DATA PROCESSING WITH UNCERTAIN TRANSFORMER TAPS

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ABSTRACT

The ultimate function of the data processor for power system control is to obtain reliable data of the system's state and parameters. Existing techniques of data processing assumed that the transformer taps in the system are known. However, there are many uncertainties associated with an actual system. Among these uncertainties could be the actual positions of transformer taps. Therefore, modified algorithms with various methods and cases, using the weighted least squares approach and assuming no knowledge of the transformer tap positions, are presented and implemented in this paper. Digital simulations have been performed and the results are presented.

INTRODUCTION

Automatic control of power systems requires the display of relevant information that indicates the operating conditions of the system. This information must be inferred from system measurements captured at the control center. If enough measurements could be obtained timely, accurately, and reliably, this would provide all information needed for control. However, a perfect data-acquisition system is technically and economically impossible so that the control must depend on measurements that are incomplete, delayed, or unreliable. Because it is uneconomical to obtain all measurements, part of this information can be calculated using the...
real-time measurements as input data to the model of the power system.

There are many uncertainties associated with an actual system using meter readings telemetered in real time to a digital computer. Uncertainties arise because of meter and communication errors, incomplete metering, errors in mathematical models, unexpected system changes, etc. The algorithms used to process these measurements, handle these uncertainties, and produce reliable data about the state and parameters of the system, is known as the data processing algorithm. It involves four basic operations namely: modelling, processing, detection and identification. The incoming data to this algorithm may consist of raw noisy measurements, line parameters, and structural information (such as circuit breaker and transformer tap positions). There are four approaches to the design of a data processor. Recent publications [5,7] have shown that the weight least squares approach works very well especially when sparsity, decoupling and bad data suppression are applied. However, existing techniques of solving data processing problems assumed that the position of transformer tap are known. But, as already discussed above, a data processing algorithms is coupled with many uncertainties. One of the information, among others, that could be affected is the actual tap positions of tap-changing transformers in the system. Therefore, this paper describes modifications to the basic data processing algorithm in which it is assumed that transformer tap positions are unknown. The aim is to study the general effect of this situation in implementing this algorithm, and also to try and devise a means of dealing with this situation to obtain a reliable data of the state variables and the transformer tap positions. Hence, the tap positions are included in the modified algorithms as variables rather than as constant. Various methods and cases of implementation of the modified algorithms have been programmed and examined in the hope of arriving at an acceptable solution. The basic and modified algorithms have been run on the standard IEEE 14 bus system. The results, discussion and conclusion are given.

2. THE BASIC ALGORITHM

The problem consists of finding the state variables \( \mathbf{X} \), which is of order \( n \), based on \( m \) of measured variables \( \mathbf{Z} \) in the presence of an error \( \mathbf{E} \). Thus, using a vector of the non-linear function \( \mathbf{h} (\mathbf{X}) \), a set of non-linear equations can be formulated to describe the relation between \( \mathbf{Z} \) and \( \mathbf{X} \):

\[
\mathbf{Z} = \mathbf{h} (\mathbf{X}) + \mathbf{E} \quad \text{.......................... (1)}
\]

Since some of the measurements are corrupted with noise it is obvious that there will be a difference between the true state (unknown) and the evaluated state. So the aim being to minimize that difference or some function of it. There are numerous approaches to handle this problem, and all can be broadly classified into linear programming approach, and weighted least squares (WLS) approach. Past experiences [7] have shown that the most common and widely accepted one is the WLS, which has proven to give reliable convergence. It minimizes the weighted sum of the squared residuals, \( J(x) \). The final model of the iterative process is given as follows:

\[
\mathbf{X}^{(n+1)} = \mathbf{X}^{(n)} + \left( \mathbf{X}^{(n)} - \mathbf{X} \right) \mathbf{C} \quad \text{.................................. (2)}
\]

\[
\mathbf{E}^{(n+1)} = \mathbf{X}^{(n)} - \mathbf{X} \quad \text{.................................. (3)}
\]

\[
\mathbf{J}^{(n+1)} = \mathbf{E}^{(n+1)} \mathbf{C}^{(n+1)} \quad \text{.................................. (4)}
\]

\[
\mathbf{X}^{(n+1)} = \mathbf{X}^{(n)} + \mathbf{E}^{(n+1)} \quad \text{.................................. (5)}
\]

where \( r_k = f_k - y_k \) and \( C_k = 1 \) if \( y_k \in \{ y_k \} \)
\[ f_i = W_i r_i \] and \( C_i = 0 \) if \( \beta_i \Delta (r_i/\delta_i) \)

\[ \beta_i = \beta_{i-1} - \alpha \quad \text{and} \quad \beta_{i+1} \leq \beta_i \leq \beta_{i-1} \]

\[ r_i = L_i - \mu_i(k) \]

\[ \delta_i = (R_i)^{0.5} \times \text{variance of the } i \text{th measurement} \]

\( a, r, k \) refer to active element, reactive element and iteration count, respectively. The matrices \( G \), \( H \) and \( R \) are the gain, Jacobian and error covariance matrices, respectively. The vectors \( \beta \) and \( \mu \) are the phase angles and the voltage magnitudes, respectively. Where \( \Delta a_{\alpha} \), \( \Delta a_{\beta} \), \( \Delta \beta \) are the initial and final minimum breaking points, and the step length, and \( W \) is the weighting factor. The iterative process of equations (2-5) is repeated until all \( \Delta \beta \) and \( \Delta v \) are less than or equal to the specified tolerances.

3. **The Modified Algorithm**

Existing techniques of data processing assumed that the position of the transformer taps are known. This section describes how the modified data processing algorithm can be implemented when it is assumed that the position of the transformer taps in the system are not known. In the case here the tap positions are treated as variables where they (the taps) are being recalculated at each iteration. The proposed algorithm implemented here uses the basic algorithm as described in section 2. The main algorithm steps various methods and cases in implementing equation (4) of appendix (A) are outlined below.

3.1 **Algorithmic Steps**

The main algorithmic steps are:

**Step 1.** Evaluate the initial tap positions for all tap-changing transformers in the system using the equation from the model and a starting voltage magnitude of 1.0 p.u. and voltage angle of 0.0 radian.

**Step 2.** Using the initially computed tap position (as obtained in step 1), a modelling of the system is then performed in which line parameters and other structural information are evaluated. Subsequently, the gain matrix is computed, inverted and kept constant (i.e., is not recalculated at each iteration).

**Step 3.** Evaluate the state variables using equations (2, 3) to solve for \( \Delta \beta \) and equations (4, 5) to solve for \( \Delta v \). The processing is then performed to determine the state variables (i.e., the voltage magnitude and angle at all nodes).

**Step 4.** Evaluate the new tap positions using the updated values of the state variables.

**Step 5.** Re-evaluate the admittances of lines containing tap-changing transformers using the new value of tap position obtained in step 4. For every change in tap setting of a transformer connecting two buses, the self and mutual admittances are recalculated before continuing the iterative solution, as the values of the admittances have changed.

**Step 6.** Repeat steps 3-5 until solution has converged. The solution is said to have converged when \( \Delta \beta \) and \( \Delta v \) are equal to or less than a specified tolerance, or when tap limits have been reached. The specified tolerance for the voltage is 0.0001 p.u. and for the voltage angle is 0.0001 radian.
3.2 Methods of Implementation

The steps outlined in the previous section are the general steps expected to be followed in implementing the algorithm in the modified form. However, further modifications were necessary in order to achieve reliable values of the state variables and the tap positions. Therefore, the main equation for calculating the tap position was modified several times, and implementation of the equation (2) of Appendix A (the overall expression for the tap position) was carried out with the following variants:

Method 1: Solve tap expression (equation 5 of Appendix A) using total power without due consideration as to the direction of flow of power. It is the initial method of implementation.

Method 2: Consider that the flow of power is from node K to node L; thus from the derivation of the expression for the tap position, the direction of flow of power will need to be reversed as appropriate. This will obviously be expected to affect the position of the tap, i.e., whether the tap positions are positive or negative.

Method 3: Based on the principle that a change in the difference of bus voltage magnitudes (due to a change in transformer tap ratio) is associated mainly with a change in the reactive power transfer, the active power was neglected in the tap expression, and without due regard to taking the conjugate of the reactive power. The value of the reactive power used is that obtained from the solution disregarding the direction of flow of power. No other consideration is given.

Method 4: In this method, it is considered that active power is neglected and the flow of the reactive power is from node K (designated the sending end) to node L (the receiving end). Subsequently, it was necessary to change the direction of power flow as appropriate.

Method 5: An alternative way is first to neglect the active power term and take the conjugate of the reactive power taking the magnitude, i.e., the conjugate of the reactive power is taken into account. In this method, the reactive power as obtained from the solution is used.

Method 6: It was decided that the real part of the admittance could be neglected in the expression for the tap position. As for other methods the results of the power obtained from the solution is used.

Method 7: The method of implementation attempted here involves expressing the tap position in a "complex" form. This means that the real and imaginary parts of the variables are treated separately and the tap position is then expressed in the form

\[ a = \frac{(B + jC)}{D + jE} \]

The procedure here is that eventually the magnitude of the tap position has to be taken, i.e.,

\[ a = \sqrt{\frac{(B^2 + C^2)}{(D^2 + E^2)}} \]

and which will always give a positive tap position. However, it was hoped that by obtaining the resultant angle of the complex tap expression a decision might be reached as regards on which side the tap position is. That is, if the resultant angle is between 0 to \(\pi\) radian the tap position is considered positive, and if between \(\pi\) to \(2\pi\) radian the tap position is considered negative. This method was found not to work for some systems because the resistance of the lines containing tap-changing transformers were zero, and treating an overflow problem in the computer program. Consequently, it was decided that the real part of the overall expression for the tap position be neglected.
3.3 Cases of Implementation

From appendix B, the active and reactive line flow are expressed as functions of transformer tap ratios. Therefore, each method of the above seven methods is implemented in three cases as outlined below.

Case A: The outlined 6 algorithmic steps are followed without further modification, where admittances of lines containing tap-changing transformers were re-evaluated at each iteration.

Case B: In this case, the admittance elements for lines containing tap-changing transformers are evaluated using the initial tap position. They are then kept constant and are not re-evaluated in step 5 of the algorithmic steps. However, the transformer ratio term $\psi_k$ in the active and reactive line flow equations is updated at each iteration.

Case C: Omit step 5 (re-evaluating the admittances) in the algorithmic steps. This means that the tap position is not updated in the calculation of line power flows. Effectively, this implies that the initial tap position is used to do the processing problem, although the tap position is being recalculated at each iteration using the updated values of the state variables.

4. TEST RESULTS

The basic algorithm (where tap positions are specified) and the various methods and cases of implementing the modified algorithm (where the taps are unknown) have been programmed. Digital simulations have been performed on the standard IEEE-14 bus system, having 21 lines and 3 transformers [5,72]. Appendix (c) gives data and system configuration for this system.

Firstly, the basic algorithm was implemented and the solution converged after 6 iterations (in 3.3 seconds). The active and reactive weighted residuals were 0.0276 and 1.093, respectively. The values of the state variables are obtained from this test (i.e. taps are specified) will be shown in this section as the true values. Again, the basic algorithm has been run with all tap settings of tap-changing transformers set to zero. The solution is obtained after 6 iterations, the active and reactive weighted residuals were 0.3581 and 4.0220, respectively. Comparisons between the results show that the standard deviations from the true values are of 0.04 for the voltage magnitudes and of 0.006 for the voltage angles. As can be seen, tap positions have a significant effect on the final solution obtained.

Secondly, various methods and cases of implementing the modified algorithm have been run on the same system. The assumption made here is that positions are not specified. In order to make easy comparison, table (1) summarises the results obtained for all methods and cases. Generally, when the algorithm was implemented using the form of tap expression as derived, i.e. with total power, very wide varying tap positions were obtained when compared to those specified for the system. The use of reactive power in the tap expression in place of total power gave an improvement in tap positions and state variables thereby confirming the principle on which the neglecting of active power was based. That is, reactive power is coupled to bus voltage magnitude (and hence tap ratios) while active power is coupled to bus voltage angle. The new equation was further modified in an attempt to improve the state variables and tap positions. The results show that some busses are significantly affected and these correspond to the buses of the lines containing tap-changing transformers. Also, the cost of performing a processing problem where tap positions are unknown is the extra time required to compute the tap positions and the re-evaluation of admittances. This process took 112 milliseconds
and total execution time was 3.9 seconds, giving an increase of 0.6 seconds over the case where the processing is performed with known tap positions.

In selecting the most acceptable method of implementation, the following points were considered: fast convergence, ultimately less execution time, and best values of the state variables with minimum deviations from the true solution. This question of convergence depends on which case is used, i.e., whether case A, B or C. Hence in comparing the convergence rate, cases A are compared for all methods, the same thing being true for cases B and C.

In considering case A, it is seen that all methods of implementation failed to converge except for method 7 when the tap position is expressed in terms of the real part and neglected. The non-convergence of case A may be explained by the fact that processing based on the weighted least squares approach is a probabilistic problem rather than a deterministic one, which involves curve fitting. It should be recalled that the iterative process until J(x) approaches a minimum. The iteration is normally stopped when \( x^k \) is equal to or less than a predetermined value, where the superscript k denotes the iteration count. It is possible for J(x) to have local minima and flat spots, and thus \( x^k \) may converge but not to the value of \( x^* \) that minimizes J(x). It is also possible for \( x^k \) to never converge. This depends on the method that may be used in the function J(x).

For case B, smooth convergence was obtained only when method 7 was implemented, without bad data being present. However, subsequent methods indicated the presence of bad data thereby increasing the execution time when bad data are replaced. It should also be noted that the use of case B has been shown to lead to misidentification and/or misidentification of bad data, and thus may be pointed to unsuitability of this case. Convergence for case C has been obtained in all the methods of implementation attempted, but best values have been obtained when methods 4 and 7 are implemented.

An examination of Table 1 shows that the best solutions were obtained when method 4 was implemented. Two methods, however, produced good results, these being methods 3 and 7. It should also be noted that convergence of method 7 is fastest for all cases A, B, and C.

5. CONCLUSION

Using the weighted least squares approach, and with the assumption that information about tap positions of tap-changing transformers are not available, various methods and cases of implementing the data processing algorithm for power system control were presented. These methods and cases were implemented and examined on the standard IEEE-14 bus system. The results obtained for all these methods and cases of implementation were presented. On the whole, the results of this work have highlighted the effect on the performance of data processors when vital information about the system is not exactly known.
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Table 2: The results obtained from various cases and methods.
REFERENCES

(2) E. HANDSCHIN (editor), "Real-Time control of electric power systems, session I: Real time data processing using state estimation in electric power systems", ELSEVIER, 1972.

APPENDIX A: EXPRESSION FOR TRANSFORMER TAP POSITION

A transformer with off-nominal turns ratio, fig (a), can be represented by an impedance or admittance connection in series with an ideal transformer, from which an equivalent circuit can be obtained as shown in figure (b). The elements of the equivalent circuit can then be treated in the same manner as other elements in the system, it should be noted that the values of these elements are not constant as they depend on tap position. Thus for any change in tap setting of the transformer these elements also change and should appropriately be recalculated in any iterative process using figure (a) an expression can be derived for the tap position of such a transformer. Consider a 5phase transformers tap position with reference to the nominal position, and is normally expressed in per unit, and the turns ratio is given by

\[ T_{n} = 1/100 \]

From this figure the following relationships can be established.

1. \[ P_{1} + jQ_{1} = V_{1} \] (a)
2. \[ P_{a} + jQ_{a} = V_{a} \] (b)
3. \[ V_{c} = V_{a} \] (c)

where \( I \) denotes complex conjugate.

Solving for tap position \( a_{n} \) gives

\[ a_{n} = \frac{V_{n} V_{c}}{(V_{c} V_{n} + V_{n} V_{c} - P_{c} + jQ_{c})(V_{n} V_{c} + jQ_{c})} \]

The above equation may be rewritten as

\[ T_{n} = \frac{V_{c} V_{n} + V_{n} V_{c} - P_{c} + jQ_{c}}{V_{c} V_{n} + jQ_{c}} \]

The above equation gives the final expression for the transformer tap position of a tap-changing transformer as represented by figure (b).

APPENDIX B: POWER EQUATIONS

(a) The active and reactive node injections at node \( k \) are given by the

\[ S_{k} = P_{k} + jQ_{k} \]

\[ = V_{k} I_{k}^{*} (B_{kl} - C_{kl} C_{kl} - B_{kl} B_{kl} - C_{kl} C_{kl}) \]

\[ Y_{kl} = \text{admittance of line \( k \) between node \( k \) and \( l \) = } \]

\[ B_{kl} = \text{ admittance of line \( k \) between node \( k \) and \( l \) = } \]
**Hanoi University of Science and Technology, Hanoi, Vietnam**

**Title:** The active and reactive line flow

**Abstract:** The complex power at the sending end is

\[ S_{k} = P_{k} + jQ_{k} \]

where

\[ P_{k} = V_{k}V_{l}^{*} (S_{B} + B_{k} + S_{C}) \]

and

\[ Q_{k} = V_{k}V_{l}^{*} (B_{k} + S_{C}) \]

The complex power at the receiving end is given by

\[ S_{l} = P_{l} + jQ_{l} \]

where

\[ P_{l} = V_{l}V_{k}^{*} (S_{B} + B_{l} + S_{C}) \]

and

\[ Q_{l} = V_{l}V_{k}^{*} (B_{l} + S_{C}) \]

**Figure (a)**

**Figure (b)**

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**Legend:**

- **Active power injection**
- **Reactive power injection**

**Graph:**

- **Active power injection**
- **Reactive power injection**