ABSTRACT - As the performance of an E-Core homopolar linear synchronous motor (HLSM) is affected by the air-gap reluctance, it can be enhanced by shaping its pole droitly. Therefore, the primary design choice relates to the pole geometry, which can be simply studied by ignoring saturation and considering only the field excitation.

This paper presents the theoretical optimisation of three different track-pole shapes. This optimisation is based on a magnetic field study using the 3-dimensional finite-difference method (FDM) of scalar magnetic potential as well as the average of zero and infinite permeability boundaries. The shape which maximises the armature flux per pole will be thought of as the optimum shape. So, the flux per pole in the middle lobe of the E-core is taken as a "goodness factor" for pole shape. It is found that the L-shape was the optimum track-pole shape. The results of the experimental investigations which are carried out on a static model of this motor confirm the computed results.

1- INTRODUCTION

The E-core HLSM has passive track-pole, and both of the D.C. and A.C. windings are carried on the middle limb of the armature core, as shown in the schematic diagram of figure (1). According to this construction the feasibility of change of the pole shape will be carried out simply. The dimensions of a given pole-shape must meet the conflicting
requirements of:

(i) high permeance, at a given air-gap, to provide substantial armature flux per unit field excitation.

(ii) minimum leakage flux to reduce the amount of iron in the field core.

To resolve these conflicts the 3-dimensional finite-difference method is applied, for given dimensions of the E-core, using scalar magnetic potentials as the main field parameter. Difficulties with boundary conditions have been overcome by averaging the solutions obtained for both infinite and zero permeability boundaries.

The following pole shapes have been examined the theoretically:

(i) A variety of rectangular-shape

(ii) TEE-shape (i.e one-shoe pole).

(iii) I-shape (i.e double-shoe pole)

The shape which maximizes the armature flux per pole will be thought of as the optimum shape. So, the flux per pole in the middle limb of the E-core is taken as a "goodness factor" for pole shape.

Practical measurements of armature flux using a simple magnetic model have confirmed the theoretical investigation of optimum track-pole shape.

![Diagram of E-core homopolar linear synchronous motor with optimum I-shape pole](image)

**Figure 1**: E-core homopolar linear synchronous motor with optimum I-shape pole

2- FIELD COMPUTATION

The volume of the HLSM is enclosed in a 3-dimensional region as shown in figure 2. This is seen to extend 10 cm beyond the iron surface. The 3-dimensional region can be formulated in differential form in terms of scalar magnetic potential $\phi$ as follows:

\[ \frac{d^2 \phi}{dx^2} + \frac{d^2 \phi}{dy^2} + \frac{d^2 \phi}{dz^2} = 0 \]  

... (1)
Considering filling up this region with a set of uniformly spaced nodes of spacing unit length. Figure (3) shows one cubic element from a large mesh containing 6 nodes. Each of six elements connected to any node is taken to have unit permeance (unless an iron boundary is less than 1 cm distance when the reluctance is proportionately less).

The network, which containing all nodes, is solved by representing it in a computer program and applying a technique known as successive overrelaxation (S.O.R.) method [6]. Relaxation of a network consists of treating all nodes in sequence but one node at a time. Taking the example of Figure [3], the node equation is simply based on Kirchoff's current law:

\[ P_1 + P_2 + P_3 + P_4 + P_5 + P_6 - 6P_0 = 0 \]  \hspace{1cm} \ldots (2)

If the left hand side of this equation is evaluated for an arbitrary choice of node potentials a quantity not equal to zero will most likely result. This is called a "residual" and is proportional to the total flux converging on the central node. For a satisfactory solution the residual at all nodes should, for successive iterations, be very small compared with the flux passing through any one element. The node potentials are obtained when the computer program has achieved convergence to a highest residual of a magnitude less than \( 10^{-4} \).

The computer results are held in a large array of magnetostatic potentials in 3-dimensional specifying potential for pole pitch of 125 mm at 1 cm intervals. This is the most suitable form for permanent file storage. It is possible by simple steps to compute the armature flux and the pole flux distribution from the magnetic potential and the permeance of elements of the mesh. Each branch of finite-difference mesh is associated with a flow quantity.
carried between two points at either end of the branch. This is directly related to the flux in the magnetic field. The flow, equivalent to the flux, in each branch, is calculated from the potential difference across the branch

\[ \Phi = \left( \frac{\mu_0 h^2}{L} \right) (P_1 - P_2) \]  

(3)

where \( S \) is the branch length, \( P_1 \) and \( P_2 \) are the branch potentials and \( \mu_0 h^2/L \) is the permeance (PM). The branches have possible directions, hence the fluxes are the individual flux components in 3-dimensions. The calculations are normalised for simplicity taking the basic mesh as one unit (L = 1) and working with a nominal excitation of 100 H. The fluxes are calculated taking \( p = 1 \) and \( h = 1 \) to give normalised units as follows:

Centre-core flux:

\[ \Phi_a = PM \left( \sum_{x=1}^{\infty} \sum_{y=1}^{\infty} P(x, y, z_2) \right) \]  

(4)

Left hand side outer-core flux:

\[ \Phi_L = PM \left( \sum_{x=1}^{\infty} \sum_{y=1}^{\infty} P(x, y, z_1) \right) \]  

(5)

Right hand side outer-core flux:

\[ \Phi_R = PM \left( \sum_{x=1}^{\infty} \sum_{y=1}^{\infty} P(x, y, z_2) \right) \]  

(6)

Leakage factor:

\[ \delta = \frac{\text{centre-core flux}}{\text{L.H.S. flux} + \text{R.H.S. flux}} \]  

(7)

where:

- PM: permeance
- P: node potential
- cw: centre core width
- ow: outer core width
- \( z \): surface of armature level
- pp: pole pitch

In case of HLSM a full pitch region is considered, and the interfaces with adjacent regions (a,b) are treated as zero permeability boundaries giving a "positive mirror image" field on the remote side of the boundary. The remaining boundaries (c,d,e,f) taking, for first solution, as zero permeability and for second solution an infinite permeability, then taken the averaging of both the first and second solutions \([7]\). The HLSM iron-core and track-pole are assumed to have infinite permeability and not to be saturated. Also the effect of open slots is neglected.
3- ADJUSTMENT OF TRACK-POLE POTENTIAL

The pole of the HLSM is a block of iron whose potential is not known at the start of the finite-difference solution. Its potential must be derived as the solution proceeds to converge, but an estimate of say 20 units may speed the initial calculations. For a single block of iron the derivation of the block potential is a simple extension of the relaxation process. The block is simply treated as a gain node within the mesh and is relaxed in the following way.

The total flux entering the block is calculated by summing the contributions from 'n' nodes connected to the block. The result is the block residual \( R_B \), see figure (4).

\[
R_B = \sum_{r=1}^{n} (P_r - P_o) S_r \quad \ldots (8)
\]

where \( S_r \) is the permeance of the \( r \)th element. The amount of potential adjustment \( \Delta P_o \) required to reduce the block residual to zero is derived from the equation:

\[
\Delta P_o = \sum_{r=1}^{n} (P_r - (P_o + \Delta P_o) S_r = 0 \quad \ldots (9)
\]

i.e. \( \Delta P_o = \frac{\sum_{r=1}^{n} (P_r - P_o) S_r}{\sum_{r=1}^{n} S_r} \quad \ldots (9)\)

\[
\Delta P_o = R_B / S_B \quad \ldots (10)
\]

If all branches connected to the block have unit permeance the block residual is given by:

\[
R_B = \sum_{r=1}^{n} (P_r - P_o) \quad \ldots (11)
\]

and \( S_B \), the characteristic permeance of the block, is given by:

\[
S_B = n \quad \ldots (12)
\]

The block residual is not evaluated in a single stage as a node residual would be. Instead, for the nodes inside the block, the residual is calculated as if they were nodes in free space, but instead of being relaxed, the residual is added to a cumulative sum to form the block residual, \( R_B \). This residual is used at the end of each set of iterations to calculate the block potential adjustment.

The value of \( R_B \) derived in this way would be slightly different to that derived by evaluating the flux contributions to the block at the end of individual iterations since node values next to the block are being relaxed one at a time and are therefore being changed while the block residual calculation is in progress.

The validity of the technique is, of course, not in question since the same conditions for convergence apply but the rate of convergence may be altered. It is possible to use an acceleration factor (relaxation factor) in changing the block potential and one is used to obtain the current solutions. By the above technique the potential of the block is readjusted and all nodes within it get to the new potential \( P_o \).
4- DISCUSSION OF COMPUTED AND LABORATORY RESULTS

The computer program is used to study various track-pole shapes, using it to calculate additionally the flux per pole in the centre core, and treating this as a "goodness factor" for pole shape.

The following pole shapes are examined theoretically:

(i) A variety of rectangular,
(ii) TEE-shape (i.e. one-shoe pole),
(iii) I-shape (i.e. double-shoe pole).

The length of the pole-shoe is chosen 1.5 pole-pitch and its width equal to outside core-width, to minimise the outer-gap reluctance.

The physical model is constructed from mild steel, and used the D.C. excitation coils only. The centre core is laminated with open slots. The air-gap is held at 10 mm and the U.C. coils are excited with d.c. supply available. The armature flux is measured using a search coil and fluxmeter as shown in figure (9). The comparison between the computer predicted armature flux values with those measured on this physical model is shown in figure (6) with a variety of pole width. Also the leakage factor comparison is shown in figure (7). There are some remarkable points are arising from these figures-

(i) There is a difference between the predicted values of armature flux and that obtained by experimental measurements. This is due to the effects of open slots and the magnetic saturation in the iron core, which are neglected in the theoretical investigations.

(ii) As the pole width increases the armature flux increases, then decreases when the pole width becomes greater than 0.7 of the pole pitch. This is due to more leakage flux from pole to pole. Therefore it is recommended that the ratio of the pole width to pole pitch may not greater than 0.7.

(iii) The optimum pole shape is the I-shape, because it allows the flow of more flux through the armature core by 20% greater than the rectangular shape (at 0.7 of the pole width to pole pitch ratio). This is due to minimising the outer air-gap reluctance.

5- CONCLUSIONS

The performance of HLSM is influenced by the air-gap reluctance which depends on the pole-shape geometry. Therefore, it is important during an early stage of design of such type of motors to optimise its pole shape. For this purpose, a theoretical study, applying the 3-dimensional finite-difference method (FDM) has been suggested, using the scalar magnetic potentials as the main field parameter. In this study the D.C. excitation has been only considered and the magnetic saturation is neglected. The study results in predicting the flux per pole in the centre core for different pole width ratios of each the following track pole
shapes: the rectangular-shape, T-shape and the I-shape. This flux is taken as a "goodness factor" during the choice of the best pole shape.

Laboratory measurements which are carried on the experimental model show, for good approximation, a reasonable agreement with the theoretical investigations. It can be concluded that the I-shape is the optimum one. It allows the flow of more flux through the armature core by 20\% greater than the rectangular shape at 0.7 pole width ratio. This ratio is recommended in order to have minimum leakage flux.

![Diagram]

Figure 5: Static flux measurement of armature flux using search coil and fluxmeter.

REFERENCES

Figure 6: Comparison between computed and measured armature flux per pole (for 100 AT, 10 mm air-gap and track depth 20 mm).

Figure 7: Comparison between computed and measured leakage factor (for 100 AT, 10 mm air-gap and track depth 20 mm).