EFFECT OF WEAVE ANGLE ON THE GEOMETRICAL PROPERTIES OF SQUARE PLAIN FABRICS OF CIRCULAR AND ELLIPTICAL THREAD CROSS SECTIONS

By

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ABSTRACT - A general theoretical approach is presented for a geometrical model of square plain woven fabric, allowing the threads to be compressed to an elliptical cross section provided that their circumference is constant. The limitation of the assumed model i.e. jamming condition represented by weave angle was computed and the relation between wrap angle and weave angle was also obtained for different values of flattening factor. Mathematical relations between cover factor, crimp percentage, relative fabric weight and weave angle were then possible to be computed at different values of flattening factor.

1. NOMENCLATURE

\( d \) : Thread diameter.
\( p \) : Thread spacing.
\( \theta \) : Weave angle, i.e. maximum angle of the thread axis to plane of cloth, degrees.
\( K \) : Thread cover factor.
\( K_n \) : Fabric cover factor.
\( C \) : Crimp percentage.
\( L \) : Length of thread axis between planes containing the axes of consecutive cross threads (crimped length).
\( W \) : Fabric weight, gm / M².
\( n \) : Number of threads per cm.
\( T \) : Thread count, tex.
\( r \) : Thread radius.
\( a \) : Semi-major axis of the elliptical thread cross-section.
\( b \) : Semi-minor axis of the elliptical thread cross-section.
\( c \) : Flattening factor (b/a).
\( P_c \) : Perimeter of circular and elliptical thread cross section respectively.
\( A_c \) : Areas of circular and elliptical thread cross section respectively.
\( B \) : (90 - wrap angle), degrees.
\( h \) : Maximum displacement of the thread axis, normal to the plane of the cloth.
\( D \) : Sum of the two diameters of warp and weft threads.

In the succeeding paragraphs the subscripts 1, 2 represent the warp and weft directions respectively.
2. INTRODUCTION

The problem of fabric geometry has been the subject of study by a number of workers [1-4]. Who have studied mainly plain fabric of circular or race-track thread cross sections.

When threads are wound onto a package or woven into a fabric the ends and picks will exert pressure on each other at intersections points and the threads tend to flatten each other. Woven fabrics may also be exposed to external flattening forces during end use or ironing. Low-twist threads flatten very easily, but as the twist increases it becomes progressively more difficult to distort its cross-section under pressure [5].

Thread flattening during and after fabric formation has a great influence on some fabric properties such as cover, crimp, weight, thickness and maximum thread setting, which may affect in turn on fabric thermal conductivity, air permeability, crease-resistance, stiffness, appearance, handle, shrinkage, tensile properties and abrasion resistance.

As most of the world's textiles are woven fabric and most of that woven fabric are mainly nearly square plain weaves, the theoretical study of such geometry is of wide application. None of the previous studies have considered compressibility in thread cross section during flattening specially in an elliptical form.

In this work an assumption has been made to restrict threads compressibility by the constancy of the perimeter of the elliptical cross section as a result of the mutual normal forces between warp and weft threads. In actual practice the perimeter also may be extended due to fibre elasticity.

The present theoretical analysis of the assumed elliptical model was superseded by a study of square plain fabric geometry of circular cross section, for ensuring a correct mean of checking the results of the derived equations [i.e., when \( e=1 \)].

The results of this study could include the relations between cover, crimp, weight and fabric weave angle under different flattening values.

3. GEOMETRY OF SQUARE PLAIN FABRIC:

3.1 Circular Thread Cross Section:

Assumptions:
- Warp and weft thread diameters are equal;
- Warp and weft thread settings are equal;
- The bending resistance of the threads is negligible;
- No consideration of internal forces;
- The threads are inextensible;
- The threads are uncompressible;
- Interference between two succeeding threads is zero as shown in Fig. 1.

Fig. 1 shows a warp cross section of plain weave using the above assumptions. From this geometrical model the following properties such as \( K \), \( C \) and \( w^* \) can be calculated as a function of weave angle ranging from \( \Theta = 0^\circ \) to \( \Theta = 60^\circ \) (jamming condition) as follows:

3.1.1 Relation between cover factor (K) and weave angle (Θ):

The ratio of thread diameter to its spacing (d/p) expresses the relative closeness of threads, this ratio also expresses the fractional area of the cloth covered by the warp or weft threads. Therefore it can be called the fractional cover [5], and calculated as follows:

From triangle \( OAB \), Fig. (1)
Fig. 1: Geometry of circular thread cross section.

\[
\sin \theta_2 = \frac{AB}{QN} = \frac{\frac{dl - \frac{dj}{2}}{2}}{\frac{di}{2} - \frac{dj}{2}} \sec \theta_2
\]

Hence
\[
\frac{1}{2} \sin \theta_2 = \frac{\frac{dl}{2}}{\frac{di}{2}} \tan \theta_2 \sin \theta_2 = \frac{dl}{pi} - \frac{dj}{2pi} \sec \theta_2
\]

Now Equation (1) becomes
\[
\frac{1}{2} \sin \theta_2 \cos \theta_2 + \frac{dj}{2pi} \cos^2 \theta_2 = \frac{c}{p} \cos \theta_2
\]

Then \( \frac{dj}{pi} = \sin \theta_2 / (2 - \cos \theta_2) \) \( \ldots (2) \)

and warp cover factor \( K_1 = 2 \sin \theta_2 / (2 - \cos \theta_2) \) \( \ldots (3) \)

and similarly \( (K_2) \) can be obtained.

Hence fabric cover factor \( (K_c) \) can be calculated from the following equation.
\[ K_c = K (2 - \frac{c}{28}) \]

3.1.2 Relation between crimp percentage \( (C) \) and weave angle \( (\Theta) \):

The equation for the crimp percentage can be written as follows:
\[ C_2 = \frac{L_2}{P_1} \times 100 \quad (\%) \]

From Fig. 1, \( L_2 \) is given as follows
\[ L_2 = \text{straight line (OK) + two arcs OB and JK} = 2OB + 2BD = 2\left(\frac{P_1}{2} - \frac{d_1}{2} \sin \theta_2 \right) \sec \theta_2 + \frac{\pi d_1}{180} \]

Then
\[ C_2 = \left(\frac{\sec \theta_2 - \frac{d_1}{2}}{P_1} \tan \theta_2 + \frac{\pi d_1}{180} \right) \times 100 \quad \ldots (5) \]
Substituting Equation (2) into Equation (5) will give the thread crimp percentage for a given weave angle:

\[
\sin \theta_2 = \sqrt{2 - \cos \theta_2 - 2 \tan \theta_2 \sec \theta_2 - 1} \times 100
\]

Thus, \( C_2 = \left[ 2 - \cos \theta_2 - 2 \left( \tan \theta_2 \sec \theta_2 \right) - 1 \right] \times 100 \)  ... (6)

1.3 Relation between relative Fabric weight \( (W^*) \), and weave angle (\( \Theta \)):

Fabric weight per unit area can be determined as follows;

\[
W = \frac{W}{W_{\text{max}}} = \frac{n}{n_{\text{max}}} \cdot \frac{(100 + C)}{(100 + C_{\text{max}})}
\]

\[
W^* = \frac{W}{W_{\text{max}}} = \frac{n}{n_{\text{max}}} \cdot \frac{(100 + C)}{(100 + C_{\text{max}})}
\]

\[
= \frac{\min p}{p_{\text{max}}} \cdot \frac{(100 + C)}{(100 + C_{\text{max}})}
\]

\[
= \frac{d}{d_{\text{max}}} \cdot \frac{(100 + C)}{(100 + C_{\text{max}})}
\]

From Table I:

\[
\frac{\sin \theta_2}{2 - \cos \theta_2} \cdot \frac{1}{0.5974 \cdot (100 + C_{\text{max}})} = 0.01434 \cdot \frac{\sin \theta_2}{2 - \cos \theta_2} \cdot (100 + C)
\]

Values of \( K, K_C \) and \( C \) against weave angle (\( \Theta \)) could be calculated from Equations 1, 4 and 6 respectively and tabulated in Table I. Fig. 2 shows these values represented as a relative percent of their maximum values at \( \Theta = 60^\circ \) beside relative fabric weight \( (W^*) \) which is calculated from Equation (6).

Table I: Values of \( \frac{d}{d_{\text{max}}} \), \( K, K_C \) and \( C \) at different values of \( \Theta \):

<table>
<thead>
<tr>
<th>( \Theta ) ((^\circ))</th>
<th>( \frac{d}{d_{\text{max}}} )</th>
<th>( K )</th>
<th>( K_C )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.0868</td>
<td>2.431</td>
<td>4.651</td>
<td>0.3757</td>
</tr>
<tr>
<td>10</td>
<td>0.1710</td>
<td>4.759</td>
<td>8.739</td>
<td>1.4774</td>
</tr>
<tr>
<td>15</td>
<td>0.2503</td>
<td>7.008</td>
<td>12.262</td>
<td>3.2221</td>
</tr>
<tr>
<td>20</td>
<td>0.3226</td>
<td>9.032</td>
<td>15.151</td>
<td>5.4587</td>
</tr>
<tr>
<td>25</td>
<td>0.3864</td>
<td>10.820</td>
<td>17.459</td>
<td>8.0255</td>
</tr>
<tr>
<td>30</td>
<td>0.4409</td>
<td>12.346</td>
<td>19.248</td>
<td>10.7303</td>
</tr>
<tr>
<td>-</td>
<td>0.4857</td>
<td>13.691</td>
<td>20.595</td>
<td>13.9688</td>
</tr>
<tr>
<td>35</td>
<td>0.5209</td>
<td>14.356</td>
<td>21.374</td>
<td>15.8538</td>
</tr>
<tr>
<td>40</td>
<td>0.5669</td>
<td>15.314</td>
<td>22.252</td>
<td>17.9471</td>
</tr>
<tr>
<td>45</td>
<td>0.6044</td>
<td>15.804</td>
<td>22.683</td>
<td>19.5500</td>
</tr>
<tr>
<td>55</td>
<td>0.5703</td>
<td>16.508</td>
<td>22.925</td>
<td>20.5603</td>
</tr>
<tr>
<td>60</td>
<td>0.5774</td>
<td>16.166</td>
<td>22.993</td>
<td>20.9135</td>
</tr>
</tbody>
</table>

3.2 Elliptical Thread Cross section:

Assumptions:

- Warp and weft thread diameters are equal;
- Warp and weft thread settings are equal;
- The bending resistance of the threads is negligible;
- The threads are inextensible;
- Interference between two succeeding threads is zero as shown in Fig.3.
Fig. 2: Relation between relative percent of crimp, thread cover, fabric cover, weight and weave angle (θ).

Fig. 3: Geometry of elliptical thread cross section.

Assuming a woven thread with a circular cross section was flattened under the effect of either internally or externally flattening force to an elliptical shape. Two separate possibilities could be imagined to define such behaviour either constancy of area or perimeter of thread cross section.

The first assumption means that the threads are incompressible more over it implies that the thread perimeter will inevitably increases (i.e. $P_e/P_c \geq 1$) as shown from the following equations:

If there is no change in area

\[ R \pi r^2 = \pi ab \]
which means that \( e = b/a = r^2/a^2 \) \( \ldots (9) \)

\[ P_e / P_c = \left[ \frac{1}{2} \left( \frac{a^2 + b^2}{r^2} \right) \right]^{1/2} \geq 1 \]

as it easy to prove that \( \frac{a^2 + b^2}{r^2} \geq 2 \)

The second possibility means that the area of cross section will inevitably decreases allowing compresibility to take place which is nearer to reality i.e. \( (A_e / A_c \leq 1) \) as shown from Table II which is based on the following equation:

At constant thread perimeter

\[ 2 \pi r = 2 \pi \left[ \frac{1}{2} \left( a^2 + b^2 \right) \right]^{1/2} \]

\[ b/r = \left( \frac{2 - \left( \frac{b}{a} \right)^2}{2} \right) \ldots (10) \]

Table II: Values of flattening factor (e) and \((A_e / A_c)\) against (a/r)

<table>
<thead>
<tr>
<th>a/r</th>
<th>b/r</th>
<th>e = b/a</th>
<th>( A_e / A_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0250</td>
<td>0.9756</td>
<td>0.9506</td>
<td>0.9987</td>
</tr>
<tr>
<td>1.0500</td>
<td>0.9474</td>
<td>0.9023</td>
<td>0.9948</td>
</tr>
<tr>
<td>1.0750</td>
<td>0.9189</td>
<td>0.8548</td>
<td>0.9878</td>
</tr>
<tr>
<td>1.1000</td>
<td>0.8800</td>
<td>0.8080</td>
<td>0.9680</td>
</tr>
<tr>
<td>1.1250</td>
<td>0.8570</td>
<td>0.7617</td>
<td>0.9641</td>
</tr>
<tr>
<td>1.1500</td>
<td>0.8230</td>
<td>0.7157</td>
<td>0.9465</td>
</tr>
<tr>
<td>1.1750</td>
<td>0.7870</td>
<td>0.6698</td>
<td>0.9247</td>
</tr>
</tbody>
</table>

In reality either change in area and perimeter could occurred simultaneously but this is out of scope of this study.

3.2.1 Calculation of Maximum Weave Angle (\( \Theta \)):

From Fig. 3

\[ h_1 = \left( L_2 - 2 \overrightarrow{AC} \right) \sin \theta + 2 \left( 2b - \overrightarrow{AC} \sin \beta \right) \ldots (11) \]

where

\[ \overrightarrow{AC} = \sqrt{\left( a + b \cos \beta \right)^2 + \left( 2b \sin \beta \right)^2} \ldots (12) \]

In the case of jamming, \( L_2 - 2 \overrightarrow{AC} = 0 \)

Substituting Eq. (12) and Eq. (13) into Eq. (11)

\[ h_1 = 4b - 2b \sin \beta \sqrt{\frac{(1 + e)^2}{e^2} \cdot \cos^2 \beta + 4 \sin^2 \beta} \]

\[ \therefore D = 2h_1 = 4b \]

\[ h_1 = D \left( 1 - \frac{1}{2} \sin \beta \sqrt{\frac{(1 + e)^2}{e^2} \cdot \cos^2 \beta + 4 \sin^2 \beta} \right) \]

Hence

\[ \frac{(1 + e)^2}{e^2} \cdot \cos^2 \beta + 4 \sin^2 \beta = \frac{1}{\sin^2 \beta} \ldots (14) \]
It is necessary to find the relationship between $\Theta$ and $\beta$ as follows;

From Fig. 3, the equation of straight line $CB$ can be written as the following:

$$y_1 = x_1 \tan \beta$$

and

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$$

... (13)

$$x_1 = \left\{ \frac{1}{\left( \frac{1}{a^2} + \frac{\tan^2 \beta}{b^2} \right)^{1/2}} \right\}$$

... (16)

\[ \tan \beta = y_1/x_1 \]

Substituting Eq. (16) into Eq. (15)

\[ y_1 = \tan \beta \left( \frac{1}{\left( \frac{1}{a^2} + \frac{\tan^2 \beta}{b^2} \right)^{1/2}} \right) \]

\[ \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1 \]

... (17)

Differentiating Eq. (17) gives

\[ \frac{dy_1}{dx_1} = \frac{b^2}{a^2} \frac{x_1}{y_1} \]

... (18)

also

\[ \frac{dy_1}{dx_1} = \tan (\pi - \Theta) \]

... (19)

With equality of Eq. (18) and Eq. (19)

\[ \therefore \tan \beta = \frac{e^2}{\tan \Theta} \]

... (20)

Substituting Eq. (20) in Eq. (14) will give

\[ \frac{(1 + e^2) \tan^2 \Theta}{e^2 (1 + e^2 \tan^2 \Theta)} + \frac{e^2}{b^2} \frac{\tan^2 \Theta}{e^2 (1 + e^2 \tan^2 \Theta)} = 0 \]

... (21)

By solving Eq. (21) graphically, it was found that the maximum values of weave angle (\(\Theta\)) corresponded to the different values of flattening factor (e) are \(\Theta = 60^\circ\) (at \(e = 1\)), \(58^\circ\) (at \(e = 0.9906\)), \(56^\circ\) (at \(e = 0.9923\)), \(54^\circ\) (at \(e = 0.9548\)), \(52^\circ\) (at \(e = 0.8508\)) and \(49.5^\circ\) (at \(e = 0.7617\)) respectively.

3.2.2 Relation Between Cover Factor (K) and Weave Angle (\(\Theta\)) at Different Values of Flattening Factor (e):

From Fig. 3:

\[ \tan \Theta = \frac{AD}{OD} = \frac{2b \sin \beta - \beta}{\frac{2}{2} - (a + b) \cos \beta} \]

i.e.

\[ \tan \Theta = \frac{2a \sin \beta - \beta}{\frac{2}{2} - \frac{2(1 + e)}{p} \cos \beta} \]

Thus

\[ \frac{1}{2} \tan \Theta = \frac{2a \sin \beta - \beta}{p} \cos \beta, \tan \Theta = \frac{2a}{p} \sin \beta - \frac{\beta}{p} \]

...
\[
\tan \Theta = \frac{2a}{p} (1 + e) \cos \beta, \quad \tan \Theta + 2e \sin \beta = e
\]

\[
\frac{2a}{p} = \frac{\tan \Theta}{(1 + e) \cos \beta}, \quad \tan \Theta = 2e \sin \beta - e
\]

Substituting Eq. (20) in Eq. (22)

\[
\frac{2a}{p} = \frac{\tan \Theta}{(1 + e) \tan^2 \Theta + 2e^3 - e}
\]

\[
\frac{2a}{p} = \frac{\tan \Theta}{\sqrt{e^2 + \tan^2 \Theta}}
\]

Therefore, thread cover factor (K) is

\[
K = \frac{2a \tan \Theta \sqrt{e^2 + \tan^2 \Theta}}{(1 + e) \tan^2 \Theta + 2e^3 - e}
\]

The results of thread and fabric cover factor against weave angle (\(\Theta\)) at various flattening factors are shown in Fig. 4.

3.2.3 Relation Between Crimp Percentage (C) and Weave Angle (\(\Theta\)) for Different Values of Flattening Factor (e):

3.2.3.1 Calculation of the ratio of straight thread length to the thread spacing (2OA/P):

From Fig. 3

\[b = 2b = 2OA \sin \Theta + 2(2b - AC \sin \beta)\]

or

\[OA = \frac{1}{\sin \Theta} (AC \sin \beta - b)\]

where

\[AC = \frac{[(a + b) \cos \beta]^2 + (2b \sin \beta)^2}{\sin \Theta}\]

Substituting Eq. (26) into Eq. (25)

\[\frac{2OA}{p} = \frac{1}{p} \frac{2 \sin \beta}{\sin \Theta} \left( (a + b) \cos \beta \right)^2 + (2b \sin \beta)^2 - \frac{2b}{\sin \Theta}\]

Now Equation (27) becomes

\[\frac{2OA}{p} = \frac{1}{p} \frac{2 \sin \beta}{\sin \Theta} \left( (1 + e)^2 \cos^2 \beta + 4e^2 \sin^2 \beta - e \right)\]

3.2.3.2 Calculation of the ratio of curved thread length to thread spacing (2AC/P):

From Fig. 3, the perimeter (P) of the shown ellipse can be calculated [6] approximately as the following equation.

\[P_e = 4(a + b) \int_0^{\pi/2} \frac{\beta}{\sin \beta} d\beta = \frac{\pi}{2}\]

\[
\frac{B}{\beta} = \frac{\pi}{2}
\]

\[
\int_0^{\pi/2} \frac{\beta}{\sin \beta} d\beta = \frac{\pi}{2}
\]

\[
C = (a + b)
\]

\[
\beta = 0
\]

\[
\beta = \frac{\pi}{2}
\]
Fig. 4: Relation between thread cover factor, fabric cover factor and weave angle (θ) at different values of flattening factor (e).

where $K = \sqrt{\frac{(a+b)^2 - (2b)^2}{(a+b)}}$

i.e. $K^2 = \left[1 - \frac{4e^2}{(1+e)^2}\right]$

Therefore $\frac{P}{q} = a(1+e) \int_{\beta=0}^{\beta=\pi/2} \left(1 - \sin^2 \beta \left[1 - \frac{4e^2}{(1+e)^2}\right]\right) \cdot d\beta$
Then arc length $\Delta \xi$ (at angle $\beta$) = \(a(1 + e)\int_0^\beta \sqrt{1 - \sin^2 \beta \left[1 - \frac{e^2}{(1 + e)^2}\right]} \, d\beta \) \quad \ldots (29)

By using the numerical integration method of Eq. (29), arc length $\Delta \xi$ could be calculated at different values of $\beta$-correspondent to weave angle ($\Theta$) from Eq. 21-until $\beta = 90^\circ$.

Therefore
\[
\frac{2\Delta \xi}{p} = \frac{2a}{p} (1 + e) \int_0^{\pi/2} \sqrt{1 - \sin^2 \beta \left[1 - \frac{e^2}{(1 + e)^2}\right]} \, d\beta - \int_0^\beta \sqrt{1 - \sin^2 \beta \left[1 - \frac{e^2}{(1 + e)^2}\right]} \, d\beta \quad \ldots \ldots (30)
\]

Equations (28) and (30) which are both illustrated in Fig. 3 shows clearly that the increase of weave angle ($\Theta$) increase the thread length which wraps over the crossing thread while decrease the straight line portion which join the two curve parts. Fig. 6 shows the crimp percentage ($C$) against ($\Theta$) using the simple crimp equation

\[
C = \left[\frac{20\Delta \xi}{p} + \frac{2\Delta \xi}{p}\right] - 1 \times 100 \quad \ldots (31)
\]

The crimp percentage ($C$) could also be illustrated against ($\beta$) using the previous derived relation between ($\Theta$, $\beta$), where both the increase of weave angle ($\Theta$) and the decrease of angle ($\beta$) have a similar effect on increasing the crimp ($C$), while the effect of flattening factor ($e$) is contradicting as shown in Figures 6 and 7.

3.2.4 Relation Between Thread Cover Factor ($K$) and Correspondent Crimp ($C$) at Different Values of ($\Theta$):

From Fig. 4 and Fig. 6 the thread crimp ($C$) could be plotted against thread cover factor ($K$) as shown in Fig. 8 at different values of ($\Theta$) which could be of practical importance i.e. predicting any of the three mentioned parameters ($K$, $C$, $e$) by knowing the other two. Also both thread length ($L$) and its correspondent spacing ($p$) related to the semi-major axis of the elliptical thread cross section ($a$) are plotted against each other in Fig. 9 which shows almost straight line ($r = 0.999$) irrespective of flattening factor ($e$), this means that ($L$) increases by the increase of thread spacing ($p$) irrespective of what so ever the value of flattening factor ($e$).

3.2.5 Relation Between Relative Weight ($W^*$) and Weave Angle ($\Theta$) at Different Values of Flattening Factor ($e$):

Equation (7) can be rewritten as follows:
\[
W^* = \frac{2a}{p} \left(\frac{p_{\min}}{-2a}\right) \cdot \frac{(100 + C)}{(100 + C_{\max})} \quad \ldots (32)
\]

By substituting in Eq. (32) from Eq. (23) will give
\[
W^* = \frac{\tan \Theta}{\left[1 + \frac{e^2}{\tan^2 \Theta}\right] \cdot \left[\frac{p_{\min}}{2a}\right] \cdot \frac{(100 + C)}{(100 + C_{\max})}} \quad \ldots (33)
\]
Fig. 3: Relation between straight line length of thread axis relative to thread spacing \((2\Delta A/P)\), wrapped arc length of thread axis relative to thread spacing \((2\Delta C/P)\) and weave angle \((\theta)\) at different values of flattening factor \((e)\).

Fig. 6: Relation between crimp percent \((C)\) and weave angle \((\theta)\) at different values of flattening factor \((e)\).
Fig. 7: Relation between crimp percent (C) and wrap angle (90-β*) at different values of flattening factor (e).

Fig. 8: Crimp percent (C) plotted versus thread cover factor (K) at different values of flattening factor (e).
Fig. 9: Crimped length relative to semi-major axis length of elliptical thread cross section \( \frac{L}{a} \) plotted against thread spacing relative to semi-major axis length of elliptical thread cross section \( \frac{d}{a} \) for different values of flattening factor \( e \).

Fig. 10: Relation between relative fabric weight \( W^* \) and weave angle \( \Theta \) at different values of flattening factor \( e \).
The values of \( P_{\text{min}} / 2a, C_{\text{max}} \) (at jamming conditions) could be obtained from Equations (23), (31) at different values of flattening factor \( e \).

Values of \( W^* \) could be plotted against weave angle \( \Theta \) at different values of \( e \) as shown in Fig. 10.

4. DISCUSSION AND CONCLUSION

- The decrease in maximum weave angle \( \Theta_{\text{max}} \) with more thread flattening—which is expected—means that with threads of high ability to be flattened e.g., low twisted continuous filament yarn will reach maximum thread setting at lower density than those of highly twisted spun yarns.

- Both thread cover factor \( (K) \) and consequently fabric cover factor \( (Kc) \) increase by flattening degree \( (1/e) \) depending on weave angle \( \Theta \), this is considered as an advantage to gain more cover with the same thread density or the same cover with less thread density.

- The weave angle \( \Theta \) decreases with degree of flattening \( (1/e) \) depending on the value of thread cover \( (K) \). Such decrease in \( \Theta \) may result in decreasing the normal mutual forces between warp and weft and hence the frictional forces which keeps the firms. This is not considered as an advantage.

- The wrap angle \( \phi (\theta) \) increases with flattening degree \( (1/e) \) which may improve yarn to yarn friction through the increase of contact area. This effect seems to be contradicted to the previous one.

- The crimp \( % (C) \) increased very slightly with the increase of flattening degree \( (1/e) \) up to weave angle \( 65^\circ \) then begins to decrease until reaching their maximum weave angle \( \Theta_{\text{max}} \).

- The flattening factor seems to have no effect on thread length \( (L) \) that is to say when the fabric is exposed to external flattening forces the thread length stay constant under any pressure and hence fabric dimensions. This is due to the approximate equivalent increase and decrease of arc length and straight length of thread respectively. This makes a distinction between fabric and other solid matter which increase in dimensions with increasing of flattening degree such behavior is considered to be a great advantage.

- The relative weight \( (W^*) \) seem to increase linearly with weave angle \( \Theta \) up to \( 40^\circ \) approximately with very less effect with flattening factor \( e \). Fabric weight— in this case—as a function of cover and crimp may be the reason, it may be also important to remember that constant \( \Theta \) (does not mean constant thread spacing \( (P) \) as it is function in both thread spacing \( (P) \) and thread minor axis \( (2b) \) of the elliptical cross section which is in turn function in flattening factor \( e \).

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