NUMERICAL TECHNIQUES FOR LOAD FLOW STUDIES

BY

Prof. Dr. S. A. Hassan*, Dr. F. S. Thalouth**, Eng. A. A. Abo El-Ela***

Abstract

The paper presents a comprehensive study for the different digital load flow techniques. Through numerical application on 16-bus power system, a comparison between the different methods regarding the computer time taken to obtain the problem final solution, the number of iterations, the core requirement, and the time per iteration.

* Head of Electrical Engineering Department, Faculty of Engineering & Technology, Monoufia University Shebin El-Kom, Egypt.

** Electrical Engineering Department, Faculty of Engineering & Technology, Helwan University, Egypt.

*** Demonstrator, Electrical Engineering Department, Faculty of Engineering & Technology, Monoufia University Shebin El-Kom, Egypt.
List of Symbols:

A. Complex quantities at bus P

\[ E_p = E_p \quad \theta_p = \theta_p + J f_p \]

- \( E_p \) = Nodal voltage.
- \( \theta_p \) = Nodal injected current.
- \( f_p \) = Net nodal injected power.
- \( \theta_p \) = Power mismatch.
- \( \theta_p \) = Current mismatch.
- \( \theta_p \) = Correction to nodal voltage.

B. Matrices:

\[ Y = G + J B \]

- \( Y \) = Nodal admittance matrix.
- \( G \) = Nodal impedance matrix.
- \( B \) = Jacobian matrix.

C. General:

- \( n \) = Number of buses in system.
- \( x_{pq} \) = Reactance of branch between buses p & q.
- \( s \) = Slack bus index.

Introduction:

Load flow (or power flow) is the solution for the static operating condition of an electric-power transmission system, and is the most frequently performed of routine digital-computer power network calculations.

Over the last 26 years an enormous amount of effort has been expended in research and development on the numerical calculation process. The aim is to present the underlying principles and techniques of the popularly accepted approaches, those more recent methods that seem to offer particular promise, and a selection of other methods that contain ideas of practical or theoretical interest.

Adopted Methods for Load-Flow Studies:

1. Gauss-Seidel Method Using \( Y_{BUS} \):

The \( Y \)-matrix iterative methods of load-flow calculation are based on the iterative solution of the linear equation (1):

\[ I_{Bus} = Y_{Bus} \quad E_{Bus} \] (1)
E. S. Hassan et al.

Assuming values for the voltages at different buses except the slack bus, then, currents are calculated from the bus loading equation as:

\[
I_p = \frac{\sum_{q=1, q \neq p}^{n} Y_{pq} I_q}{Y_{pp}} \quad p = 1, 2, \ldots, n \quad (2)
\]

Selecting the ground as the reference bus, a set of \( n - 1 \) simultaneous equations can be written in the form:

\[
E_p = \frac{1}{Y_{pp}} \left( I_p - \sum_{q=1}^{n} Y_{pq} I_q \right) \quad p = 1, 2, \ldots, n \quad (3)
\]

From (1)(3):

\[
E_p = \frac{1}{Y_{pp}} \left( \frac{V_p - J \angle Q_p}{Y_{pp}} - \sum_{q=1, q \neq p}^{n} Y_{pq} I_q \right) \quad (4)
\]

It is noted that, the admittance matrix iterative methods converge slowly, because of the loose mathematical coupling between the buses at each iteration cycle. Acceleration techniques are invariably used in practice to speed up the convergence. A fixed empirically determined acceleration factor \( \alpha \) (1 < \alpha < 2) is applied to each voltage change reevaluation thus:

\[
E_p = \alpha \frac{V_p^{*}}{Y_{pp}} \quad (5)
\]

2. Gauss-Seidel Method Using \( \bar{Z}_{bus} \): The major difference in principle between the \( Y \)-matrix iterative methods and the \( Z \)-matrix methods is that in the latter, the linear equation (1) is solved directly for \( \bar{Z}_{bus} \) in terms of \( \bar{I}_{bus} \) using the inverse of \( \bar{Y}_{bus} \) as:

\[
\bar{Z}_{bus} = \bar{Y}_{bus}^{-1} \bar{I}_{bus} = \bar{Z}_{bus} \bar{I}_{bus} \quad (6)
\]

The bus current \( (Q \neq s) \) is evaluated from

\[
I_p = \left( Y_{SP} \right) \frac{\bar{Z}_{p}}{\bar{Y}_{p}} \quad (7)
\]

where:

- \( Q_p \) = calculated values of reactive power for a PV-node (equivalent to setting \( \Delta Q_p = 0 \))

The bus voltages are reevaluated by the iterative application of:
\[ E_{k+1}^p = E_s + \sum_{q=1}^{n} \frac{Z_{pq}}{E_q^k} \cdot V_q^{k+1} + \sum_{q \neq p}^{n} \frac{V_p^k}{E_q^k} \cdot V_q^{k+1} \]

where:
\[ V_q^{k+1} = \frac{P_q - J Q_q}{E_q^{k+1}} \]

3. **Newton-Raphson Method Using \( V \) Bus in Rectangular Coordinates [4]**

The load flow problem can be solved by the Newton-Raphson method using a set of nonlinear equations to express the specified real and reactive powers in terms of bus voltages.

Substituting from the network performance equation (1) for \( I_p \) in (2):
\[ P_p - J Q_p = E_p^* \sum_{q=1}^{n} \gamma_{pq} E_q \]  

(8)
since
\[ Z_p = E_p + J f_p \]
and
\[ \gamma_{pq} = \frac{e_p^2 - e_q^2}{E_p E_q} \]

Equation (8) becomes:
\[ P_p - J Q_p = (e_p - J f_p) \sum_{q=1}^{n} \gamma_{pq} (e_q - J f_q) \]

Separating the real and imaginary parts:
\[ P_p = \sum_{q=1}^{n} e_p (e_q G_{pq} + f_q B_{pq}) + f_p (f_q G_{pq} - e_q B_{pq}) \]
\[ Q_p = \sum_{q=1}^{n} f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \]

Newton-Raphson method requires that a set of linear equations be formed expressing the relationship between the changes in real and reactive powers and the components of bus voltages as follows:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q \\
\end{bmatrix}
= 
\begin{bmatrix}
v_1 & J_2 \\
J_2 & J_4 \\
\end{bmatrix}
\begin{bmatrix}
\Delta E \\
\Delta f \\
\end{bmatrix}
\]

(9)

Equation (9) can be solved for \( \Delta E \) and \( \Delta f \).
To reduce the computation time and to increase the rate of the convergence, a multiplication factor \( \lambda \) for the J-matrix can be introduced (\( \lambda \ll 1 \)).

So

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \lambda
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta e \\
\Delta f
\end{bmatrix}
\]

4. Newton-Raphson Method Using \( Y_{bus} \) in Polar Coordinates \([2,5]\)

The power flow of the general n-node power system can also be described by a set of n-simultaneous complex equation(8).

If the voltage vector \( E_p \) at node \( p \) be expressed in polar form as:

\[
E_p = |E_p| e^{-j\Theta_p}
\]

Then equation (9) is reduced to a set of 2n real simultaneous equations:

\[
P_p = |E_p| \sum_{q=1}^{n} |E_q| (G_{pq} \cos (\Theta_p - \Theta_q) + B_{pq} \sin (\Theta_p - \Theta_q)) \tag{11}
\]

\[
Q_p = |E_p| \sum_{q=1}^{n} |E_q| (G_{pq} \sin (\Theta_p - \Theta_q) - B_{pq} \cos (\Theta_p - \Theta_q)) \tag{12}
\]

The elements of the Jacobian are calculated from equations(11&12).

The equations relating changes in power to changes in voltage magnitudes and phase angles for the N-R method is:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \Theta_p \\
|E|
\end{bmatrix} \tag{13}
\]

Equation (13) can be solved for \( \Delta \Theta_p \) and \( |E| \)

In general, for a small change in the magnitude of bus voltage the real power at the bus does not changes appreciably.
Likewise, for a small change in the phase angle of the bus voltage the reactive power does not change appreciably.

Therefore, using polar coordinates, a solution for the load flow problem can be obtain assuming the elements of the submatrices \( J_2 \) and \( J_3 \) to be zero.
So, the simplified matrix equation is:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_1 & 0 \\
0 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta |E|
\end{bmatrix}
\]  

(14)

5. Decoupled Methods [8]:

An inherent characteristic of any practical electric-power transmission system operating in the steady state is the strong interdependence between active powers and voltage magnitudes. Correspondingly, the coupling between "P-\theta" and "Q- |E| " components of the problem is relatively weak. In the load-flow problem, recent trends towards this objective by decoupling (solving separated) the P-\theta and Q- |E| problems are:

5.1: The Fast Decoupled Load-Flow Method Using \( Y_{Bus} \) in Polar Coordinates [7,8]

The first step in applying the P-\theta & Q- |E| decoupling principle is to neglect the coupling submatrices \( J_2 \) and \( J_3 \), giving two separated equations:

\[
\begin{bmatrix}
\Delta P/V \\
\Delta Q/V
\end{bmatrix} =
\begin{bmatrix}
B' \\
B''
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix}
\]  

(16)

(17)

where:

\[
B'_{pq} = -1/x_{pq} \quad (p \neq q)
\]

\[
B''_{pp} = \sum_{p \neq q} 1/x_{pq} \quad \text{and} \quad B''_{pq} = -B'_{pq}
\]

5.2: A New Decoupled Load Flow (Voltage Vectors) Method [6]

The two separated equations are:

\[
\Delta P^k = \tau^k \Delta \theta^{k+1}
\]  

\[
\Delta Q^k = \tau^k \Delta \theta^{k+1}
\]  

(18)

(19)

where:

\[
T_{pq} = -\frac{V_p V_q}{Z_{pq}^2} / x_{pq}, \quad T_{pp} = -\sum_{q=0}^{n} T_{pq} \quad p \neq q
\]

\[
U_{pq} = -\frac{1}{Z_{pq}^2} / x_{pq}, \quad U_{pp} = -\sum_{q=0}^{n} U_{pq} \quad p \neq q
\]
The relations (18 & 19) represent the new decoupled load-flow method for solving the load-flow problem.


The Taylor series expansion of quadratic equations corresponding to the load flow equations is given by:

\[
\begin{bmatrix}
Y_s
\end{bmatrix} = \begin{bmatrix}
Y(x_e)
\end{bmatrix} + \begin{bmatrix}
J
\end{bmatrix} \begin{bmatrix}
\Delta x
\end{bmatrix} + \frac{1}{2} \begin{bmatrix}
H
\end{bmatrix} \begin{bmatrix}
\Delta x
\end{bmatrix} \begin{bmatrix}
\Delta x
\end{bmatrix}
\]

where:

\[
J = \begin{bmatrix}
\frac{\partial Y_1}{\partial x_1} & \cdots & \frac{\partial Y_1}{\partial x_n} \\
\frac{\partial Y_2}{\partial x_1} & \cdots & \frac{\partial Y_2}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial Y_n}{\partial x_1} & \cdots & \frac{\partial Y_n}{\partial x_n}
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
\frac{\partial^2 Y_1}{\partial x_1^2} & \cdots & \frac{\partial^2 Y_1}{\partial x_n^2} \\
\frac{\partial^2 Y_1}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 Y_1}{\partial x_n \partial x_2} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 Y_1}{\partial x_1 \partial x_n} & \cdots & \frac{\partial^2 Y_1}{\partial x_n^2}
\end{bmatrix}
\]

The most important fact is that no terms above third, or the second derivative, exist because the original equation is quadratic. It is seen that the third term which is complicated and of high dimensionality can be expressed as a vector as:

\[
\begin{bmatrix}
Y_s
\end{bmatrix} = \begin{bmatrix}
Y(x_e)
\end{bmatrix} + \begin{bmatrix}
J
\end{bmatrix} \begin{bmatrix}
\Delta x
\end{bmatrix} + \begin{bmatrix}
\frac{1}{2} Y(\Delta x)
\end{bmatrix}
\]

(21)

Rewriting (21), we have:

\[
J \Delta x^{(k+1)} = Y_s - Y(x_e) - Y(\Delta x^k)
\]

and defining \( \Delta y^k \) as below:

\[
\Delta y^k = y_s - Y(x_e) - Y(\Delta x^k)
\]

the final numerical expression can be written as:

\[
J \Delta x^{(k+1)} = \Delta y^k
\]

In order to solve the equation for \( \Delta x \), the matrix \( J \) is triangular factorized to produce the triangulated matrix \( J' \) and new vector \( \Delta y'^k \) (the sign ' denotes triangular-factorized matrix).

\[
J \begin{bmatrix}
\Delta x^{(k+1)}
\end{bmatrix} = \begin{bmatrix}
\Delta y^k
\end{bmatrix}
\]

and \( \Delta x \) is obtained by the back substitution.
Numerical Application:

The different mentioned techniques are applied to have the load flow solution for the power system shown in Fig. 1 which is the 220 KV part of Egyptian unified electric power system and with system data as mentioned in tables 1 & 2.

Results:

The problem solution using the fast load-flow method retaining nonlinearity[10] is given in tables 3-1 & 3-2. Table 4 illustrates a comparison between the different techniques for having the problem solution. The results obtained are from computer programmes on ICL computer [12].

Comments:

Gauss-Seidel method using \( Y_{Bus} \) is an efficient and simple method. It requires smaller computer storage (6.72 K) and small time to obtain the final solution (18 Sec), although it uses the largest number of iterations (13), so that the convergence is low but the time per iteration is very small (1.4 Sec).

Gauss-Seidel method using \( Z_{Bus} \) uses small space in the computer (11.712 K), small number of iterations (3) and small time to obtain the final solution (20 Sec). The time per iteration is large (6.6 Sec) comparing with the last method, but the convergence is high.

M-R method using \( Y_{Bus} \) in rectangular coordinates is a complex method and uses large space in the memory of the computer (24.576 K), the overall time which is used to obtain the final solution is large (27 Sec) but the number of iterations is small (3), the time per iteration is large (9 Sec) comparing with other methods.

M-R method using \( Y_{Bus} \) in polar coordinates is nearly similar to last method (in rectangular coordinates) but the main difference is that, the voltage magnitude for polar coordinates (1.07 P.U) is higher than that for rectangular coordinates (1.04 P.U) for the second iteration in each bus [12]. Although the convergence region obtained with polar coordinates may be wider than one obtained with rectangular coordinates and the convergence with polar coordinates is less stable than that with rectangular coordinates.
The relations (18 & 19) represent the new decoupled load-flow method for solving the load-flow problem.


The Taylor series expansion of quadratic equations corresponding to the load flow equations is given by:

\[
\left[ Y_s \right] = \left[ Y(x_0) \right] + \left[ J \right] \left[ \Delta x \right] + \frac{1}{2} \left[ H \right] \left[ \Delta x \right] \left[ \Delta x \right] (20)
\]

where:

\[
J = \begin{bmatrix}
\frac{\partial Y_1}{\partial x_1} & \cdots & \frac{\partial Y_1}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial Y_n}{\partial x_1} & \cdots & \frac{\partial Y_n}{\partial x_n}
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
\frac{\partial^2 Y_1}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 Y_1}{\partial x_1 \partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 Y_n}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 Y_n}{\partial x_1 \partial x_n}
\end{bmatrix}
\]

The most important fact is that no terms above third, or the second derivative, exist because the original equation is quadratic. It is seen that the third term which is complicated and of high dimensionality can be expressed as a vector as:

\[
\left[ Y_s \right] = \left[ Y(x_0) \right] + \left[ J \right] \left[ \Delta x \right] + \left[ Y(\Delta x) \right] (21)
\]

Rewriting (21), we have:

\[
J \Delta x^{(k+1)} = Y_s - Y(x_0) - Y(\Delta x^k)
\]

and defining \( \Delta y^k \) as below:

\[
\Delta y^k = Y_s - Y(x_0) - Y(\Delta x^k)
\]

the final numerical expression can be written as:

\[
J \Delta x^{(k+1)} = \Delta y^k
\]

In order to solve the equation for \( \Delta x \), the matrix \( J \) is triangular factorized to produce the triangulated matrix \( J' \) and new vector \( \Delta y'^k \) (the sign ' denotes triangular-factorized matrix).

\[
\begin{bmatrix}
J' \\
\Delta x^{(k+1)}
\end{bmatrix} = \Delta y'^k
\]

and \( \Delta x \) is obtained by the back substitution.
Numerical Application:

The different mentioned techniques are applied to have the load flow solution for the power system shown in Fig. 1 which is the 220 KV part of Egyptian unified electric power system and with system data as mentioned in tables 1 & 2.

Results:

The problem solution using the fast load-flow method retaining nonlinearity[10] is given in tables 3-1 & 3-2. Table 4 illustrates a comparison between the different techniques for having the problem solution. The results obtained are from computer programs on ICL computer [12].

Comments:

Gauss-Seidel method using $Y_{bus}$ is an efficient and simple method. It requires smaller computer storage (6.72 K) and small time to obtain the final solution (18 Sec), although it uses the largest number of iterations (13), so that the convergence is low but the time per iteration is very small (1.4 Sec).

Gauss-Seidel method using $Z_{bus}$ uses small space in the computer (11.712 K), small number of iterations(3) and small time to obtain the final solution (20 Sec). The time per iteration is large (6.6 Sec) comparing with the last method, but the convergence is high.

N-R method using $Y_{bus}$ in rectangular coordinates is a complex method and uses large space in the memory of the computer (24.576 K), the overall time which is used to obtain the final solution is large (27 Sec) but the number of iterations is small (3), the time per iteration is large (9 Sec) comparing with other methods.

N-R method using $Y_{bus}$ in polar coordinates is nearly similar to last method (in rectangular coordinates) but the main difference is that, the voltage magnitude for polar coordinates (1.07 P.U) is higher than that for rectangular coordinates (1.04 P.U) for the second iteration in each bus [12]. Although the convergence region obtained with polar coordinates may be wider than one obtained with rectangular coordinates and the convergence with polar coordinates is less stable than that with rectangular coordinates.
The approximation method of N-R in polar coordinates, uses large number of iterations (7), large overall time to obtain the final solution (49 Sec), and the convergence is low but the storage required is small (18,304 K).

The modification of N-R method by the use of factor $\lambda \leq 1$, reduces the total time to obtain the final solution (17 Sec). The time per iteration (4.25 Sec) and the time per bus (1.13 Sec) is also reduced, but the storage required is similar to the normal N-R method (24,448 K).

The fast decoupled method using $Y_{bus}$ in polar coordinates uses a large number of iterations (6), small time per iteration (3.3 Sec) and the storage required is small (17,472 K) compared with the normal N-R method [9].

The new decoupled (voltage vectors) method using $Y_{bus}$ in polar coordinates is similar to the fast decoupled method.

The fast load-flow method retaining nonlinearity with triangular technique is similar to the normal N-R method but the only difference is that the storage required is high (34,368 K) and the total time required to obtain the final solution is small (21 Sec). If the elements of the J-matrix are stored using the sparsity techniques, it will save the space taken in the memory of the computer.

The algorithm used for this method is the exact expression, which takes into account all the terms of the Taylor series expansion and retains the nonlinearity of the original equation.

The modification of the fast load-flow method retaining nonlinearity without using triangular technique, it is similar to the last method (with triangular technique) but the only difference is that the storage requirement is small (25,024 K) comparing to the last method, and the total time taken to obtain final solution is small (16. Sec), therefore, the time per iteration is also small (4.5 Sec).
Conclusions:

From the above analysis and the results obtained by applying the different techniques on the actual power system shown in Fig. 1, it could be concluded that:

1- Gauss-Seidel method using $Y_{Bus}$ is preferred when the network is small compared with other methods, added to this Gauss-Seidel method using $Z_{Bus}$ is preferred when it is wanted to make high convergence, small computation time, and small computer storage.

2- N-R method in rectangular coordinates is preferred when the network is large but the space required in the computer and the time per iteration are large.

3- N-R method in polar coordinates is preferred for the same reasons mentioned before but the problem of stability will appear.

4- The approximation method can be considered as a modification of N-R method. It may be used when it is wanted to decrease the computation time and space in the computer.

5- The multiplication of J-matrix by factor $\lambda$ (in N-R method) is preferred when it is wanted to decrease the computation time and the time per iteration.

6- The fast and new decoupled methods are preferred when it is wanted to decrease the computation time, the time per iteration and the storage in the computer.

7- The fast load-flow method with triangular technique is used to decrease the computation time and the time per iteration, but the storage required is large.

8- The fast load-flow method without using triangular technique is preferred when it is wanted to decrease the computation time, the time per iteration and computer storage compared with last method.
REFERENCES:


Table 1  Data for branches of the moderate size real electric network.

<table>
<thead>
<tr>
<th>Branch number</th>
<th>From node</th>
<th>To node</th>
<th>Cross-section mm$^2$/cond</th>
<th>length km.</th>
<th>Resistance Ω/ph.</th>
<th>Inductive reactance Ω/ph.</th>
<th>Permissible continuous current Load Amp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>400</td>
<td>37.29</td>
<td>2.9459</td>
<td>14.6672</td>
<td>865</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>240</td>
<td>2.94</td>
<td>0.3822</td>
<td>1.2111</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
<td>400</td>
<td>46.6</td>
<td>1.864</td>
<td>9.2582</td>
<td>1700</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>5</td>
<td>240</td>
<td>135</td>
<td>8.775</td>
<td>27.805</td>
<td>1200</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>8</td>
<td>240</td>
<td>19.24</td>
<td>2.5012</td>
<td>7.9254</td>
<td>600</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>240</td>
<td>19.24</td>
<td>2.5012</td>
<td>7.9254</td>
<td>600</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>9</td>
<td>400</td>
<td>60</td>
<td>4.8</td>
<td>23.8409</td>
<td>850</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>9</td>
<td>400</td>
<td>60</td>
<td>4.8</td>
<td>23.8409</td>
<td>850</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>9</td>
<td>400</td>
<td>60</td>
<td>4.8</td>
<td>23.8409</td>
<td>850</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>11</td>
<td>400</td>
<td>50</td>
<td>4.0</td>
<td>19.8674</td>
<td>850</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>6</td>
<td>400</td>
<td>16.4</td>
<td>0.656</td>
<td>3.2583</td>
<td>1700</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>6</td>
<td>400</td>
<td>24</td>
<td>0.96</td>
<td>4.7682</td>
<td>1700</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>1</td>
<td>400</td>
<td>11</td>
<td>0.44</td>
<td>2.1654</td>
<td>1700</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>11</td>
<td>400</td>
<td>43</td>
<td>3.14</td>
<td>17.086</td>
<td>850</td>
</tr>
<tr>
<td>15</td>
<td>13</td>
<td>12</td>
<td>400</td>
<td>63</td>
<td>2.52</td>
<td>12.5165</td>
<td>1700</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>13</td>
<td>400</td>
<td>69</td>
<td>2.79</td>
<td>13.7085</td>
<td>1700</td>
</tr>
<tr>
<td>17</td>
<td>12</td>
<td>14</td>
<td>400</td>
<td>47</td>
<td>1.88</td>
<td>9.3377</td>
<td>1700</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>16</td>
<td>400</td>
<td>130.5</td>
<td>5.22</td>
<td>25.927</td>
<td>1700</td>
</tr>
<tr>
<td>19</td>
<td>16</td>
<td>15</td>
<td>240</td>
<td>35</td>
<td>2.275</td>
<td>7.2087</td>
<td>1700</td>
</tr>
</tbody>
</table>

* These branches are double circuit, the others are single circuit.
### Table 2. Node data for the moderate network

<table>
<thead>
<tr>
<th>Node No.</th>
<th>Name of country</th>
<th>Permissible generation</th>
<th>local consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MW</td>
<td>MVAR</td>
</tr>
<tr>
<td>1</td>
<td>Cairo West</td>
<td>175</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>Cairo - 500</td>
<td>125</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>Cairo south</td>
<td>120</td>
<td>60</td>
</tr>
<tr>
<td>4</td>
<td>Helwan</td>
<td>—</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>El-Suez</td>
<td>75</td>
<td>96</td>
</tr>
<tr>
<td>6</td>
<td>Cairo North</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>7</td>
<td>Cairo North (II)</td>
<td>—</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>Heliopolis</td>
<td>—</td>
<td>220</td>
</tr>
<tr>
<td>9</td>
<td>Zagazig</td>
<td>—</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>Tanta</td>
<td>—</td>
<td>210</td>
</tr>
<tr>
<td>11</td>
<td>Demanhor</td>
<td>160</td>
<td>100</td>
</tr>
<tr>
<td>12</td>
<td>Tahrir I</td>
<td>—</td>
<td>68</td>
</tr>
<tr>
<td>13</td>
<td>Aboes</td>
<td>—</td>
<td>120</td>
</tr>
<tr>
<td>14</td>
<td>El-America</td>
<td>—</td>
<td>200</td>
</tr>
<tr>
<td>15</td>
<td>Abo-El-Matameer</td>
<td>—</td>
<td>200</td>
</tr>
</tbody>
</table>
Table 3-1

<table>
<thead>
<tr>
<th>VA</th>
<th>VM</th>
<th>DELT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.060000</td>
<td>1.060000</td>
<td>0.000000</td>
</tr>
<tr>
<td>1.043465</td>
<td>1.043473</td>
<td>0.000075</td>
</tr>
<tr>
<td>1.053539</td>
<td>1.053568</td>
<td>0.000030</td>
</tr>
<tr>
<td>1.035998</td>
<td>1.035999</td>
<td>-0.000001</td>
</tr>
<tr>
<td>1.032202</td>
<td>1.032235</td>
<td>-0.000033</td>
</tr>
<tr>
<td>1.047407</td>
<td>1.047417</td>
<td>-0.000010</td>
</tr>
<tr>
<td>1.018962</td>
<td>1.019617</td>
<td>-0.000655</td>
</tr>
<tr>
<td>1.032352</td>
<td>1.034058</td>
<td>-0.001706</td>
</tr>
<tr>
<td>1.072286</td>
<td>1.075765</td>
<td>-0.003479</td>
</tr>
<tr>
<td>1.005878</td>
<td>1.009468</td>
<td>-0.003590</td>
</tr>
<tr>
<td>1.013897</td>
<td>1.013995</td>
<td>-0.000098</td>
</tr>
<tr>
<td>1.020904</td>
<td>1.021493</td>
<td>-0.000589</td>
</tr>
<tr>
<td>1.001455</td>
<td>1.002528</td>
<td>-0.001073</td>
</tr>
<tr>
<td>0.874969</td>
<td>0.880167</td>
<td>-0.005198</td>
</tr>
<tr>
<td>0.897654</td>
<td>0.901679</td>
<td>-0.004025</td>
</tr>
</tbody>
</table>

Table 3-2

<table>
<thead>
<tr>
<th>CIIR</th>
<th>SBIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.015315</td>
<td>0.015948</td>
</tr>
<tr>
<td>0.222819</td>
<td>0.231846</td>
</tr>
<tr>
<td>-0.023295</td>
<td>-0.026471</td>
</tr>
<tr>
<td>0.016831</td>
<td>0.017281</td>
</tr>
<tr>
<td>0.181367</td>
<td>0.190347</td>
</tr>
<tr>
<td>-0.019833</td>
<td>-0.018428</td>
</tr>
<tr>
<td>0.084689</td>
<td>0.081829</td>
</tr>
<tr>
<td>-0.034895</td>
<td>-0.031229</td>
</tr>
<tr>
<td>0.027243</td>
<td>0.034862</td>
</tr>
<tr>
<td>0.165710</td>
<td>0.175653</td>
</tr>
<tr>
<td>-0.112307</td>
<td>-0.127687</td>
</tr>
<tr>
<td>-0.019504</td>
<td>-0.020674</td>
</tr>
<tr>
<td>0.122999</td>
<td>0.125864</td>
</tr>
<tr>
<td>0.046978</td>
<td>0.047118</td>
</tr>
<tr>
<td>0.217595</td>
<td>0.220727</td>
</tr>
<tr>
<td>0.088651</td>
<td>0.092853</td>
</tr>
<tr>
<td>0.339779</td>
<td>0.353110</td>
</tr>
<tr>
<td>0.171732</td>
<td>0.170277</td>
</tr>
</tbody>
</table>

10.52.33 FREE ACROSS 34 TRANSFERS
10.52.33 FREE XLPD 41 TRANSFERS
0.47 : DELETED : 00
10.52.34 0.47 DELETED, CLOCKED 0.18
END OF MACRO
PARAMETER NUMBER 4 UNACCESSSED
END OF MACRO
MAXIMUM ONLINE BS USED : 295 KBWORDS
10.52.35 0.47 FINISHED : 1 LISTFILES
BUDGET USED LEFT
TIME(6) 48 998890
MONEY 133 992419