SAFETY CONSIDERATIONS
IN
UNTRANSPOSED OVERHEAD TRANSMISSION LINES
BY
M. Tantawy* & S. Farrag**

ABSTRACT:

For untransposed H.V. T.L. the electro magnetic and electrostatic effects are so large that affecting the balance of T.L. phase impedances. So, if an ungrounded object in the nearby of such lines, induced voltage will appear on it causing a flow of current from the object to ground through the object's capacitance.

This paper presents formulae for predicting the induced currents passing through an object in the vicinity of such lines. Also, formulae for predicting the voltage gradient are developed.

1- INTRODUCTION:

For an ungrounded object in the nearby of extra high voltage transmission line, induced voltages appear on it due to the electromagnetic and electrostatic induction, causing a flow of current from the object to ground through the object's capacitance to ground which will be energised.

These induced voltages are more severe in the vicinity of untransposed transmission lines.

If a low impedance such as a person's body shorts the object to ground, there will be an initial pulse of current to discharge the energy stored in the object's capacitance to ground. This discharge may be painful or even dangerous to humans.

2- Induced Current In An Object:-

For the object X of length $A_x$, in the vicinity of a set of conductors, we have

\begin{equation}
C_x = \frac{A_x}{\sqrt{1 + \left(\frac{A_x}{A_y}\right)^2}}
\end{equation}

where,

\* Assistant Prof., Electrical P
El-Mansoura University.

\** Assistant Lecturer, Electrical
El-Monofia University.
V\_1, \ldots, V\_i, \ldots, V\_n and carrying charges Q\_1, \ldots, Q\_i, \ldots, Q\_n then the object will have an induced voltage V\_x, and will carry a charge Q\_x.

The voltage equations can be written as:

\[
\begin{pmatrix}
V\_1 \\
V\_2 \\
\vdots \\
V\_n \\
V\_x
\end{pmatrix} =
\begin{pmatrix}
P\_{ll} & \cdots & P\_{lx} \\
\vdots & \ddots & \vdots \\
0 & \cdots & P\_{nn}
\end{pmatrix}
\begin{pmatrix}
Q\_1 \\
Q\_2 \\
\vdots \\
Q\_n \\
Q\_x
\end{pmatrix}
\]

\[\ldots\ldots(1)\]

Symbolically,

\[\begin{bmatrix} V \end{bmatrix} = \begin{bmatrix} P \end{bmatrix} \cdot \begin{bmatrix} Q \end{bmatrix} \]

\[\ldots\ldots(2)\]

Generally, the current flow from the \(i\) th conductor to the object is,

\[I\_ix = J \cdot w \cdot C\_ix \cdot (V\_i - V\_x) \cdot A\_x \text{ amps} \quad \ldots\ldots(3)\]

where,

\[C\_ix = \text{is the mutual capacitance between \(i\) th conductor and object \(X\) in the absence of the other conductors.}\]

So, the total current flow to the object from \(n\) conductors is,

\[I\_x = \sum_{i=1}^{n} I\_ix = J \cdot w \cdot A\_x \cdot \sum_{i=1}^{n} C\_ix \cdot (V\_i - V\_x) \ldots\ldots(4)\]

If the object is grounded; \((V\_x = 0)\) therefore,

\[I\_x = J \cdot w \cdot A\_x \cdot \sum_{i=1}^{n} C\_ix \cdot V\_i \quad \ldots\ldots(5)\]

\[(-1)^{i+(n+1)} \cdot \frac{\text{cofactor} P\_{ix}}{\text{determinant of} \ P} \text{ Farad/meter}\]
To simplify the expression of \( V_i \), a new set of voltages \( (V_i') \) may be introduced as:
\[
\begin{bmatrix}
V_1' \\
V_2' \\
\vdots \\
V_n'
\end{bmatrix}
= \begin{bmatrix}
P_{11} & \cdots & P_{1n} \\
\vdots & \ddots & \vdots \\
P_{n1} & \cdots & P_{nn}
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
Q_2 \\
\vdots \\
Q_n
\end{bmatrix}
\]

or in expanded form,

\[
\begin{bmatrix}
V_1' \\
V_2' \\
\vdots \\
V_n'
\end{bmatrix}
= \begin{bmatrix}
P_{11} & \cdots & P_{12} & \cdots & P_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
P_{n1} & \cdots & P_{n2} & \cdots & P_{nn}
\end{bmatrix}
\begin{bmatrix}
Q_1 \\
Q_2 \\
\vdots \\
Q_n
\end{bmatrix}
\]

Solving eqn. (6) for \( Q_1, Q_2, \ldots, Q_n \) and substituting in eqns. (1) gives:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix}
= \begin{bmatrix}
P_{12}/P_{22} & \cdots & P_{1n}/P_{nn} \\
\vdots & \ddots & \vdots \\
P_{n1}/P_{11} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
V_1' \\
V_2' \\
\vdots \\
V_n'
\end{bmatrix}
\]

Symbolically:

\[
[v] = [H] \cdot [v']
\]

\[
\therefore [v'] = [H^{-1}] \cdot [v]
\]

where,

\[
H_{ii} = 1
\]

and \( H_{ij} = P_{ij}/P_{jj} \)

So, the voltage of conductor \( i \) is:

\[
V_i = \sum_{j=1}^{n} \frac{P_{ij}}{P_{jj}} \cdot V_j'
\]

\[
\therefore \text{substituting in eqn. (5)}
\]

\[
I_x = J \cdot W \cdot A_x \sum_{i=1}^{n} C_{ix} \cdot V_i'
\]
FIG. 1: OBJECT IN VICINITY OF OVERHEAD TRANSMISSION LINE

FIG. 2: OBJECTS CONSIDERED FOR APPROXIMATE INDUCED CURRENT FORMULA
where,

\[ C_{ix} \text{ can be approximated to a high degree of accuracy as:} \]

\[ C_{ix} = \frac{P_{ix}}{P_{ii} + P_{xx} - P_{ix}^2} \text{ Farad/meter.} \]

3. Energy Stored:

The energy stored in the object's capacitance to ground is:

\[ SE_x = \frac{1}{2} C_x \cdot V_{xp}^2 \text{ joules} \]

where,

\[ V_{xp} \text{ is the object's peak open circuit voltage.} \]

\[ R_x \text{ is the leakage resistance,} \]

\[ V_{xp} = 2 I_x \cdot R_x / (1 + R_x \cdot J w \cdot C_x) \text{ volts} \]

The current \( I_x \) can be calculated from eqn. (10) and the value of \( C_x \) (the object-to-ground capacitance) can be approximately computed (5) as:

\[ C_x = \left( \frac{A_x}{3.048} - 1 \right) \cdot 1000 \text{ picofarads} \]

4. Voltage Gradient (3):

At the vicinity of an O. H. T. L., the voltage gradient is given by:

\[ E_x = \frac{1}{2 \pi \epsilon_0} \sum_{i=1}^{n} \frac{Q_i \cdot 2 H_i \cdot 10^{-3}}{H_i^2 + S_{ix}^2} \text{ KV/m.} \]

and \[ [Q] = [P]^{-1} \cdot [V] \]

where:

\[ H_i \text{ is the phase bundle height above ground in meters.} \]

\[ S_{ix} \text{ is the distance in meters from the ith bundle to the point of interest.} \]

\[ n \text{ is the number of phase bundles.} \]

\[ Q_i \text{ is the charge of phase bundle in coulombs.} \]

\[ \epsilon_0 \text{ is the permittivity of free space = 8.854 \cdot 10^{-12} F/m.} \]
5. Approximate Formulae For Predicting Electrostatic Effects:

5.1. Induced Current:

For a long truck and an automobile of dimensions as shown in Fig. (2), Formulae are developed to have the maximum induced current \( I_{xp} \), the cut-off current \( I_{xc} \), the cut-off distance \( L_c \) (the distance from the center phase to where this current level exists), and the slope \( \alpha_{xc} \).

For T.L. with horizontal configurations, different phase spacing, different operating voltages, and different heights, the induced current defined by eqn.(10) can be evaluated and plotted as a function of the distance Fig. (3).

Imperically the maximum r.m.s. induced current is given by:

\[
I_{xp} = a \cdot (KV) \cdot (V_{p.u.}) \cdot \sqrt{S} \cdot h^{-b} \cdot mA \quad \ldots\ldots\ldots(16)
\]

where,

- \( a \) is constant, depends on the line voltage and object size.
- \( KV \) is the nominal line-to-line voltage of the transmission circuit.
- \( h \) is the height in meters of the flat conductor configuration above the ground.
- \( b \) is constant which depends also on line voltage and object size.
- \( S \) is the phase spacing in meters.
- \( V_{p.u.} \) is the per unit operating voltage of the line.

Table (1-A) gives the constants \( a \) and \( b \) for predicting maximum r.m.s. induced current \( I_{xp} \).

The cut-off distance \( L_c \) was found to be a function of phase spacing \( S \). It can be calculated as:

\[
L_c = C \cdot S + d \quad \ldots\ldots\ldots(17)
\]

where, constants \( C \) & \( d \) are given in Table (1-B).

The cut-off current \( I_{xc} \) depends on line voltage and object size

\[
I_{xc} = V_{p.u.} \cdot I_{xco} \quad mA \quad \ldots\ldots\ldots(18)
\]

where,

- \( I_{xco} \) is the cut-off current at nominal voltage.

Table (1-C) gives the values of \( I_{xco} \) for a long truck and an automobile.
FIG. 3.1 ZONE BOUNDARIES FOR INDUCED CURRENT

FIG. (4): ZONE BOUNDARIES FOR THE NUMERICAL APPLICATION
The slope $\alpha_{xc}$ is the greatest slope of a straight line through the cut-off point, defined by $L_c$ and $I_{xc}$, and tangent to the induced current curve. It is a function of conductor height and is given by:

$$\alpha_{xc} = g \cdot h^{-2} \quad \text{mA/m}$$

where,

$g$ is a function of line voltage and object size, and given in Table (1-D).

5.2. Voltage Gradient:

Similarly, the maximum r.m.s. voltage gradient is given by the following formula:

$$E_{xp} = m \cdot (KV) \cdot (V_{p.u.}) \cdot \sqrt{S} \cdot H^{-n} \quad \text{KV/m}$$

where, $m$ and $n$ depend on line voltage and are given in Table (2-A) for various voltages.

The cut-off distance $M_c$ is given by:

$$M_c = p \cdot S + q$$

The values of $p$ & $q$ are given in Table (2-B) for various values of line voltage.

The slope $B_{xc}$, is given by:

$$B_{xc} = R \cdot H^{-2} \quad \text{KV/m/m}$$

where Table (2-C) determines the values of $R$ for various voltages.

The cut-off voltage gradient, $E_{xc}$, is given by

$$E_{xc} = V_{p.u.} \cdot E_{xco}$$

where,

$E_{xco}$ is the cut-off gradient at nominal voltage and given in Table (2-D).

6. Numerical Application:

If an automobile with dimensions shown in Fig.(2) in near-by a 765 KV line having a phase spacing $S = 15$ m., conductor height $h = 15$ m., two 46 m.m. diameter conductors per bundle and subspacing of 0.49 m. The line was assumed to be operating at 800 KV. It is required to establish areas along the right-of-way at which electrostatic effects may be a problem.
Solutions:

From Table (1-A), for an automobile, and at 765 KV,

\[ a = 0.0497 \quad \text{&} \quad b = 1.635 \]

Substituting in eqn. (16)

\[ I_{xp} = (0.0411) \times (765) \times \left(\frac{-800}{765}\right) \times 15 \times (15)^{-1.635} = 1.52 \text{ m.A.} \]

From Table (1-B)

\[ C = 1.491 \quad \text{&} \quad d = 20.20 \]

Substituting in eqn. (17)

\[ L_c = (1.491) \times (15) + 20.20 = 42.56 \ \text{m} \]

From Table (1-C)

\[ I_{xc} = 0.364 \ \text{m.A.} \]

Substituting in eqn. (18)

\[ I_{xc} = \left(\frac{800}{765}\right) \times (0.364) = 0.38 \ \text{m.A.} \]

From Table (1-D)

\[ g = 12.97 \]

Substituting in eqn. (19):

\[ \alpha = (12.97) \times (15)^{-2} = 0.057 \]

So, we can plot the zone boundary values from the foregoing results as shown in Fig. (4).

Substituting in eqn. (13) to have the automobile's capacitance to ground,

\[ C_x = \left(\frac{5.78}{3.048} - 1\right) \times 1000 = 899.6 \ \text{picofarad assuming that its leakage resistance is infinite, and substituting in eqn. (12) so,} \]

\[ V_{xp} = \frac{2 \times (1.25 \times 10^{-3}) \times E_x}{2 \times (50) \times E_x (899.6 \times 10^{-12})} = 6254.97 \ \text{volts} \]

Substituting in eqn. (11) to have the discharge energy,

\[ S\delta_x = \frac{1}{2} (899.6 \times 10^{-12}) \times (6254.97)^2 = 17.59 \ \text{milli-Joules} \]
### Table (1)

<table>
<thead>
<tr>
<th>Line-to-Line Voltage KV</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Automobil (5.79 m)</td>
<td>Long Truck (11.9 m)</td>
<td>Automobile &amp; Long Truck</td>
<td>$I_{xco} (mA)$</td>
</tr>
<tr>
<td>345</td>
<td>0.0453</td>
<td>1.652</td>
<td>0.217</td>
<td>1.763</td>
</tr>
<tr>
<td>500</td>
<td>0.0506</td>
<td>1.689</td>
<td>0.211</td>
<td>1.743</td>
</tr>
<tr>
<td>765</td>
<td>0.0497</td>
<td>1.635</td>
<td>0.194</td>
<td>1.669</td>
</tr>
<tr>
<td>1100</td>
<td>0.0404</td>
<td>1.545</td>
<td>0.157</td>
<td>1.579</td>
</tr>
</tbody>
</table>

### Table (2)

<table>
<thead>
<tr>
<th>Line-to-Line Voltage KV</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>n</td>
<td>p</td>
<td>q</td>
</tr>
<tr>
<td>345</td>
<td>0.2550</td>
<td>1.6923</td>
<td>2.055</td>
<td>12.21</td>
</tr>
<tr>
<td>500</td>
<td>0.3068</td>
<td>1.7425</td>
<td>1.604</td>
<td>17.89</td>
</tr>
<tr>
<td>765</td>
<td>0.3076</td>
<td>1.6819</td>
<td>1.491</td>
<td>20.20</td>
</tr>
<tr>
<td>1100</td>
<td>0.2532</td>
<td>1.5876</td>
<td>1.640</td>
<td>17.01</td>
</tr>
</tbody>
</table>
7. Conclusions:

Formulas for predicting the maximum induced current in an object in the vicinity of EHV and UHV lines have been presented which enable the designer to establish the zone at which the electrostatic effects may be a problem.

Also, formulas for predicting the voltage gradient at ground level have been developed.

8. References


