AN ALGORITHM FOR DIRECT CALCULATION OF MAXIMAL OVERVOLTAGES AT TRANSMISSION LINE ENERGIZATION

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Abstract:

The paper is based on closed form analytical solution for switching overvoltages obtained by s-domain techniques. The sequential closing of the switch poles energizing a transmission line is reflected in expressions of the open-end voltages in terms of time and three parameters related to the moments of switching. These functions are then maximized by constrained optimization techniques.

Introduction:

A new approach for the calculation of electromagnetic transients in power system networks, based on state equations obtained by rational approximation in the s-domain, has been recently developed[1-5]. Its main advantage with respect to travelling wave approach methods[6,7], of which an up-to-date overview is given by Marti[8], consists in the fact that a transient voltage or current can be expressed and calculated from closed, form, analytical expression. Thus a transient can be calculated at discrete times without step-by-step integration procedures; and, perhaps, it is even more important that a closed form solution permits analytical calculations for instance maximization of overvoltages with respect to time and some other parameters, as it is shown in this paper. Without claiming that s-domain solutions are in general perferable to travelling wave methods, it is intened to examine here one particular application which is possible only due to the analytical form of the solution. It is felt that other analytical calculations, related perhaps to control and optimization, may eventually also be performed.

When energizing a transmission line the overvoltages at the far end depend in addition to time, on three parameters related to the closing time of the three switch poles. In order to obtain maximized values, a large number of simulations has to be performed. On the other hand, a numerical optimization algorithm will approach in large steps the solution point in the four dimensional space. This is of course at the price of longer preparatory work (as compared to simulations using a travelling wave approach) required for producing the closed form expressions of the objective functions to be optimized.

Closed form calculation of voltages:

This section will review the s-domain procedures of references 1,2,3,4,5, on hand of the problem of sequential energization of a three-phase transmission line (Fig. 1-a). The problem consists of three sequential problems, each with a single input: the compensating voltage for the closing switch pole (Fig. 1-b). There are each time several outputs: the voltages at the line end, and the voltages at the still-open switch poles.

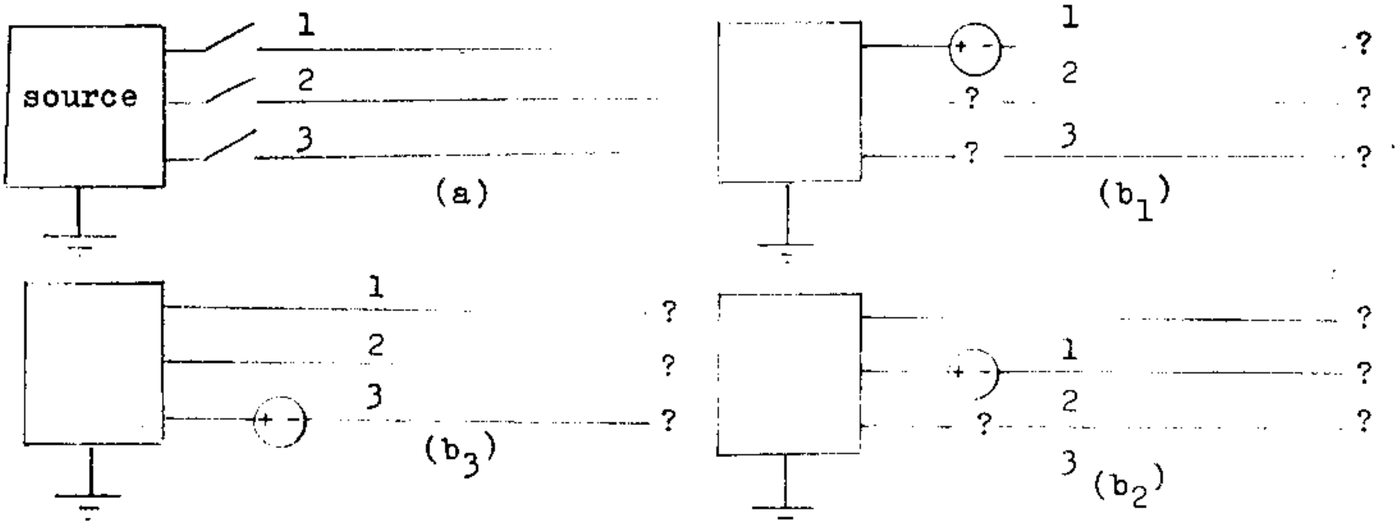

Fig. 1: Line energization

(a) Basic circuit.

(b) Compensating voltages sources when closing switch pole 1,2 and? indicate outputs of interest.

Scalar transfer functions H(s) are set up for all input-output conditions (see Appendix 1). This is possible by using the concept of complex depth $p = \frac{1}{\sqrt{s\mu_o\sigma}}$ in terms of the complex frequency s for the calculation of line impedances and replacing everywhere else as well jw by s.

The transfer functions H(s) are then approximated by rational polynomials in s. These are expanded in simple fractions with around 20 mostly complex conjugate poles p_k and associated residues r_k:

$$H(s) = \sum_k \frac{r_k}{s - p_k} + d \qquad (1)$$

The equivalent state equations are:

$$x_k = p_k x_k + u \qquad (2)$$

$$y = \sum_k r_k x_k + du \qquad (3)$$

If the input is exponential, i.e.:

$$u = \sum_i a_i e^{\gamma_i t} \qquad (4)$$

The equations (2) yield by integration from $t = t_o$ with $x_k(t_o) = 0$:

$$x_k = \sum_i A_i e^{\gamma_i t} + A_k e^{p_k t} \qquad (5)$$

where: $$A_i = \frac{a_i}{i - p_k} \qquad (6')$$

$$A_k = - \sum_i \frac{a_i}{\gamma_i - p_k} e^{(\gamma_i - p_k)t_o} \qquad (6'')$$

Once the state variables are calculated by equation (5), they can be used in equations as (3) to obtain any number of outputs y (see Fig. 1-b):

$$y = \sum_i (B_i + d)a_i e^{\gamma_i t} + \sum_k r_k A_k e^{p_k t} \qquad (7)$$

where:

$$B_i = \sum_k \frac{r_k}{\gamma_i - p_k} \qquad (8)$$

Subscripts for indicating which output is considered have been and will be ommitted for simplicity.

For the first pole to close γ_i of, equation (4) is $\pm$ jw.

In general γ_i represent (complex) input frequencies. The state variables (5) and the outputs contain the input frequencies and the (complex) natural frequencies p_k. At the next switching the output voltages become part of the compensating voltages for the closing pole, and thus all previous γ_i and p_k become input frequencies.

Thus all p_k from the previous stage have to be renamed as γ_i and new natural frequencies p_k will exist in the input. The details of these operations will be shown in the next section.

Overvoltages at sequential energization nomenclature:

v_i : Voltage across switch pole i, of i = 1,2,3; at receiving end of i = 4,5,6 (Fig. 2).

e_i : Thevenin voltage at the switches, for power frequency source.

e_{is}: Sine-input (i = 1,2,3)

e_{ic}: Cos - input (i = 1,2,3)

v_{is}: Sine - responce (i = 1,...,6)

v_{ic}: Cos - responce (i = 1,...,6)

θ : Switching angle

t_i : Closing time of switch pole i

v_i, v''_i, v_i''': Voltage due to the application of compensating voltage for closing:

pole 1 at $t_1 = 0$ (i = 2,...,6)

pole 2 at t_2 (i = 3,...,6)

pole 3 at t_3 (i = 4,5,6)

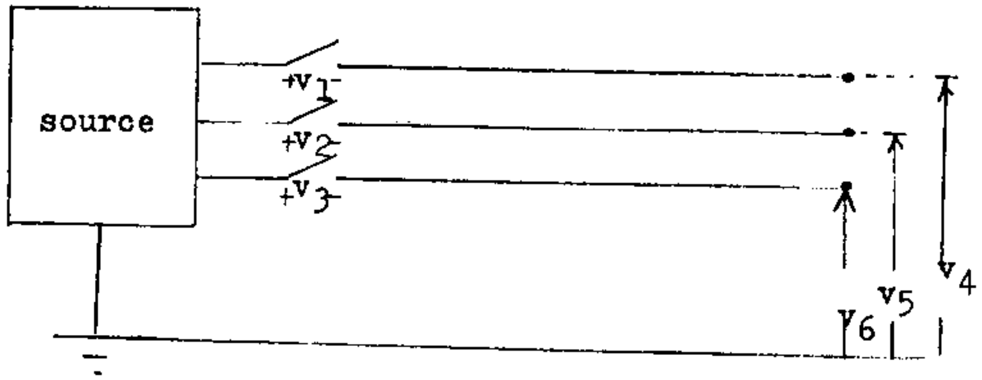

Fig. 2: Reference directions for voltages $v_1,\ldots,v_6$.

The parameters of the problem:

Phases a,b and c are associated to particular conductors of the line; whichever phase voltage is applied first to the line will be called 1; the numbers 1,2 and 3 are assigned to correspond to the sequence of pole closure; 4,5 correspond to the respective receiving end voltages. The source Thevenin voltage as seen between the switch terminals is:

$$e_1 = \sin(wt + \theta) \qquad (9')$$

$$e_2 = \sin(wt + \theta \mp \frac{2\pi}{3}) \qquad (9'')$$

$$e_3 = \sin(wt + \theta \pm \frac{2\pi}{3}) \qquad (9''')$$

The switch poles close at $t_1 = 0$, t_2, t_3.

The receiving and voltages v_4, v_5 and v_6 are thus functions of t, θ, t_2 and t_3. They can be maximized with respect to these four parameters, subject to the constraints: $t_1 > 0$ and $t_3 > t_2 > 0$.

Since - and cos-Responses:

Equations (9) can be rewritten as

$$e_i = \sin(wt + \theta + (i-1)\frac{2\pi}{3})$$

$$e_i = e_{is}.\cos\theta + e_{ic}.\sin\theta (i = 1,2,3) \qquad (10)$$

where $e_{is} = \sin(wt \mp (i-1)\frac{2\pi}{3})$ (11')

$$e_{ic} = \cos(wt \pm (i-1)\frac{2\pi}{3}) \qquad (11'')$$

are a sine-and a cos-input, respectively. The input voltage e_i of equation (10) is a linear combination, with coefficients cos θ and sin θ, of these two particular inputs.

By superposition, all outputs will be obtained superimposing the responses v_{is} and v_{ic} due to there two inputs. They will be called sine-and cos-responses respectively. The complete response, due to the input (10) and a givenset-of switching operations, is:

$$v_i = v_{is} \cos \theta + v_{ic} \sin \theta \qquad (12)$$

Thus, up to the last step, it is sufficient to consider separately the sine-and cos-inputs and to calculate independently their responses. This will be shown in more detailed in the following sections.

<u>One pole closed</u>:

The first phase closes at $t_1 = 0$. Before switching the voltage across pole 1 is $v_1 = e_1$ of equation (9').
The switching operation is simulated by the application across pole 1 of the compensating voltage $-v_1 = -e_1$, or, more exactly, of its sine-and cos-components. The other two poles are open. The result of the calculations are the closed form expressions (see Appendix 2):

$$v_{is}(t) \text{ and } v_{ic}(t), \text{ for } i = 2,\ldots,6 \qquad (13)$$

Now the resultant voltage at pole 2 has the components

$$e_{2s}(t) + v_{2s}(t) \text{ and } e_{2c}(t) + v_{2c}(t) \qquad (14)$$

<u>Two poles closed</u>:

The second pole closes at $t = t_2$. This is simulated by applying across pole 2 a compensating voltage equal to the negative of the given switch voltage given in equation (14). The first pole is closed, the third still open. We obtain the responses (see Appendix 3):

$$v''_{is}(t,t_2) \text{ and } v''_{ic}(t,t_2), \text{ for } i = 3,\ldots,6 \qquad (15)$$

The resultant voltage at pole 3 has the components:

$$e_{3s}(t)+v'_{3s}(t)+v''_{3s}(t,t_2) \text{ and } e_{3c}(t)+v''_{3c}(t,t_2)$$

At the open line end ($i = 4,5,6$) the voltage components are:

$$v_{is}(t,t_2) = v'_{is}(t) + v''_{is}(t,t_2) \qquad (17')$$

$$v_{ic}(t,t_2) = v'_{ic}(t) + v''_{ic}(t,t_2) \qquad (17'')$$

Three poles closed:

The third pole closes at $t=t_3$. We apply here the negative of the voltages given by equation (16). Poles 1 and 2 are closed. The responses obtained are (see Appendix 4):

$$v'''_{is}(t,t_2,t_3) \text{ and } v'''_{ic}(t,t_2,t_3), \text{ for } i=4,5,6 \qquad (18)$$

Thus the resultant sine-and cos responses at the open line end ($i = 4,5,6$) are :

$$v_{is}(t,t_2,t_3)=v'_{is}(t)+v''_{is}(t,t_2)+v'''_{is}(t,t_2,t_3) \qquad (19')$$

$$v_{ic}(t,t_2,t_3)=v'_{ic}(t)+v''_{ic}(t,t_2)+v'''_{ic}(t,t_2,t_3) \qquad (19'')$$

where the results of equations (13), (15) and (18) are used.

Resultant voltages:

Equation (12) indicates how the resultant voltage is obtained at the line end. It is applied for the components obtained in equations (13), (17) and (19).

Maximization of v_i given in equation (12) with respect to θ is straightforward; by requesting that

$$\frac{\partial v_i}{\partial \theta} = 0 \qquad (20)$$

We obtain the maximizing value of θ from:

$$\tan \theta = \frac{v_{ic}}{v_{is}} \qquad (21)$$

So that the maximum value of v_i in terms of θ becomes

$$v_i = \sqrt{v_{is}^2 + v_{ic}^2} \qquad (22)$$

Substituting the particular expressions of equations (13), (17) and (19), equation (22) yields the objectives, for $i=4,5,6$,

1- for $t \geqslant 0$: $v_1(t) \longrightarrow \max.$ (23')

2- for $t \geqslant t_2 \quad 0$: $v_i(t,t_2) \longrightarrow \max.$ (23'')

3- for $t \geqslant t_3 \quad t_2 \quad 0$: $v_i(t,t_2,t_3) \longrightarrow \max.$ (23''')

These are will scaled problems of conestrained optimization.

The optimization process gradients:

The objective functions and constraints for the optimization problems are shown in equation (23) and Appendices 3 and 4 indicate how the parameters t_2 and t_3 enter into the functions $v_i(t,t_2)$ and $v_i(t,t_2,t_3)$. For actually calculating the gradient of the general output function y of (7) equations (6) and (7) have to be used.

Derivative with respect to t:

From equation (7):

$$\frac{\partial y}{\partial t} = \sum_i \gamma_i (B_i + d) a_i \, e^{\gamma_i t} + \sum_k p_k \, r_k \, A_k \, e^{p_k t} \qquad (24)$$

The equation (24): A_k and B_i are obtained from equations (6") and (8).

Derivative with respect to t_3:

The parameter t_3 appears only in $v_i(t,t_2,t_3)$ of equation (23"'). Therefore, it enters into (7) only through the coefficient A_k of (6"):

$$A_k = - \sum_i \frac{a_i}{\gamma_i - p_k} \, e^{(\gamma_i - p_k)t_3} \qquad (25)$$

and we obtain

$$\frac{\partial y}{\partial t_3} = \sum_k r_k \frac{\partial A_k}{\partial t_3} \, e^{p_k t} \qquad (26)$$

where, from (25)

$$\frac{\partial A_k}{\partial t_3} = - \sum_i a_i \, e^{(\gamma_i - p_k)t_3} \qquad (27)$$

Derivative with respect to t_2:

The parameter t_2 appears first in $v_i(t,t_2)$ of equation (23") valid after closing the second switch pole. It is then carried over into the subsequent stage of switching. For the derivative of $v_i(t,t_2)$ with respect to t_2 equations similar to (26) and (27) are obtained:

$$\frac{\partial y}{\partial t_2} = \sum_k r_k \frac{\partial A_k}{\partial t_2} \, e^{p_k t} \qquad (26')$$

where $$\frac{\partial A_k}{\partial t_2} = -\sum_i a_i \, e^{(\gamma_i - p_k)t_2} \qquad (27')$$

In the third stage, after the third pole is closed, t_2 enters in expression (7) of y through those coefficients a_i of the input which carry over the new natural output terms obtained in the previous stage:

$$\frac{\partial y}{\partial t_2} = \sum_i (B_i+d)\,\frac{\partial a_i}{\partial t_2}\, e^{\gamma_i t} + \sum_k r_k \frac{\partial A_k}{\partial t_2}\, e^{p_k t} \qquad (28)$$

We will now denote by a prime the variables of the previous stage. Therefore, from (7):

$$a_i = r_{k'} A_{k'} \qquad (29)$$

where, from (6"):

$$A_{k'} = -\sum_{i'} \frac{a_{i'}}{\gamma_{i'} - p_{k'}}\, e^{(\gamma_{i'} - p_{k'})t_2} \qquad (30)$$

For equation (20) we calculate, using (29) and (30)

$$\frac{\partial a_i}{\partial t_2} = r_{k'} \frac{\partial A_{k'}}{\partial t_2} = -r_k \sum_{i'} a_{i'}\, e^{(\gamma_{i'} - p_{k'})t_2} \qquad (31)$$

and, using (25):

$$\frac{\partial A_k}{\partial t_2} = -\sum_i \frac{\frac{\partial a_i}{\partial t_2}}{\gamma_i - p_k} \cdot e^{(\gamma_i - p_k)t_3} \qquad (32)$$

where $\frac{\partial a_i}{\partial t_2}$ of (31) is used.

The end results are:

- for the second stage: equation (26') with (27)
- for the third stage: equation (28) with (31) and (32).

Conclusions:

By representing the transfer functions of a transmission line and the source network in terms of the complex variable s, an accurate rational approximation can be determined. Thus, transient voltages can be calculated from closed form expressions.

Simple and accurate algorithm for direct calculation of maximal overvoltages due line energization has been developed. Important computational advantages result from the fact that the method gives the maximized overvoltage directly without a large number of simulations has to be performed.

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Appendices

Appendix 1: Calculation of scalar transfer function:

Let V: denote a vector of line voltages, and I a vector of line currents. Then the following matrix equations can be written in the s-domain:

$$I = Y_{line}(s).\ V_{line} \qquad (A.1)$$

$$I = Y_{source}(s).\ V_{source} \qquad (A.2)$$

$$V_{switch} = V_{source} - V_{line} \qquad (A.3)$$

Equations(A.1) relate the line currents to the sending end voltage if the receiving end is open. Equations(A.2) relate the source currents to source terminal voltages; with the Thevenin voltage being removed and placed between the switch poles. Equations(A.3) represent voltage relations at the switch poles. From the last equations only those (n) are used where the poles are closed in the switch voltage is either zero or equal to the single compensating voltage $V_{comp.}$ For the poles which are still open we set the currents equal to zero. Thus the number of equations is (6+n) with (7+n) variables: $V_{line}(3)$, $V_{source}(3)$, $V_{comp}(1)$, I(n). Consequently one can calculate V_{line} and I in terms of $V_{comp.}$ and, with these expressions the receiving end voltages V_R can also be obtained in terms of $V_{comp.}$ Thus there are n-basic calculations of H(s) to be performed (for n = 1, n=2 and n=3). The actual number is larger because of closing switch can be on any of the three phases a,b or c and there are several outputs as shown in Fig. 1-b.

Appendix 2: Voltages resulting from closing the first pole:

The sine-and cos-components of the compensating voltage, applied at $t_o = 0$, are:

$$u_s = -e_{1s} = -\sin wt \qquad (A.4')$$

$$u_c = -e_{1c} = -\cos wt \qquad (A.4'')$$

Comparing equations (A.4') and (4) we obtain:

$$a_1 = -\frac{1}{2j}\ ,\quad a_2 = \frac{1}{2j} \qquad (A.5')$$

$$_1 = jw \quad , \quad _2 = -jw \qquad \text{(A.5")}$$

Consequently x_k of (5) contains in complex form a single forced oscillatory response to the sine-input u of (A.4') with the same frequency w, and an attenuating response with the complex neutral frequency $p_k = \alpha_k + j\,\beta_k$.
This remark serves only for interpreting results which are obtained by the computer in complex form.

Appendix 3: Voltages resulting from closing the second pole:

The second pole closes at $t_o = t_2$. All compensating complex output frequencies due to the closure of the first pole together with the jw of the source voltage are now renamed $_i$ and the coefficients of the exponential functions are renamed a_i. The responses obtained using equations (4) to (8) will have a forced part with complex frequencies γ_i and a natural part with the natural frequencies p_k of the circuit being switched.

Only the natural response is function of the closing time t_2, which appears in the exponent in each term of coefficient A_k, given in equation (6"). This is reflected in equation (16) where t_2 is present only through the coefficient of the new natural frequencies p_k (of this stage of switching). However, these terms become inputs in the next switching operations.

Appendix 4: Voltages resulting from closing the third pole:

The third pole closes at $t_o = t_3$. Again all results of the previous step, which now-become inputs, are renamed denoting by γ_i the complex frequencies and a_i the input coefficient as required by equation (4). However, now, those coefficients a_i which result from the natural frequencies. The previous switching are functions of t_2. The new parameter t_3 will appear in the exponent in each term of the coefficients A_k of the new natural frequencies p_k. Thus, in equation (6") a_i will contain t_2 in exponential form and t_3 will appear in the exponent replacing t_o. These remarks should serve as guidance for the evaluation of partial derivatives with respect to t_2 and t_3 needed in the optimization process.