HEAT AND MASS TRANSFER FROM MOIST AIR FLOWING INSIDE COLD HORIZONTAL CIRCULAR DUCT
I-THEORETICAL STUDY

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Abstract

Heat and mass transfer from moist air flowing inside cold horizontal circular duct is investigated theoretically. In the present theoretical model, the flow of air inside the cold horizontal circular duct is assumed to be laminar and steady in cylindrical coordinates. The governing equations are the continuity, momentum, energy and concentration equations. These governing equations are put in the dimensionless form. By introducing new proper independent and dependent variables, the governing equations are transformed into a set of dimensionless differential equations. A computer program in FORTRAN language is developed to solve this set of equations to determine the distribution of the dimensionless velocity, temperature and concentration. Also, local values of Nusselt and Sherwood numbers for different values of process parameters are calculated. These parameters are presented by the following dimensionless physical quantities; Prandtl, Schmidt, and Reynolds numbers.

The present results show that, the average values of Nusselt and Sherwood numbers increase with increasing Reynolds number. Nusselt number increases with increasing Prandtl number. Also, Sherwood number increases with increasing Schmidt number. Comparison between the obtained theoretical results and the previous experimental works show good agreement.

Keywords: Heat and mass transfer, Steady, Laminar Flow, Moist air and Horizontal circular duct.

NOMENCLATURE

- C : Concentration of water vapor in the flowing air kg/m³
- C' : Dimensionless concentration, C' = \frac{C - C_{\infty}}{C_{\infty} - C_{\infty}}

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**Greek Symbol**

- \( \alpha \): Thermal diffusivity
- \( \psi \): Stream function
- \( \mu \): Dynamic viscosity of fluid
- \( \nu \): Kinematics viscosity of fluid
- \( \rho \): Density
- \( \theta \): Dimensionless temperature; \( \theta = \frac{T - T_w}{T_a - T_w} \)

**Subscript**

- \( av \): average
- \( w \): wall
- \( v \): vapor
- \( i \): inlet
- \( \infty \): free stream
- \( l \): humid air
- \( 2 \): condensate

1. **INTRODUCTION**

Humid-air flows in ducts are often used in various technical applications. Humidity control systems, air cleaning and conditioning systems, and also apparatus for drying materials in humid gases are some examples of these applications. If the
temperature of the streamline surface is lower than the saturation point, then the steam contained in the air undergoes condensation. In this case, the total wall heat flux includes the component due to convection, spent on cooling the vapor-gas mixture in the boundary layer, and the phase-transition heat, spent on vapor condensation. Heat and mass transfer processes through air-water interfaces are of a major importance in many engineering applications.

Lee et al. (2006) studied a series of experiments was performed to investigate the local heat transfer phenomenon inside a U-tube in a reflux condensation mode. A total of 477 data (108 for pure steam flow conditions and 369 for steam-air flow conditions) for the local heat transfer coefficients were obtained for various inlet flow rates of steam and air under atmospheric conditions. Based on the steam-air experimental results, a new correlation applicable to the reflux condensation mode is developed using the degradation factor.

Jenny Lindblom and Bo Nordell, (2006) studied the condensation irrigation (CI) which is a combined system for desalination and irrigation. By evaporating seawater in, for example, solar stills and letting the humidified air transport the formed vapour into an underground pipe system, fresh water will precipitate as the air is cooled by the ground. By using drainage pipes for underground air transportation, perforations in the pipes enable the water to percolate into the soil.

Revankar and Ollock, (2005) studied a two-dimensional, steady state model of convective film-wise condensation of a vapor and condensable gas mixture flowing downward inside a vertical tube. The mass transfer at the film and gas interface is treated as diffusion controlled process. The non-condensable effect on the condensation is taken into account through boundary layer analysis of species concentration and energy balance. Numerical predictions were obtained for the condensation heat transfer coefficient of turbulent vapor flow associated with laminar condensate. The predicted results were compared with the experimental data in the literature to assess the model. Non-condensable mass fraction and vapor non-condensable mixture temperature were presented in the form of radial and axial profiles.

Volchkov et al. (2004) studied the numerical solution of the system of differential energy, diffusion, and boundary-layer flow equations for the laminar and turbulent flows of humid air with surface. Steam condensation shows that the momentum- and matter-transfer processes and the convective heat transfer are similar with allowance for the standard corrections for the Prandtl and Lewis numbers in the range of steam concentrations in the flow core 0<C_d<0.2. Heat and mass transfer in humid air with such steam concentrations obeys the “dry-wall” regularities. The total heat transfer on the wall can be substantially increases due to the phase-transition heat; with increasing moisture content, the heat-transfer intensification ratio also increases.

Thome et al. (2003), studied a new flow pattern map and flow pattern based heat transfer model for condensation inside horizontal plain tubes. In Part I, a new version of a two-phase flow pattern map, originally for flow boiling, is presented for condensation inside horizontal tubes while a new heat transfer model is presented in Part II. The new flow pattern map incorporates a newly defined logarithmic mean void fraction (LMe) method for calculation of vapor void fractions spanning from low pressures up to pressures near the critical point. Several other modifications are also made that are appropriate for condensation as opposed to evaporation. In the absence of void fraction data at high reduced pressures for these conditions, the new LMe method has been indirectly validated using the convective condensation model for annular flow and
corresponding heat transfer test data at reduced pressures up to 0.8 bar. The new map has also been successfully compared to some recent

Sarma et al. (2002) solved analytically the problem of convective condensation of vapors inside a horizontal condenser tube. Homogeneous model approach is employed in the estimation of shear velocity, which is subsequently, made use of in predicting local convective condensation heat transfer coefficients. These results are compared with some of the available solutions in the literature. It is observed that the agreement is reasonably satisfactory validating the assumptions and the theory presented.

In most engineering applications, heat and mass transfer occur simultaneously and often these are strongly coupled. Therefore, in the present work, numerical study of heat and mass transfer from moving moist air flowing inside a horizontal cooled circular duct with constant wall temperature is examined.

2. MATHEMATICAL MODEL

A schematic of physical model under study is shown in Fig. (1), which shows the coordinate system and dimensional nomenclature. The fluid flow is assumed to be axi-symmetric flow, laminar, and steady with constant properties for the range of temperatures considered, two dimension, and compressible. Before contact, the velocity and temperature of fluids are uniform and correspond to free stream conditions.

The effect of gravity force and heat dissipation is neglected. Moreover, uniform velocity and temperature profiles, at the inlet of the circular duct, are assumed.

2.1 Governing Equations

It is suitable to select the cylindrical coordinate \((r, \phi, z)\) to express the flow governing equations. The used coordinate system is shown in figure (1). The flow describing equations are continuity, momentum, energy and concentration equations for axi-symmetric. They can be written as:

![Diagram of the problem](image-url)

\[
\begin{align*}
\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial w}{\partial z} &= 0 \quad (1) \\
\frac{\partial w}{\partial z} + u \frac{\partial w}{\partial r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \quad (2) \\
\frac{\partial u}{\partial z} + u \frac{\partial u}{\partial r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right) \quad (3) \\
\frac{\partial T}{\partial z} + u \frac{\partial T}{\partial r} &= \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \quad (4) \\
\frac{\partial C}{\partial z} + u \frac{\partial C}{\partial r} &= D \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) \quad (5)
\end{align*}
\]
Equations (1-5) are the continuity equation, axial and radial momentum equations, energy equation and concentration equation respectively. Where; \( w \) and \( u \) are the velocity component in axial and radial directions. The temperature, density and specific heat of the fluid at constant pressure are denoted by \( T \), \( \rho \) and \( C_p \) respectively. In the foregoing equations, \( P \), \( \mu \), \( k \), \( D \) and \( C \) are pressure, dynamic viscosity, thermal conductivity of the fluid, mass diffusivity; and concentration respectively.

The flow, heat and mass transfer inside the circular duct for laminar and steady flow are considered. In this case, the corresponding continuity, momentum, energy and concentration equations (1 – 5) in cylindrical coordinates must satisfy the following boundary conditions, as :

At \( z = 0.0 \), \( 0.0 < r < r_o \) :

\[ w = w_o \text{, } u = 0.0 \text{, } T = T_w \text{, } C = C_w \]

at \( 0.0 < z < 1 \), \( r = r_o \) :

\[ w = u = 0.0 \text{, } T = T_w \text{, } C = C_w \]

at \( z = 1 \), \( 0.0 < r < r_o \) :

\[ \frac{\partial w}{\partial z} = 0.0 \text{, } u = 0.0 \text{, } \frac{\partial T}{\partial r} = 0.0 \text{, } \frac{\partial C}{\partial r} = 0.0 \] (6) for interface

\[ k_1 \frac{\partial T}{\partial r} = k_s \frac{\partial T_w}{\partial r} + \rho_i D \frac{\partial m_r}{\partial r} \]

- At Axis of Symmetry at \( r = 0 \) , \( 0 < z < 1 \)

\[ \frac{\partial w}{\partial r} = \frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = \frac{\partial C}{\partial r} = 0.0 \]

Where; \( T_w \) and \( C_w \) are the temperature and water vapor concentration in ambient air.

Solving the governing equations (1-5) with their boundary conditions equation (6), one can obtain the velocity, temperature and concentration distributions through the flow field. Knowing hydrodynamic and thermal flow field local heat and mass transfer coefficients, and in turn, local Nusselt and Sherwood numbers can be determined. Local heat and mass transfer coefficients are defined according to the following equations;

\[ h = \frac{q}{T_w - T_w} = \frac{k}{T_w - T_w} \frac{\partial T}{\partial r} \] @ \( r = r_o \) \]

\[ h_m = \frac{m'}{C_w - C_w} = \frac{D}{C_w - C_w} \] (7-a)

And consequently local Nusselt and Sherwood numbers can be expressed as;

\[ Nu = \frac{h \cdot r_o}{k} \text{, } Sh = \frac{h_m \cdot r_o}{D} \] \) (7-b)

Average Nusselt and Sherwood numbers can be expressed as;

\[ Nu_{av} = \left( \frac{1}{l} \right) \int_0^l Nu \cdot dz \]

\[ Sh_{av} = \left( \frac{1}{l} \right) \int_0^l Sh \cdot dz \] \) (7-c)

Equation (7-a) expresses local heat transfer coefficient \( h \) as a function of heat flux at plate \( q \), the duct wall temperature \( T_w \) and the temperature of flow \( T_w \) and local mass transfer coefficient \( h_m \) as a function of mass flux of moist air \( m' \), the concentration at surface \( C_w \) and concentration of moist air flow \( C_w \). Referring to the definition of local Nusselt and Sherwood numbers, \( r_o \) is the air flow passage radius. Referring to the definition of the average Nusselt and Sherwood numbers equation (7-c), \( l \) is the distance along z-axis corresponding to duct length.

For eliminating pressure term from above momentum equations (2-3), one introduces flow vorticity \( \omega \) and stream function \( \psi \), where \( \omega \) and \( \psi \) take the following definitions:
Substitution with \( \omega, \psi \) in equations (1-5) and with some manipulations, they can be rewritten as:

\[
\begin{align*}
1 \frac{\partial \psi}{\partial z} \left( \frac{\partial \omega}{\partial r} - \frac{\omega}{r} \right) - 1 \frac{\partial \psi}{\partial r} \frac{\partial \omega}{\partial z} = & \\
\mu \left\{ \frac{\partial^2 \omega}{\partial z^2} + \frac{\partial^2 \omega}{r \partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} - \frac{\omega}{r^2} \right\} = & \\
\omega = & \left( 1 \frac{\partial^2 \psi}{\partial z^2} - 1 \frac{\partial \psi}{\partial r} \frac{\partial \psi}{\partial z} + 1 \frac{\partial^2 \psi}{r \partial r^2} \right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (9)
\end{align*}
\]

Equations (8-11) must satisfy the following boundary conditions:

- For \( z=0.0, 0.0 \leq r \leq r_a \):
  \[ \frac{1 \partial \psi}{r \partial r} = -w_0, \quad \frac{\partial \psi}{\partial z} = 0.0, \quad T = T_m, \quad C = C_m \]
- For \( 0.0 < z < 1 \), \( r = r_a \):
  \[ \frac{\partial \psi}{\partial r} = 0.0, \quad T = T_m, \quad C = C_m \]
- For \( z=0 \), \( 0 < z < 1 \), \( r = 0.0 \):
  \[ \frac{\partial^2 \psi}{\partial r^2} = 0.0, \quad \frac{\partial \psi}{\partial r} = 0.0 \]
- For \( 0.0 < z < 1 \), (interface):
  \[ \frac{\partial T}{\partial z} = \frac{1 \partial^2 T}{\partial r^2} + \rho_p D h \frac{\partial m_c}{\partial r} \]

Where \( 1 \) refer to moist air flow and \( 2 \) refer to condensate water.

### 2.2 Dimensionless Form of the Governing Equations

To put the flow describing equations in dimensionless form, one introduces the following definitions of the dependent and independent variables as:

\[
\begin{align*}
\theta &= \frac{T - T_m}{T_m - T_0}, \quad \psi = \frac{\psi}{w_0}, \quad \Omega = \frac{\omega}{w_0/r_m}, \\
W &= \frac{w}{w_0}, \quad U = \frac{u}{w_0}, \quad R = \frac{r}{r_m}, \quad Z = \frac{z}{r_m}\end{align*}
\]

\[ L = \frac{1}{r_m} \quad C' = \frac{C - C_m}{C_u - C_m} \]

Where the dimensionless temperature, the dimensionless stream function the dimensionless vorticity and dimensionless concentration are denoted by \( \theta \), \( \psi \), \( \Omega \) and \( C' \) respectively, \( W \) and \( U \) are the dimensionless axial and radial velocities respectively. The dimensionless duct length is denoted by \( L \). The dimensionless cylindrical coordinates and the dimensionless distance are denoted as \( R \) and \( Z \) respectively.

Reynolds, Prandtl Schmidt and Peclet numbers in the following sections are denoted as \( \text{Re}, \ \text{Pr}, \ \text{Sc} \) and \( \text{Pe} \) respectively, they are defined according to the following relations:

\[ \text{Re} = \frac{D w_0 r_m}{\mu}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad \text{Sc} = \frac{\nu}{D} \quad \text{and} \quad \text{Pe} = \text{Re} \cdot \text{Pr} \quad \text{or} \quad \text{Pe}_v = \text{Re} \cdot \text{Sc} \]

Using the above dimensionless forms of the dependent and independent variables (13) and the dimensionless previously mentioned numbers (14), the dimensionless form of the momentum, vorticity, energy, and concentration equations can be rewritten as:
be derived as a function of dimensionless temperature as:

\[ N_u = \left( \frac{1}{\theta - 1} \right) \left( \frac{d\theta}{dR} \right)_{R=1} \]

\[ Sh = \left( \frac{dC^*}{dR} \right)_{R=1} \]

2.3 NUMERICAL TECHNIQUE

Considering a mesh covering the domain of the flow field with lines parallel to the R-axis and Z-axis; and with uniform step size, each node of the mesh is identified by two identifiers \( i \) and \( j \), as shown in Fig. (2). Considering the \( R \)-direction, the value of \( R \) at any row \( i + 1 \), can be evaluated according to the following relations:

\[ R_{i+1} = R_i + \Delta R \]

Where \( i \) varies from 1 corresponding to \( R = 0 \), to \( M \) corresponding to \( R = 1 \), where the total number of rows equal to \( M \). \( \Delta R \) is the value of step size in \( R \)-direction. It is clear that, \( \Delta R \) depends on the selected total number of rows \( M \) and the maximum distance in \( R \) direction. In the same manner, the value of \( Z \) at any column \( j + 1 \) can be evaluated according to the following relation:

\[ Z_{j+1} = Z_j + \Delta Z \]

Where \( j \) varies from 1 corresponding to \( Z = 0 \), to \( N \) corresponding to \( Z = L \) and the step size in \( Z \)-direction which depends on the number of columns \( N \) and the maximum distance in \( Z \) direction.

The partial differential forms of the governing equations are transformed to sets of linear algebraic equations. Solving these sets of equations, using Gauss-Siedel iterative method, the hydrodynamic, the thermal and concentration flow fields can be obtained. Consequently, local and average Nusselt and Sherwood numbers can be derived with the aid of their definitions.

2.4 Numerical Procedure

The systems of algebraic equations are solved using Gauss-Siedel iteration method with the above technique. The calculations are carried out on uniform grid size.
distribution. A uniform grid with 
(6000*200) is considered to give grid 
independent results.

Because finite difference technique is used 
to solve the momentum and energy 
equations, the solution can become unstable, 
that is, as the solution proceeds; it might 
diverge increasingly from the actual 
solution. A simplified analysis of the 
conditions under which instability occurs is 
established, the results obtained indicate that 
convergence exists if:

$$\Delta R \propto \Delta Z^2 \cdot Re$$

![Fig.(2) schematic diagram of the mesh used in calculation](image)

The solution is carried out row by row. 
Seeking for rapid convergence of the 
solution, the obtained value of 
($\Omega, \psi, \theta$, and $C^*$) is used in the rest of 
equations of the rest ($\Omega, \psi, \theta$, and $C^*$)'s. 
This iteration is continued till either the 
percentage relative error is equal to or less 
than the prescribed allowable error 
(maximum error \( \leq 1 \times 10^{-3} \)).

Accordingly, the hydrodynamic and 
temperature distributions of the flow are 
obtained. The velocities and local Nusselt 
and Sherwood numbers can obtained at any 
general point $i,j$ as;

Axial velocity, $w = \frac{1}{R_i} \frac{\psi_{i+1,j} - \psi_{i-1,j}}{2\Delta R}$, radial 
velocity, $U = \frac{1}{R_i} \frac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta Z}$.

Nusselt number, $Nu = \left( \frac{1}{\theta} \right) \left( \frac{\theta_{i+1} - \theta_{i-1}}{\Delta R} \right)$

Sherwood number, $Sh = \left( \frac{C_{i+1}^* - C_{i-1}^*}{\Delta R} \right)$

3. Results and Discussion

Figure (3) illustrates the flow pattern for the 
flow issues inside a circular duct at $Re = 2500$ and 5000. The streamlines show that at 
Re=2500 the flow issues inside the duct is 
going parallel to the duct wall rapidly. At 
Re= 5000 the duct length is larger for low 
Reynolds number values of streamline 
inside the duct were found.

Figure (4) illustrates the isothermal contours 
for the flow issues inside a circular duct at $Re = 2500$ and 5000. The temperature distribution denotes that the 
flow temperature is lower at the upper wall and get 
higher far from the upper wall as expected.

Figure (5) illustrates the isomass contours for the flow issues inside a circular 
duct at $Re = 2500$ and 5000. The concentration distribution denotes that the 
flow concentration is high at the wall and get 
poor far from the lower wall as expected.

The accuracy of numerical model is verified by comparing the results from 
present model with the corresponding results 
reported by Lee et al. (2006) and Sarma et al. 
(2002). Figure (6) shows the variation of the 
average Nusselt number with Reynolds number for the present work compared with 
results obtained by Lee et al. (2006). It is 
noticed that for the present work Nusselt number increases with increasing Reynolds number. It is observed that good agreement 
was obtained.

Figure (7) shows the variation of the 
average Sherwood number with Reynolds number for the present work compared with 
results obtained by Sarma et al. (2002). It is 
observed that for the present work Sherwood number increases with increasing Reynolds number. The present results show good agreement with the results of previous work.
Figure (8) shows the variation of local Nusselt with respect to dimensionless distance $Z$ at different values of Prandtl number. It is observed that, as Prandtl number increases the value of Nusselt number also increases.

Local Sherwood number is plotted against dimensionless distance $Z$ with different values of Schmidt number, as shown in Fig. (9). It is observed that, as Schmidt number increases the value of Sherwood number increases.

The following relations correlate the theoretical results, where the Nusselt and Sherwood numbers are expressed as functions of Reynolds, Prandtl and Schmidt numbers:

$$Nu = 1.145 \, Re^{0.64} \, Pr^{0.34}$$

$$Sh = 1.252 \, Re^{0.612} \, Sc^{0.34}$$

Conclusions

In the present study, the condensation process of water vapor from moist air is, theoretically analyzed. The effects of process parameters are investigated. These parameters are presented by the following dimensionless physical quantities; Prandtl, Schmidt, and Reynolds numbers. Considering the theoretical proposed model, it is found that the increase of Prandtl and Schmidt numbers cause increase of Nusselt and Sherwood numbers.

Comparison between the obtained results and the previous works in case of steady flow shows good agreement. Two correlations for Nusselt number and Sherwood numbers are obtained.

REFERENCES


Figure (3) Stream lines for different Reynolds number

Re=2500

Dimensionless Distance, Z

Re=5000

Dimensionless Distance, Z

Figure (4) Isotherm lines for different Reynolds number

Re=2500

Dimensionless Distance, Z

Re=5000

Dimensionless Distance, Z

Re=2500

Dimensionless Distance, Z
Figure (5) Iso-mass lines for different Reynolds number

Figure (6) Comparison between Present Study and Previous work

Figure (7) Comparison between Average Sherwood number

Figure (8) Effect of Prandtl number on local Nusselt number at Re=2500

Figure (9) Effect of Schmidt number on local Sherwood number at Re=2500