that the image has been segmented and that we can concentrate on one region corresponding to a single object. As shown in Fig.(1), the coordinate system is fixed w.r.t. the camera with the Z-axis pointing along the optical axis. Also, perspective projection is used with the origin be the center of projection and Z=F be the image plane[12], where F denotes the focal length through the paper. Hence, for any object point, the z-coordinate will be restricted to be positive. If the corresponding point of the object point P, $(X, Y, Z)^T$, on the image plane is the point p with the coordinates (x, y) on the image plane coordinate system, then;

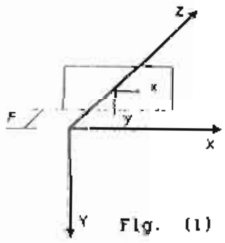

Fig. (1)

$$\frac{x}{F} = \frac{X}{Z} \qquad \frac{y}{F} = \frac{Y}{Z} \tag{1}$$

Let the translational component of the camera velocity be T and its angular velocity be ω. Where $T = (U,V,W)^T$ and $\omega = (A,B,C)^T$. The expected optical flow at the point(x,y), denoted by (u,v), is:

$$u = \dot{x} \quad \text{and} \quad v = \dot{y} \tag{2}$$

If the velocity of the point P in space is V, then;

$$V = -T - \omega \times r \quad \text{Where } r=(X,Y,Z)^T$$

and,
$$V = (\dot{X}, \dot{Y}, \dot{Z})^T$$

With these definitions and using equations (1) and (2), we can obtain the translational and rotational components of the optical flow, (u_t,v_t) and (u_r,v_r) respectively, as functions of the camera motion parameters having the form:

$$u_t = \frac{-UF + xW}{Z}, \qquad v_t = \frac{-VF + yW}{Z} \tag{3}$$

$$u_r = \frac{Axy}{F} - \frac{B(x+F^2)}{F} + Cy, \qquad v_r = \frac{A(y+F^2)}{F} - \frac{Bxy}{F} - Cx. \tag{4}$$

2.3.1 The pure translational motion

As shown by Horn[6], in the case of pure translational motion, the motion of the camera is uniquely determined if it has been determined up to a constant scaling factor[6]. Determining the values Z, U, V, and W which minimizes the $ML_{\alpha\beta}$ norm we can obtain:

$$Z = \frac{\alpha^2 + \beta^2}{u\alpha + v\beta} \qquad (5)$$

Where α and β is defined as:

$$\alpha = -UF + xW \quad \text{and} \quad \beta = -VF + yW .$$

Where Z is the Z-coordinate of the point projected at (x,y), on the image plane, the optical flow at which is (u,v). Eqn 5 is used to determine the structure of the imaged object. Also it is used to determine whether t or -t is the correct solution for the motion parameters problem where Z must be positive. Also;
For the motion parameters, keeping in mind that $U^2 + V^2 + W^2 = 1$;
Let $M = U/W$ and $N = V/W$ we find that,

$$\left.\begin{aligned} &N = \frac{fd - e(a-\lambda_1)}{(b-\lambda_1)(a-\lambda_1) - d^2} \quad \text{and} \quad M = (-e-(b-\lambda_1)N)/d . \\ &\text{Hence, } W = \cos(\tan^{-1}(\sqrt{M^2+N^2})), \quad V = N\,W, \text{ and } \quad U = M\,W. \end{aligned}\right\} \qquad (6)$$

Let's for the following assuming that the image plane is the rectangle $x \in |-L, L|$ and $y \in |-h, h|$, then the variables in (6) can be defined as:

$$a = F^2 \sum_{y=-h}^{h} \sum_{x=-L}^{L} v^2 , \qquad b = F^2 \sum_{y=-h}^{h} \sum_{x=-L}^{L} u^2 ,$$

$$c = \sum_{y=-h}^{h} \sum_{x=-L}^{L} (xv - yu)^2, \qquad d = -F^2 \sum_{y=-h}^{h} \sum_{x=-L}^{L} uv ,$$

$$e = F \sum_{y=-h}^{h} \sum_{x=-L}^{L} u\,(xv - yu) , \qquad f = -F \sum_{y=-h}^{h} \sum_{x=-L}^{L} v\,(xv - yu) .$$

Where λ_1 is the smallest root of the equation;

$$\lambda^3 - (a+b+c)\lambda^2 + (ab+bc+ca-d^2-e^2-f^2)\lambda + (ae^2+bf^2+cd^2-abc-2def) = 0.$$

Note that, this equation has three real positive roots (for more, see Horn[6]).

In the pure translational motion the "Focus of Expansion" (FOE) is defined as the point of intersection of all the image points flow vectors. As shown in Fig.(2), if the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the image plane have the optical flow vectors $[u_1, v_1]^T$ and $[u_2, v_2]^T$ respectively, then from Eqns 3 and 4 we have;

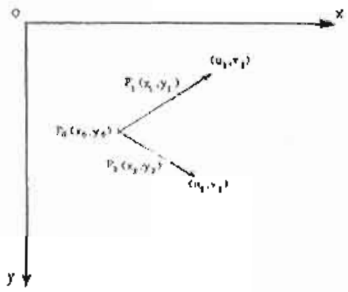

Figure (2)

$$u_1 = \frac{-UF + x_1 W}{Z_1}, \qquad v_1 = \frac{-VF + y_1 W}{Z_1},$$
$$u_2 = \frac{-UF + x_2 W}{Z_2} \quad \text{and} \quad v_2 = \frac{-VF + y_2 W}{Z_2}. \tag{7}$$

The slopes of the straight lines P_1P_0 and P_2P_0, are s_1 and s_2 respectively, where P_0 (x_0, y_0) is the FOE point. These slopes can be determined as follows:

$$s_1 = \frac{y_1 - y_0}{x_1 - x_0} \quad \text{and} \quad s_2 = \frac{y_2 - y_0}{x_2 - x_0} \tag{8}$$

Using Eqn (7), it is also possible to determine these slopes as

$$s_1 = v_1/u_1 = \frac{-VF + y_1 W}{-UF + x_1 W} \quad \text{and} \quad s_2 = v_2/u_2 = \frac{-VF + y_2 W}{-UF + x_2 W} \tag{9}$$

equating the values of s_1 and s_2 from eqns.(8) and (9) and solving for x_0 and y_0, then we have after few mathematical manipulations:

$$x_0 = U\,F/\,W \quad \text{and} \quad y_0 = V\,F/\,W \tag{10}$$

It is evident that the coordinates of FOE point are functions of the motion parameters. The coordinates of the FOE can also be obtained by solving the two equations(8), this gives the solution:

$$\left.\begin{aligned} x_0 &= (x_2 s_2 - x_1 s_1 + y_1 - y_2)/(s_2 - s_1) \quad \text{and} \\ y_0 &= (y_2 s_1 - y_1 s_2 + x_1 - x_2)/(s_1 - s_2) \end{aligned}\right\} \tag{11}$$

Replacing s_1 and s_2 by v_1/u_1 and v_2/u_2 respectively, then;

$$\left.\begin{aligned} x_0 &= [x_2 u_1 v_2 - x_1 u_2 v_1 + u_1 u_2 (y_1 - y_2)]/(u_1 v_2 - u_2 v_1) \\ y_0 &= [y_2 u_2 v_1 - y_1 u_1 v_2 + v_1 v_2 (x_1 - x_2)]/(u_2 v_1 - u_1 v_2) \end{aligned}\right\} \tag{12}$$

The use of the flow information at only two points to determine the motion parameters U,V and W is inaccurate process. This is due to the error associated with the intensity image values used to determine optical flow at each point. Another source of error is the use of the iterative scheme described previously. In order to make use of the optical flow information available at each point in the image, the following error criterion is introduced to estimate the motion parameters,

$$\sum_{y=0}^{h} \sum_{x=-L}^{L} (x_0 - U\,F/W)^2 + (y_0 - V\,F/W)^2$$

Differentiating the integral with respect to U and V, and equating the results to zero:

$$\left.\begin{aligned} U/W &= a/(2\,L\,h\,F), \\ V/W &= b/(2\,L\,h\,F). \end{aligned}\right\} \tag{13}$$

Where;

$$a = \sum_{y=0}^{h} \sum_{x=-L}^{L} x_0, \quad \text{and} \quad b = \sum_{y=0}^{h} \sum_{x=-L}^{L} y_0.$$

Taking pairs of the image points on opposite sides of an image diagonal, i.e.:

$$x_1 = x\ ,\qquad x_2 = -x\ ,\qquad y_1 = y\ ,\qquad y_2 = -y. \tag{14}$$

In fact, this method cannot detect whether t or -t is the correct velocity of the camera. So, we can use equation (5) as explained previously to detect it. Estimation accuracy can be increased by increasing the used point-pairs. This occurs by using another rule for associating this pair differs from the method used in eqn(14). This method of the motion recovery is robust in the case of more accurate flow directions against noise and small perturbations.

COMPUTATION COMPLEXITY

The modified Horn algorithm is tested against the proposed algorithm. Under similar conditions and adjusting the No. of point-pairs to give similar accuracy, the computation time of the proposed algorithm was three-eighth of the modified Horn algorithm. Taking in account that, any repeated multiplication is done once.

2.3.2 The pure rotational case

The angular velocity components in X,Y, and Z directions denoted by A,B, and C respectively are obtained as follows[6].

$$A = [k(bc-e^2/F) - l(cd-ef/F) + m(ed-bf)]/M\ ,$$
$$B = [-k(cd-ef/F) + l(ac-f^2/F) - m(ae-fd)]/M\ ,$$
$$C = [k(ed-bf)/F - l(ae-fd)/F + m(ab-d^2)]/M.$$

After reformulating the eqns in [6] taking the focal length into account and issuing an implementable form we find that:

$$a = \frac{1}{F}\sum_{y=-h}^{h}\ \sum_{x=-L}^{L}(x^2y^2 + (y^2+F^2)^2),$$

$$b = \frac{1}{F}\sum_{y=-h}^{h}\ \sum_{x=-L}^{L}((x^2+F^2)^2 + x^2y^2),$$

$$c = \sum_{y=-h}^{h}\ \sum_{x=-L}^{L}(x^2+y^2)\ ,\quad d = -\frac{1}{F}\sum_{y=-h}^{h}\ \sum_{x=-L}^{L}(xy(x^2+y^2+2F^2))\ ,$$

$$e = -F^2\sum_{y=-h}^{h}\ \sum_{x=-L}^{L} y\ ,\qquad f = -F^2\sum_{y=-h}^{h}\ \sum_{x=-L}^{L} x\ ,$$

$$k = \sum_{y=-h}^{h}\ \sum_{x=-L}^{L}(uxy + v(y^2+F^2)),\quad l = -\sum_{y=-h}^{h}\ \sum_{x=-L}^{L}(u(x^2+F^2) + vxy),$$

$$m = \sum_{y=-h}^{h}\ \sum_{x=-L}^{L}(uy - vx),\qquad M = a(bc - e^2/F) - d(cd-ef/F) + f(ed-bf)/F$$

3.EXPERIMENTAL WORK

Figure 3 shows three synthetic images of a sphere. In this figure, the images (a) and (b) represent case number 1 which is two successive images of a sphere during the motion of the camera slightly along the z-axis in the negative direction. Also, the image (a) and (c) represent case number 2 which is two successive images of a sphere during the rotation of the camera slightly around the z-axis with rotating velocity 0.01 rad/s. For the used system, we took the values, δx=1mm, δy=1mm, and δt=1s. The image (d) is the needle diagram which represents the optical flow of case 1 with λ=10. and the number of iterations is 8. Similarly, the image (e) represents the optical flow of case 2 with the same λ and number of iterations as case 1. The needle diagram is some vectors their amplitudes are indication to the amplitudes of the optical flow at the corresponding image points and theirs directions represents the directions of the optical flow at the corresponding image points. For real images we see that, image (f) and (g) represent case number 3 which is two successive images of a curved wall covered with varying brightness pattern during the translational motion of the camera. The needle diagram of case 3 is shown in image (h). For the image (h) the parameters taken as: λ=3 and the number of iterations is also 8.

The following table displays some results of the motion recovery process with different parameters. For the cases which include pure translational motion, the results are the values of the motion parameters, U, V, and W. But in the case of pure rotational motion, the results are the angular velocity components, A, B, and C. For the translational motion the asterisk to the left of the case number tells that, the used algorithm is the proposed one. Also, the FOE coordinates is valid for the translational case.

case No.	λ	F mm	U	V	W	X_0	Y_0
			A	B	C		
1	10	160	-.0504357	-.0135763	-.998635	8.08074	2.17517
*1	10	160	-.0401729	-.2876312	-.995343	-6.4576	14.08659
2	10	160	6.7779e-5	.00051745	-.0096063	——	——
3	3	160	.0698959	.0191948	.99737	11.2128	3.07927
*3	3	160	-.02173	-.0684174	.99742	-3.48579	-10.9751
3	10	160	.0781309	.0152258	.996827	12.5407	2.44389
*3	10	160	-.0884374	-.130361	.987514	-14.3289	-21.1214
3	15	160	.0796699	.0155798	.9967	12.7894	2.50102
*3	15	160	.144253	-.279073	.949373	24.3113	-47.0327
3	3	130	.0891608	.0240085	.995728	11.6406	3.1345
*3	3	130	-.0267092	-.0840946	.9961	-3.48579	-10.9751
3	3	90	.145356	.036476	.988707	13.2314	3.32034
*3	3	90	-.0384178	-.120959	.991918	-3.48579	-10.9751
3	3	110	.110003	.0289088	.993511	12.1793	3.20074
*3	3	110	-.0315168	-.0992314	.994565	-3.48579	-10.9751

From the experiments of each stage, it was found that mainly, the errors are due to the computation of the optical flow. Also, it was found that the direction of the flow vectors always tends to the direction of the brightness gradient. This is due to the restricted brightness constancy constraint. The angle between the recovered direction of motion and the X-Y plane can be estimated easily from the components U,V, and W. From the table it can be implied that this angle is changed due to the error in the value of the focal length in the translational case. Experimentally it was found that the higher the focal length the greater the angle between the recovered direction and the X-Y plane. The change in λ produces a slight change in the recovered motion parameters. This is because of the limitation in the number of iterations. Experimentally it was found also that the recovered motion parameters has less sensitivity for the change of λ in the case of pure rotational motion .

4. CONCLUSION

In this paper an iterative scheme was used to compute the optical flow using a sequence of images. It was found that, in spite of the violation of the optical flow constraint equation, the results is considerably accurate because of the developed treatment. In the case of pure translational motion, it was proved that the motion can be determined (up to a constant factor) by considering the direction of the optical flow at two points. In the same case, the relationship between the coordinates of the focus of expansion (FOE) and the motion parameters is also argued. The proposed algorithm for the recovery of motion parameters showed more simplicity than the modified Horn algorithm.

5. REFERENCES

[1] Schunck, B.G., & B.K.P. Horn " Constraints on Optical FlowComputation," *Proc. Pattern recognition and image processing conf.*, Dallas, Texas, pp. 205-210, 3-4 August, 1981.

[2] Nagel, H.-H. "On Change Detection Displacement Vector Estimation in Image Sequences," *Pattern Recognition Letters*, Vol. 1, pp. 55-59, October, 1982.

[3] Nagel, H.-H. "Displacement Vectors Derived from Second-Order Intensity Variations in Image Sequences," *Compute Vision, Graphics and Image Processing*, Vol. 21, No. 1, pp. 85-117, january, 1983a.

[4] Nagel, H.-H. "Constraints for the Estimation of Displacement Vector Fields from Image Sequences," *Proc. of the Intern. Joint Conf. on Artificial Intelligence*, Karlsruhe, West Germany, pp. 945-951, 8-12, August, 1983b.

[5] Hildreth, E.C. " The Detection of Intensity Changes by Computer and Biological Vision Systems," *Computer Vision, Graphics and Image Processing*, Vol. 22, No. 1, pp. 1-27, April, 1983.

[6] Berthold Klaus Paul Horn, " Robot Vision," *MIT Press, Massachusetts*, 1986.

[7] Neumann, B. " Exploiting Image Formation Knowledge for Motion Analysis," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. 2, No. 6, pp. 550-554, November, 1980.

[8] Tsai, R.Y., T.S. Huang, & W.-L. Zhu "Estimating Three-Dimensional Motion Parameters of a Rigid Planar Patch, II:Singular Value Decomposition," *IEEE Trans. on Acoustics, Speech, and Signal Processing*, Vol.30, No. 4, pp. 525-534, August, 1982.

[9] Hay, C.J. " Optical Motion and Space Perception - An Extension of Gibson's Analysis," *Psychological Review*, Vol. 73, pp. 550-565, 1966.

[10] Bruss, A.R., & B.K.P. Horn " Passive Navigation,", *Computer Vision, Graphics and Image processing*, Vol. 21, No. 1, pp. 3-20, January, 1983.

[11] Heel - Joachim " Dynamic Motion Vision," *Robotics*, Vol. 6, No. 3, pp. 297-314, Jul, 1990.

[12] Ken-Ichi Kanatani " Transformation of Optical Flow by Camera Rotation,", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. 10, No. 2, March 1988.

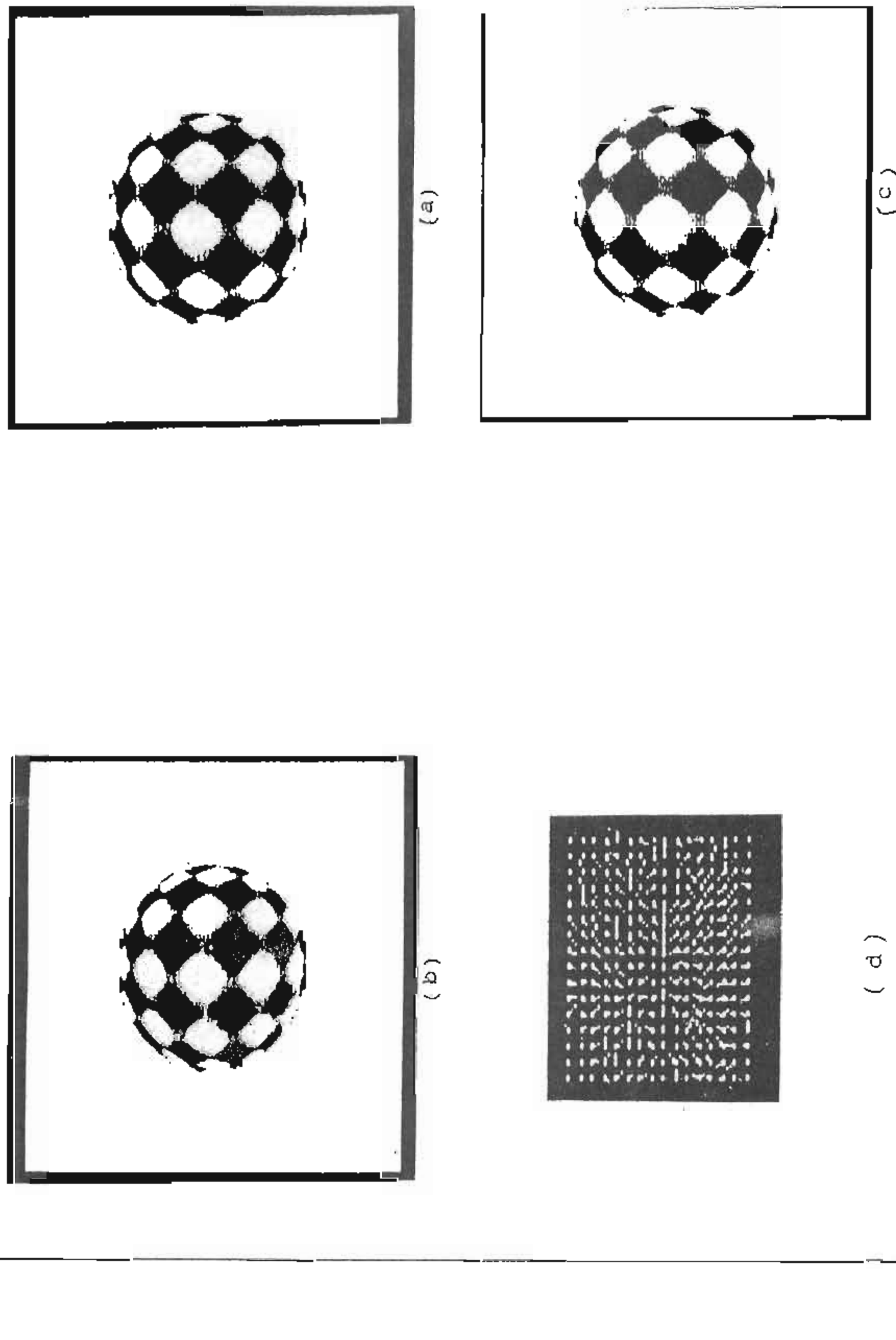

Figure (3)

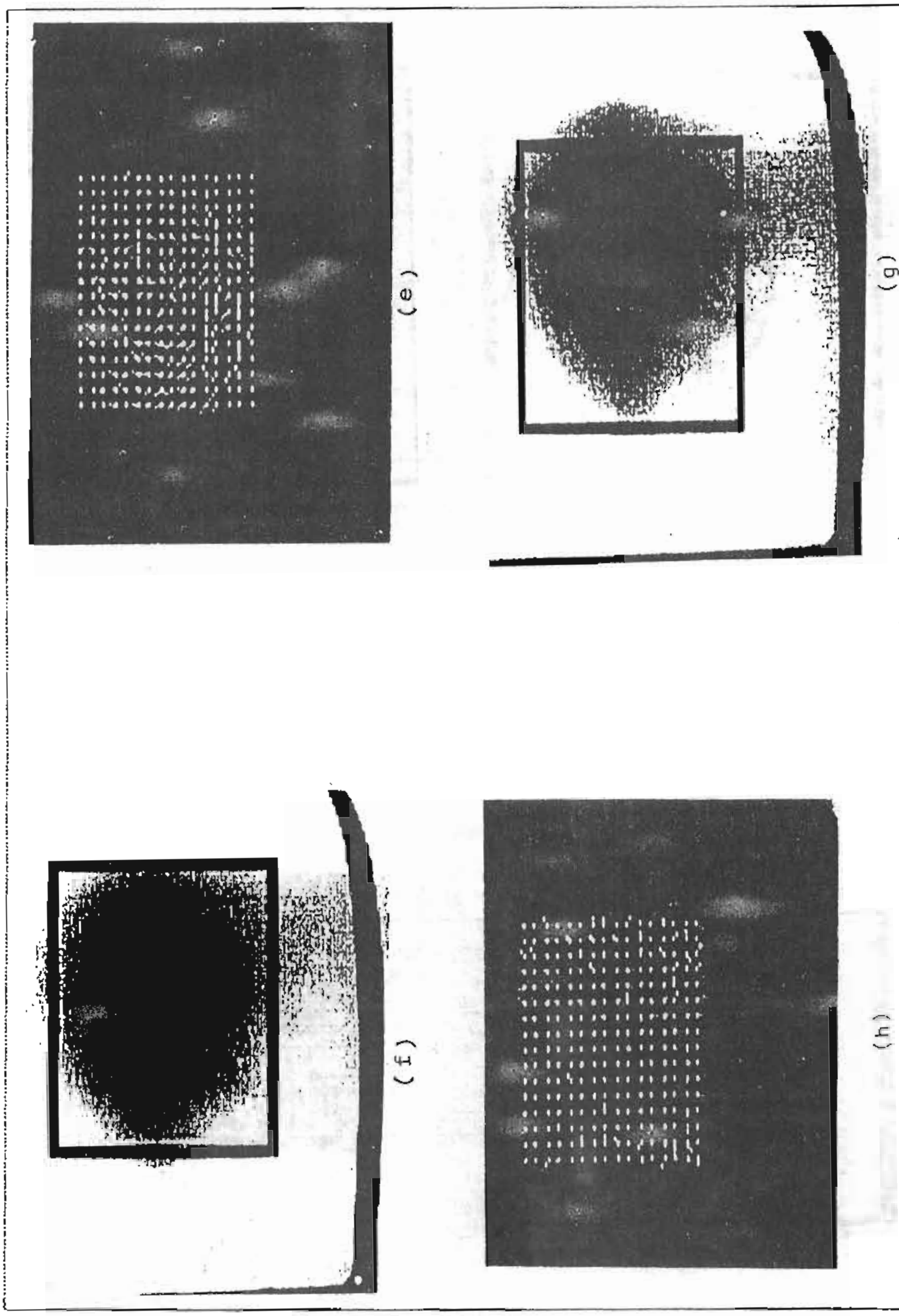

Figure 3 (Cont.)