THEORETICAL AND EXPERIMENTAL HEAT CONDUCTION IN
A CLADDED ROD WITH INTERNAL HEAT GENERATION AND
NON-UNIFORM SURFACE COOLING

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ملخــص : يهدف البحث الى اختبـار معملـى لصلاحية نموذج تحليلى ثنائى البعـــــد
وضع لحسـاب توزيع درجـات الحرارة فى أبعـاد مغلفـة، يتولد فيها حرارة داخلية والتبريـد
حولها غير منتظـــم، كما يحدث فى المفاعـلات النووية • ولقد اجريت التجربة على عمـــود
يسخــــن كهربيـــا ويبرد بالحمل الطبيعـى ، من جهة فى الهواء ، ومن جهة' فى المـاء •

ABSTRACT

The purpose of this work is to examine experimentally
the validity of a two-dimensional analytical model proposed
for prediction of the temperature distribution in a nuclear
fuel rod in steady and transient conditions. Cooling around
the fuel rod is considered non-uniform. The reactor fuel
rod is replaced by an elecrically heated rod inserted in a
cladding tube. The tube is cooled by natural convection in
air at one side and by natural convection in water at the
other side. Agreement between the theoretical and experime-
ntal temperature distribution of the tube surface prove the
adequacy of the proposed analytical model.

INTRODUCTION

Reactor safety is concerned with investigation of the
thermohydraulic problems arising in transient and/or emergency
conditions of nuclear reactors. These conditions are depart-
ures from normal operating conditions that are not expected
to occur in the lifetime of any particular reactor, but which
may be expected to occur a few times within a large number
of reactors over 30-40 year period [1]. Examples would be
minor or major loss of primary coolant or minor breaks in the
secondary steam side [1,2]. In this case, the reactor design
should be such that safe shutdown is achieved without any
offsite consequences. Decay heat in the reactor core must be
removed without overheating of the fuel to avoid formation of
hydrogen bubbles as a result of chemical reaction between zir
calloy cladding and steam [1]. In other cases, the cladding
material may be subjected to high temperatures and pressure
difference. The combined effect of such factors may lead to
local ballooing or swelling of the clad which means non-uni
cooling arround some fuel rods.

In this work, it is assumed that the fuel rod in a press-
urized water reactor is subjected to a non-uniform cooling
process(hydrogen or steam cooling and water cooling). For this
purpose a test device has been designed and constructed to
simulate this situation partly. Temperature distribution in
the test section is obtained theoretically using the proposed
model. Comparison between the obtained theoretical and exper-
imental results is than made.

THEORETICAL ANALYSIS

Soild reactor and radioisotopic fuel elements (and such
others as electrical resistance heaters) are devices in which
heat is both generated and conducted. On the other hand, there
is no heat generation in reactor fuel claddings. The problem
of temperature distribution in and heat removal from these
elements are important in evaluating reactor performance. In
the considered case (cylindrical fuel element), heat is gener-
ated and conducted in the fuel material, then conducted in a
gas gap between the fuel and cladding, and then transferred by
convection to a coolant flowing around the cladding. It is
assumed that the cooling process is not uniform around the
cladding. To solve this problem analytically, a two dimensional
model is developed which is described in details in reference
[3]. Briefly, the cross-sectional area of the fuel rod is
divided into differential elements in the two polar coordinates
r and θ as shown in Fig. 1. Differential equation governing
heat conduction with internal heat generation is applied in
each element and then integrated with respect to the directions
r and θ and the time (t).

$$\rho c(\partial T/\partial t) = (1/r)\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + (1/r)\frac{\partial}{\partial \theta}\left(\frac{k}{r}\frac{\partial T}{\partial \theta}\right) + S$$

A system of discretization equations in the following form is
then obtained:

$$A_{i,j} T_{i,j} = C R_{i,j+1} T_{i+1,j} + C R_{i-1,i} T_{i-1,i} + C T_{i,j+1} T_{i,j+1}$$

$$+ C T_{i-1,j} T_{i,j-1} + d_{ij}$$

where

$CR_{i,i+1}$ and $CT_{j,j+1}$ = radial and tangentional heat conductance
between the faces i,i+1 and j,j+1
respectively,

$A_{ij} = CR_{i,i+1} + CR_{i-1,i} + CT_{j,j+1} + CT_{j-1,j} + (\rho c \Delta v/\Delta t)$,

S = thermal source strength, and

$d_{ij} = S\Delta v + (\rho c \Delta v/\Delta t) T^{0}_{i,j}$

Solution of the system of discretization equations   is
then obtained using line-by-line method. The following bound-
ary conditions are applied:

$(\partial T/\partial r)_{r=0} = 0$ (an assumption which holds exactly for
uniform cooling) , and

$-k(\partial T/\partial r)_{r_o} = \propto (T_c - T_f)$

where
T_c = cladding surface temperature

T_f = coolant temperature

$\propto$ = heat transfer coefficient between coolant and
cladding

At the diameter AC - Fig. 1.

$$(\partial T/\partial \theta)_{\theta=0} = (\partial T/\partial \theta)_{\theta=\pi} = 0$$

As an initial condition, grid point temperatures are consid-
ered to be the same as the coolant temperature.

EXPERIMENTAL APPARATUS AND MEASUREMENTS

To examine the adequacy of the proposed two-dimensional
model some experiments are performed in which a fuel rod is
replaced by an electrically heated rod inserted in a stainless
steel tube. The tube is cooled from one side by natural conv-
ection in air and from the other side by natural convection
in water.

A schematic diagram of the experimental appearatus is
sketched in Fig. 2. The test section is consisted of a stain-
less steel rod (3) of 75 cm length and 8.1 mm diameter. The
rod is inserted and fixed within a stainless steel tube (4)
by two insulating rings (5). The stainless steel tube has
length of 50 cm, 11.1 mm outside diameter and 9.4 mm inside diameter.
The stainless steel rod acts as an electric heater by high
DC current supplied to it by a rectifier unit of the type MERA
900 with maximum current of 900 amperes and a maximum voltage
of 65 volts.

The test section is fixed horizontally at the center of
a plexiglas pool (2). The pool is kept containing distilled
water at a certain level in which it makes the lower half of
the test section immersed in the water, while the upper half
remains surrounded by the air as shown in Fig. 2. This geome-
try simulates a reactor fuel rod which may be subjected to
non-uniform cooling rates. The test section is connected to
the power supply by two copper terminals (8).

The surface temperatures of the stainless steel tube are
sensed by five thermocouples (6) distributed on its periphery

at different positions as shown in Fig. 2. Two thermocouples
out of the five; one is located at the most top (air side),
while the other is placed at the most bottom (water pool side).
The third one is fixed on the tube surface just at the level
of the water surface. The remaining two thermocouples; one
is placed in between the top one and the third one, while the
other is located between the bottom thermocouple and the third
one.

The water pool temperature as well as the ambient temper-
ature are also sensed by two thermocoples (17) as shown in
Fig. 2. The used thermocouples are copper-constant made from
30 gauge wires. These thermocouples are connected to a tempe-
rature recorder (10) with an accuracy of about 1°C.

Using the measured values of electric current and voltage
drop across the stainless steel rod, the electric power can
be estimated.

RESULTS AND DISCUSSION

The experiment is carried out under the following condi-
tions:

- Steady-state conditions.
- The electric heating power is 434 W which corresponds to a
  a volumetric thermal strength of 17 MW/m^3, and
- Ambient temperature and water pool temperature (T_a and T_w)
  are 23 and 35 C respectively.

To obtain the value of the heat transfer coefficient in
the air gap between the heating rod and the cladding tube, air
is assumed to be stagnent in the gap, which means heat is
assumed to be transferred through the gap by conduction.
Thermal conductivity of air at 150°C is about 0.03707 W/m.C,
and the air gap is 0.65 mm thick, then one obtaines 57 W/m^2.C
as a value for the heat transfer coefficient in the air gap.

On the other hand, the information which are given
as input data to get the theorctical temperature distribution
in both an actual nuclear fuel rod and in the experimental
test section are listed in the table [4, 5].

Using these data and applying the analytical model,
one obtaines the peripheral temperature distribution of the
test tube. Results are shown in Fig. 3 plotted on the same
graph with the experimentally obtained data. According to the
figure, there is a good agreement between the theory and exper-
iment. The theoretical temperature distribution is obtained
for values of the heat transfer coefficients in air and water
to be 6-10 and 100-120 W/m^2.C, where heat is transferred by
natural convection [4,6]. From Fig. 3, the theoritical temp-
erature on the top point of the test tube (air side) is found

to be 136-147°C, where the experimentally obtained value for
this temperature is 146°C. In water side, the theoretical
value of the tube surface temperature lies between 128 and
140°C where the experimental value is 128°C. These neglegable
discripances can be explained as due to the uncertinty in the
values of the heat trnasfer coefficients in air and in water.
Fig. 4 illustirates the qualitative picture of natural conve-
ction around the test section.

Data	Actual fuel rod	Test section
Density, Kg/m^3		
fuel rod	10200	7800
cladding	6300	7800
Specific heat, J/Kg.K		
fuel rod	296	480
cladding	319	480
Thermal conductivity,W/m.K		
fuel rod	2.59	55
cladding	15.13	55
Outside radius of the cladding (R_o), mm	4.68	5.55
Fuel radius (R_F), mm	4.0985	4.05
Cladding thickness, mm	0.50	0.85
Volumetric heat generation , W/m^3	7.2×10^8	1.7×10^7
Gap heat transfer coefficient , W/m^2.C	4500	57

Fig. 5 shows the theoretically obtained temperature distribu-
tion in the heating rod of the test section and in an actual
nuclear fuel rod of a 900 MWe pressuried water reactor [3].
The figure indicates that the temperature gradient in an actual
fuel rod much higher than in the experimental heating rod.
This is because the value of the thermal conductivity of the
rod material (stainless steel) is much higher than that of
the fuel material. In addition, the volumetric thermal source
strength in an actual fuel rod is much higher than in the
experimental rod. The same conclusion can be obtained using
the simple one dimensional equation for heat conduction with
internal heat generation and given in the following form [7]:

$$t_m - t_s = q^{\prime\prime\prime} R^2/4 K$$

where t_m is the temperature in the center line of a cylindrical
rod and t_s is the surface temperature of the bare rod having
a radius (R) and $q^{\prime\prime\prime}$ is the volumetric heat generation rate.
Substituting for $q^{\prime\prime\prime}$, R, and K one obtaines a temperature differ-
ence of less than 1.27 °C for the experimental heating rod.

CONCLUSION

From the above discussion, the following conclusions may be drown

1. Values of the heat transfer coefficients in air and in water affects the surface temperature distribution of the cladding tube.

2. Temperature distribution inside the experimental heated rod is more flat than that inside an actual nuclear fuel rod due to the high thermal conductivity of the experimental rod material and because of the low heat source strength in the experiment.

3. Agreement between the experimental and theoretical results prove the validity of the proposed two-dimensional analytical model in steady-state operation.

NOMENCLATURE

A,d	Occasional coefficients
c	Specific heat
CT,CR	Heat conductance in tangentional and radial direcions.
S,q'''	Volumetric thermal source strength
T	Temperature
t	Time
K	Thermal conductivity
v	Volume
ρ	density
α	heat transfer coefficient

Subscripts

c	Cladding
f	Fuel, fluid
a	air
w	water

REFERENCES

1. Marshall,W. "Nuclear Power Technology, Vol. 1, Reactor Technology", Clarendon Press. Oxford, 1983.

2. Phung, D.L. "Light water reactor safety before and after the Three Mile Island accident", Nuclear science and Engineering, 90, 509-520, 1985.

3. Mahgoub, M.M. "Two-dimensional temperature distribution in nuclear fuel rods", Bulletin of the Faculty of Engineering,EL-Mansoura University,Vol.11, No.2,December 1986

4.   Holman,J.P. "Heat Transfer", McGraw-Hill International
             Book Company, 1981.

5.   Electricte De France "EDF 900 MWe Nuclear Power Plants",
             EDF-Senex, Paris, 1983.

6.   Sukhatme, S.P. "A Textbook on Heat Transfer", Orient
     .   ·   Longman Limited, Bombay, 1983.

7.   El-Wakil, M.M. "Nuclear Power Engineering", McGraw-Hill
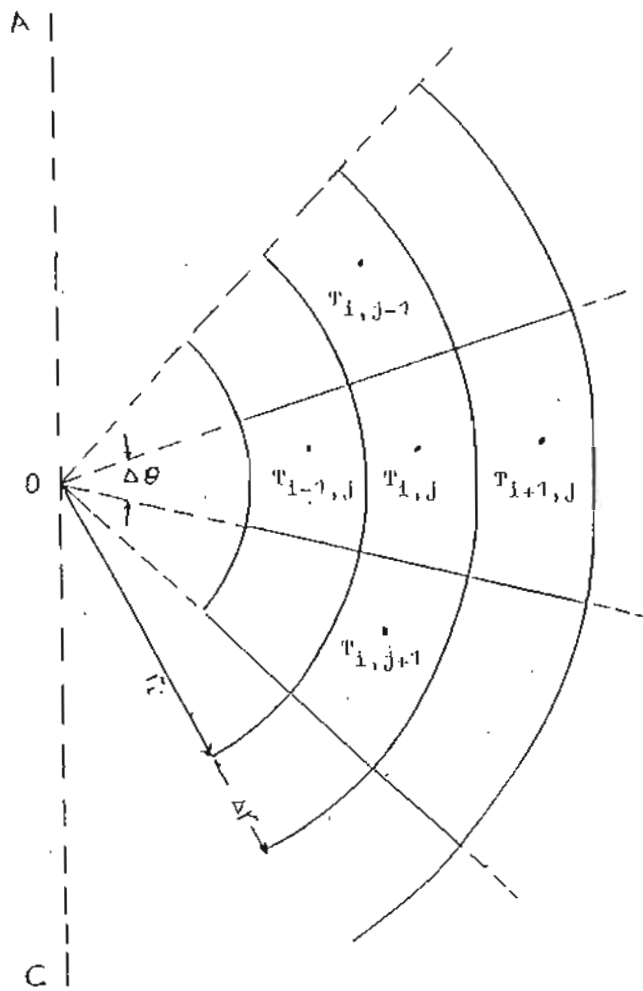   Book Company, 1962.

Fig.1 Grid design

1. wooden support
2. plexiglas pool
3. stainless steel rod
4. stainless steel tube
5. insulating ring
6. thermocouples
7. cable connection
8. electric cable

9. distilled water
10. temperature recorder
11. voltmeter
12. ammeter
13. stand table
14. air gap
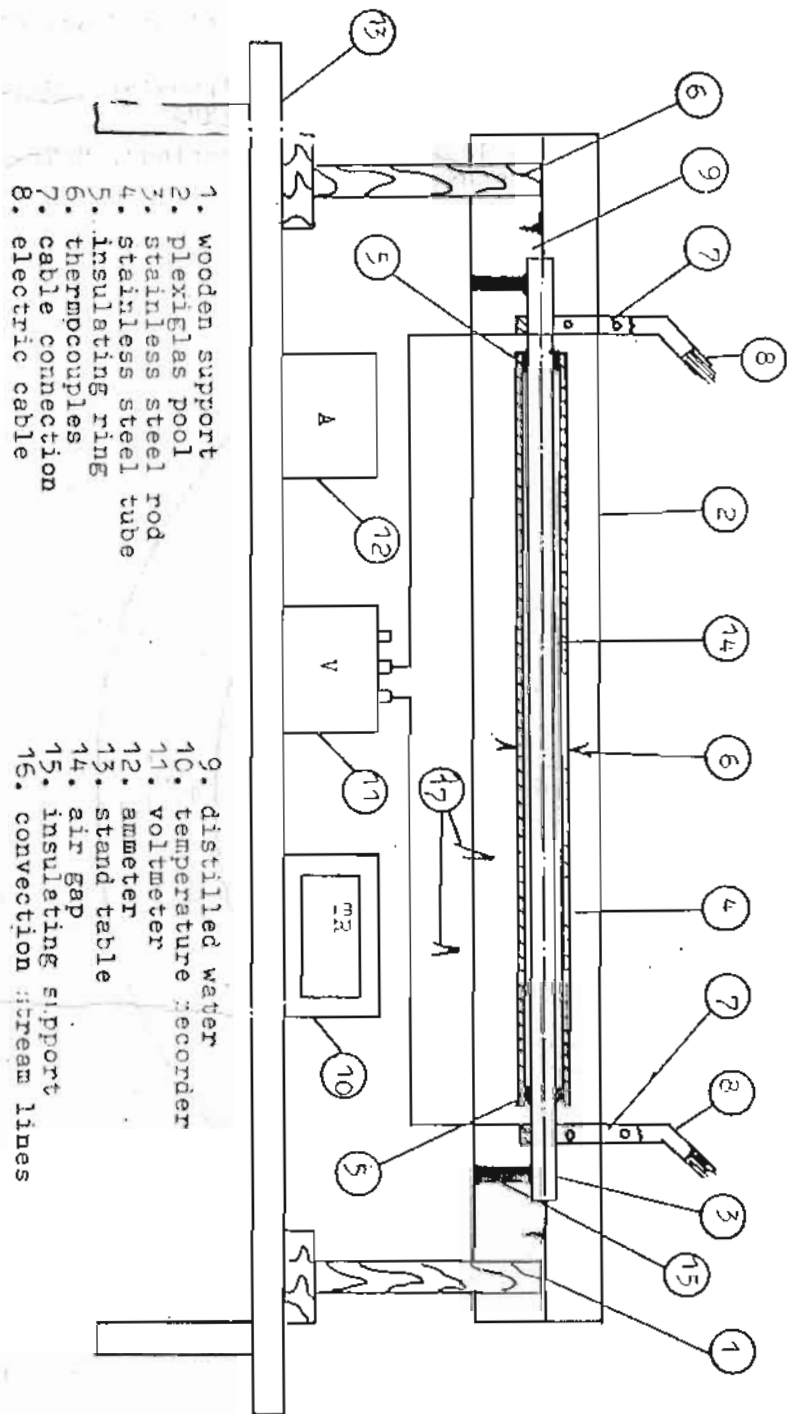15. insulating support
16. convection stream lines

Fig.2 Schematic diagram of the experimental apparatus
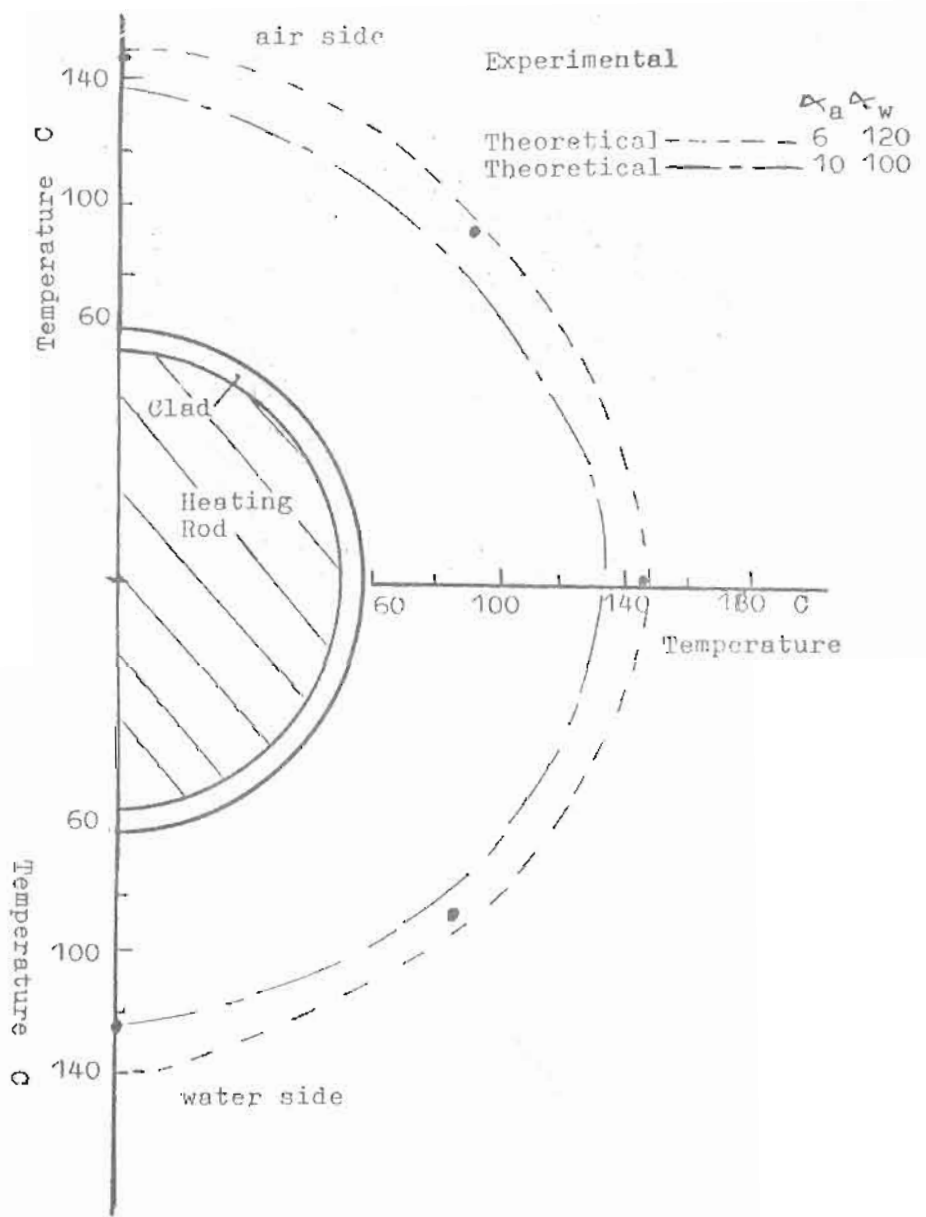
Fig.3 Experimental and theoretical temperature
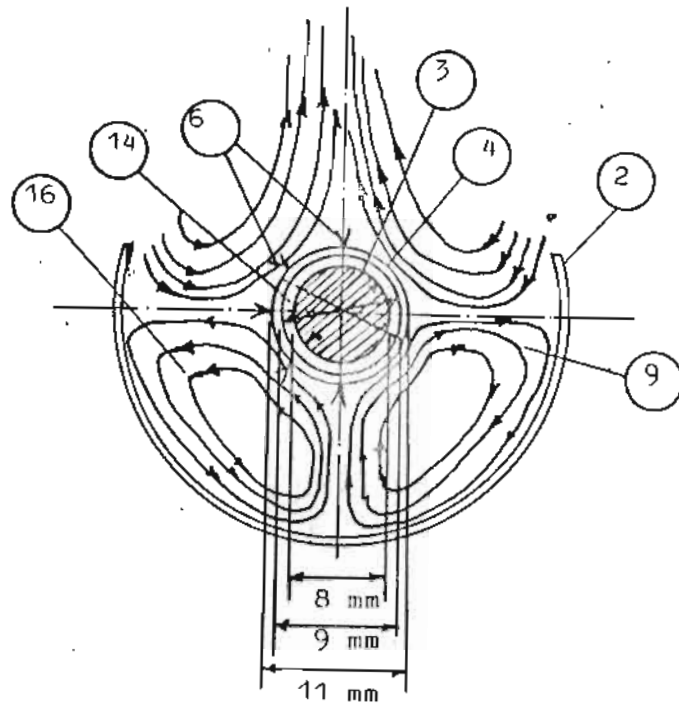distribution on the surface of the clad
tube

Fig.4   Qualitative picture of natural
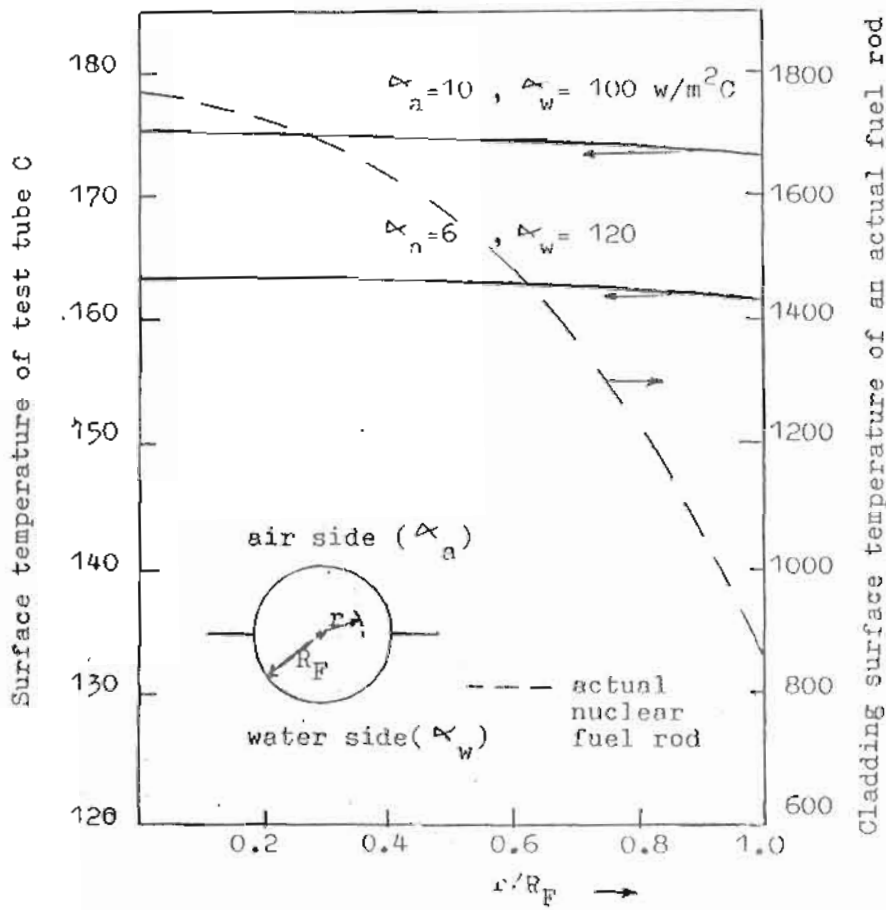   convection around the test section

Fig.5 Theoretical distribution in the heating rod and in actual nuclear fuel rod