NODAL LINE FINITE DIFFERENCE METHOD FOR THE ANALYSIS OF ELASTIC PLATES WITH TWO OPPOSITE SIMPLY SUPPORTED ENDS

BY

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INTRODUCTION

The analytical solutions of two and three dimensional structural problems with series expansion are restricted for special cases of loading and edge conditions. Alternative approaches for the solution of many cases of loading and edge conditions using of numerical solution are available. There have been a considerable number of studies aimed at clarifying the mathematical bases for the numerical methods and their applications in solving the structural problems.

The finite difference method is considered as one of the earliest numerical methods, which is successfully used for the analysis of certain class of structural problems. This method has the disadvantage that the resulting matrix has a relatively large size and does not have the nice property of banded matrices. Another numerical approach named the finite element method has become a wide spread and convenient solution technique for a wide range of complex structures. The finite element method, while powerful and versatile, has its drawbacks since a large number of simultaneous algebric equations have to be solved.

A development of the finite element method is the finite strip method, in which the actual structure is idealized into strips connected at nodal lines, while the two ends of all the strips join together to form the two opposite boundaries of the domain. The first paper on the finite strip method was presented by CHEUNG [1] on plate bending problems using a simply supported rectangular strips. This was subsequently generalized by CHEUNG [2,3] to include other end conditions. The displacement function of the strip is expressed as a product of a polynomial function across the width of the strip and a series function in the longitudenal direction. These series should satisfy a priore the boundary conditions at the ends of the strip. The most common series used are the basic functions which are derived from the solution of beam vibration differential equation. These basic functions have been worked out explicitly by VLAZOV [4] for the various end conditions.

This paper presents the formulation of a new semi analytical method named "nodal line finite difference method" (N.L.F.D). This method is similar to that of the finite strip method since both uses the same basic functions in the form of continious differentiable series in one direction. As a result a two dimensional problem reduces to a one dimensional one. The basic functions

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which fitted the boundary conditions in one direction are used in this method at a mesh of nodal lines in conjunction with simple finite difference approach in the other direction. The present approach has an advantage over the finite strip method,since the number of the unknown parameters along a nodal line is equal to the number of terms used in the basic function, and this is greatly less than that of the finite strip method.

The proposed technique is used to analyze a simple case of isotropic rectangular plates with two opposite simply supported ends. The results obtained are in very close agreement with those of the same conditions worked out by TIMOSHENKO [5]. The method can also be extended to include other material properties and other combinations of boundary conditions.

METHOD OF ANALYSIS

a - Nodal line finite difference equation

The solution of plate bending problems using the proposed technique, requires the division of the plate into a mesh of parallel nodal lines in one direction as shown in Fig.1. The displacement function at each nodal line of the mesh is expressed as a summation of the basic function terms fitting the boundary conditions at the two opposite ends of the nodal line multiplied by nodal parameters. These parameters are assumed as functions of single variable of the direction perpendicular to the nodal lines direction.

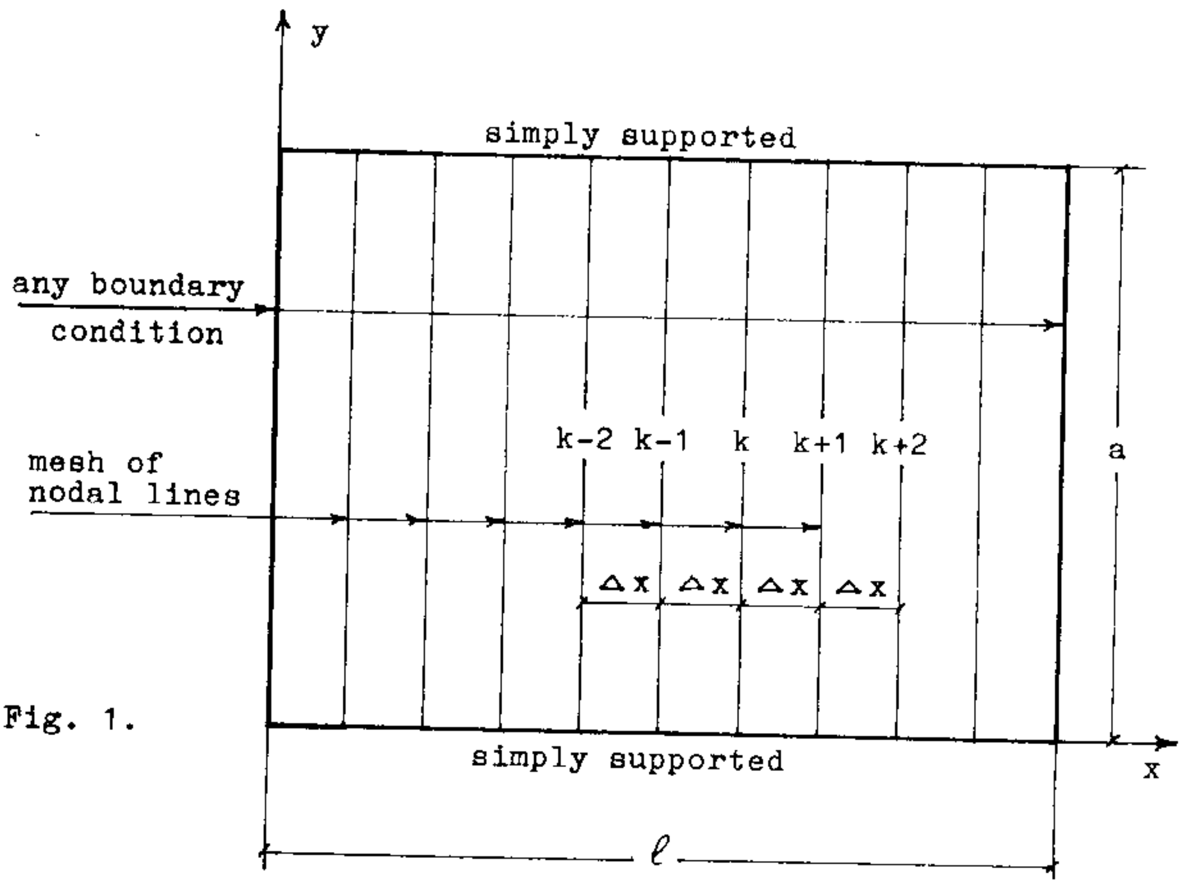


Fig. 1.

The derivation of the nodal line finite difference equation of the plate bending problems starts from the differential equation of the plate. The partial differential equation for the deflection surface of the elastic isotropic plates is governed by

$$w^{\cdot\cdot\cdot\cdot} + 2\, w^{\cdot\cdot\prime\prime} + w^{\prime\prime\prime\prime} = \frac{q}{B} \qquad (1)$$

where $(\ )^{\cdot} = \frac{\partial}{\partial x}$, $(\ )^{\prime} = \frac{\partial}{\partial y}$ and

B is the bending stiffness of the plate

$$B = \frac{E\, t^3}{12(1-\nu^2)}$$

The displacement function at any nodal line k (Fig. 1) is proposed in a series form as

$$w_k = \sum_{m=1}^{r} f_{m,k}(x) \cdot Y_m(y) \qquad (2)$$

For the case of plates with two opposite simply supported ends, the basic function is a trigonometric series in the form

$$Y_m(y) = \sin \frac{m\pi}{a} y = \sin \mu_m y \qquad (3)$$

Substitution of equation (2) into equation (1) gives

$$\sum_{m=1}^{r} [\, f^{\cdot\cdot\cdot\cdot}_{m,k} Y_m + 2\, f^{\cdot\cdot}_{m,k} Y^{\prime\prime}_m + f_{m,k} Y^{\prime\prime\prime\prime}_m \,] = \frac{q_k}{B}$$

$$\sum_{m=1}^{r} [\, f^{\cdot\cdot\cdot\cdot}_{m,k} - 2\, \mu^2_m f^{\cdot\cdot}_{m,k} + \mu^4_m f_{m,k} \,] \sin \mu_m y = \frac{q_k}{B} \qquad (4)$$

Applied loads must also be resolved into series similar to the displacement function

$$q_k = \sum_{m=1}^{r} q_{m,k} \sin \mu_m y \qquad (5)$$

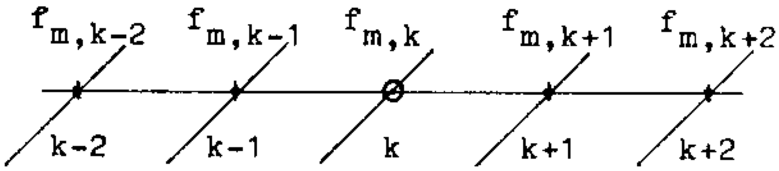
By substituting equation (5) into equation (4), we get

$$\sum_{m=1}^{r} [\, f^{\cdot\cdot\cdot\cdot}_{m,k} - 2\, \mu^2_m f^{\cdot\cdot}_{m,k} + \mu^4_m f_{m,k} \,] = \frac{1}{B} \sum_{m=1}^{r} q_{m,k} \qquad (6)$$

For each term of the basic function, the following relation can be written

$$f^{\cdot\cdot\cdot\cdot}_{m,k} - 2\, \mu^2_m f^{\cdot\cdot}_{m,k} + \mu^4_m f_{m,k} = \frac{1}{B} q_{m,k} \qquad (7)$$

By applying the central finite difference technique in the x direction, we get



$$\left.\begin{array}{lllllll} f_{m,k} = & [\ 0 & 0 & 1 & 0 & 0\ ] \\ \ddot{f}_{m,k} = \frac{1}{\Delta x^2} & [\ 0 & 1 & -2 & 1 & 0\ ] \\ \ddot{\ddot{f}}_{m,k} = \frac{1}{\Delta x^4} & [\ 1 & -4 & 6 & -4 & 1\ ] \end{array}\right\} \quad (8)$$

where Δx is a constant distance between the nodal lines in the x direction.

subistituting equation (8) into equation (7) gives

$$\frac{1}{\Delta x^4}[1 \quad -(4+2\psi_m^2) \quad (6+4\psi_m^2+\psi_m^4) \quad -(4+2\psi_m^2) \quad 1]\{f_{m,k-2} \quad f_{m,k-1} \quad f_{m,k} \quad f_{m,k+1} \quad f_{m,k+2}\} = \frac{1}{B} q_{m,k} \quad (9)$$

where $\psi_m = \frac{m\pi}{\lambda} = \mu_m \, \Delta x \quad , \lambda = \frac{a}{\Delta x}$

Equation (9) can be rewritten as

$$[1 \quad C_m^1 \quad C_m^2 \quad C_m^1 \quad 1]\{f_{m,k-2} \quad f_{m,k-1} \quad f_{m,k} \quad f_{m,k+1} \quad f_{m,k+1}\} = \frac{a^4}{B\lambda^4} q_{m,k} \quad (10)$$

Equation (10) represents the central nodal line finite difference equation in a matrix form.

Application of equation (10) at each nodal line of the plate gives

$$[S]_m\{f\}_m = \{P\}_m \quad (11)$$

where $[S]_m$ is a square matrix,

$\{f\}_m$ is the vector of the unknown nodal line parameters

and $\{P\}_m$ is the load vector.

The resulting square matrix $[S]_m$ is a band matrix of dimensions (MxM) having a band width equal to 5. Where M equal to the number of the nodal lines. When eliminating the zero elements, this matrix can be stored in a reduced rectangular matrix of dimensions (Mx5). As a result of that, the execution time of the problem is drastically reduced.

b - Internal forces

For an elastic isotropic plate, the internal forces per unit length at any point are given by

$$\left.\begin{aligned} M_x &= -B\,(w^{\cdot\cdot} + \nu w'') \\ M_y &= -B\,(w'' + \nu w^{\cdot\cdot}) \\ M_{xy} &= B\,(1-\nu)\,w^{\cdot\prime} \\ Q_x &= -B\,(w^{\cdot\cdot\cdot} + w^{\cdot\prime\prime}) \\ Q_y &= -B\,(w^{\cdot\cdot\prime} + w^{\prime\prime\prime}) \end{aligned}\right\} \quad (12)$$

Once the displacement function at each nodal line is available (equation (2)), it is a relatively simple matter to obtain the internal forces. By applying the central finite difference technique in the x direction, the internal forces at each nodal line k can be written as

$$\left.\begin{aligned} M_{x,k} &= -\frac{B\lambda^2}{a^2}\sum_{m=1}^{r}[f_{m,k-1}-(2+\nu\psi_m^2)f_{m,k}+f_{m,k+1}]\sin\mu_m y \\ M_{y,k} &= -\frac{B\lambda^2}{a^2}\sum_{m=1}^{r}[\nu f_{m,k-1}-(2\nu+\psi_m^2)f_{m,k}+\nu f_{m,k+1}]\sin\mu_m y \\ M_{xy,k} &= \frac{B\lambda^2}{2a^2}(1-\nu)\sum_{m=1}^{r}\psi_m[-f_{m,k-1}+f_{m,k+1}]\cos\mu_m y \\ Q_{x,k} &= -\frac{B\lambda^3}{2a^3}\sum_{m=1}^{r}[-f_{m,k-2}+(2+\psi_m^2)f_{m,k-1}-(2+\psi_m^2)f_{m,k+1} \\ &\qquad + f_{m,k+2}]\sin\mu_m y \\ Q_{y,k} &= -\frac{B\lambda^3}{a^3}\sum_{m=1}^{r}\psi_m[f_{m,k-1}-(2+\psi_m^2)f_{m,k}+f_{m,k+1}]\cos\mu_m y \end{aligned}\right\} \quad (13)$$

c - Boundary conditions

The proposed technique requires the application of the central nodal line finite difference equation at each nodal line of the plate. This equation can be used for all nodal lines withen the plate including the edge nodal lines. Each edge central nodal line finite difference equation will introduce two additional exterior nodal lines. According to the prescribed boundary conditions, the parameters of the exterior nodal lines have to be defined in terms of parameters of the edge and the two adjacent nodal lines. Thus the parameters of the exterior nodal lines can be written in the following forms.

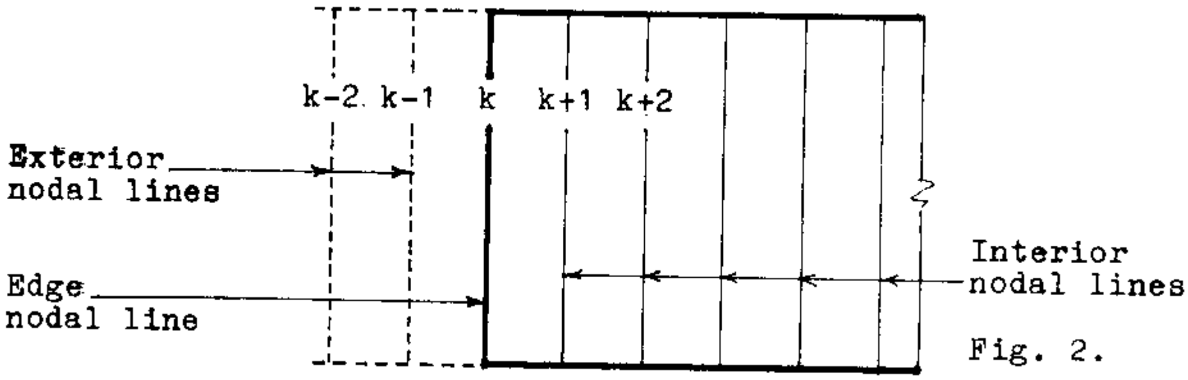


Fig. 2.

1 - Simply supported egde [$w_k = 0$, $(\ddot{w} + \nu w'')_k = 0$]

$$\left.\begin{aligned} f_{m,k} &= 0 \\ f_{m,k-1} &= -f_{m,k+1} \\ f_{m,k-2} &= -f_{m,k+2} \end{aligned}\right\} \qquad (14)$$

2 - Clamped edge [$w_k = 0$, $\dot{w}_k = 0$]

$$\left.\begin{aligned} f_{m,k} &= 0 \\ f_{m,k-1} &= f_{m,k+1} \\ f_{m,k-2} &= f_{m,k+2} \end{aligned}\right\} \qquad (15)$$

3 - Free edge [$(\ddot{w} + \nu w'')_k = 0$, $\{\dddot{w} + (2-\nu)\dot{w}''\}_k = 0$]

$$\left.\begin{aligned} f_{m,k} &\neq 0 \\ f_{m,k-1} &= \xi_1 f_{m,k} - f_{m,k+1} \\ f_{m,k-2} &= \xi_1\xi_2 f_{m,k} - 2\xi_2 f_{m,k+1} + f_{m,k+2} \end{aligned}\right\} \qquad (16)$$

where

$\xi_1 = (2+\nu\psi_m^2)$, $\xi_2 = [2+(2-\nu)\psi_m^2]$

NUMERICAL EXAMPLES

To demonstrate the accuracy of the nodal line finite difference method presented herein, the solution of some plate bending problems has been carried out. The proposed solution technique is applied to plates with two opposite simply supported ends and any combination of boundary conditions on the other two sides. The study is restricted to the elastic isotropic plates with constant thickness under symmetrical types of loading.

Study of convergence of the proposed method for a square plate under uniform distributed load has been achieved. The boundary conditions, for this case is illustrated in Figs.3.a and b. The obtained results are summarized in Tables 1 and 2. The study illustrates the effect of both number of terms used in the basic function and the mesh interval Δx on the convergence of the method.

Tables 3,4,5 and 6 include the results obtained from the analysis of rectangular plates with different ratios of rectangularity ℓ/a under a uniform distributed load. As far as the loading is concerned, types of loading other than uniform distributed are considered. The central deflection of square plate simply supported on all edges under a patch line load has been presented in Table 7. Rectangular plates having different ratios of rectangularity under central concentrated patch load has been analized. The central deflection of this case is given in Table 8.

In the above mentioned examples, only odd terms contribute to the results, because of symmetry of loading along the nodal line direction. The results obtained demonstrate the high accuracy and the rapid convergence of the proposed method.

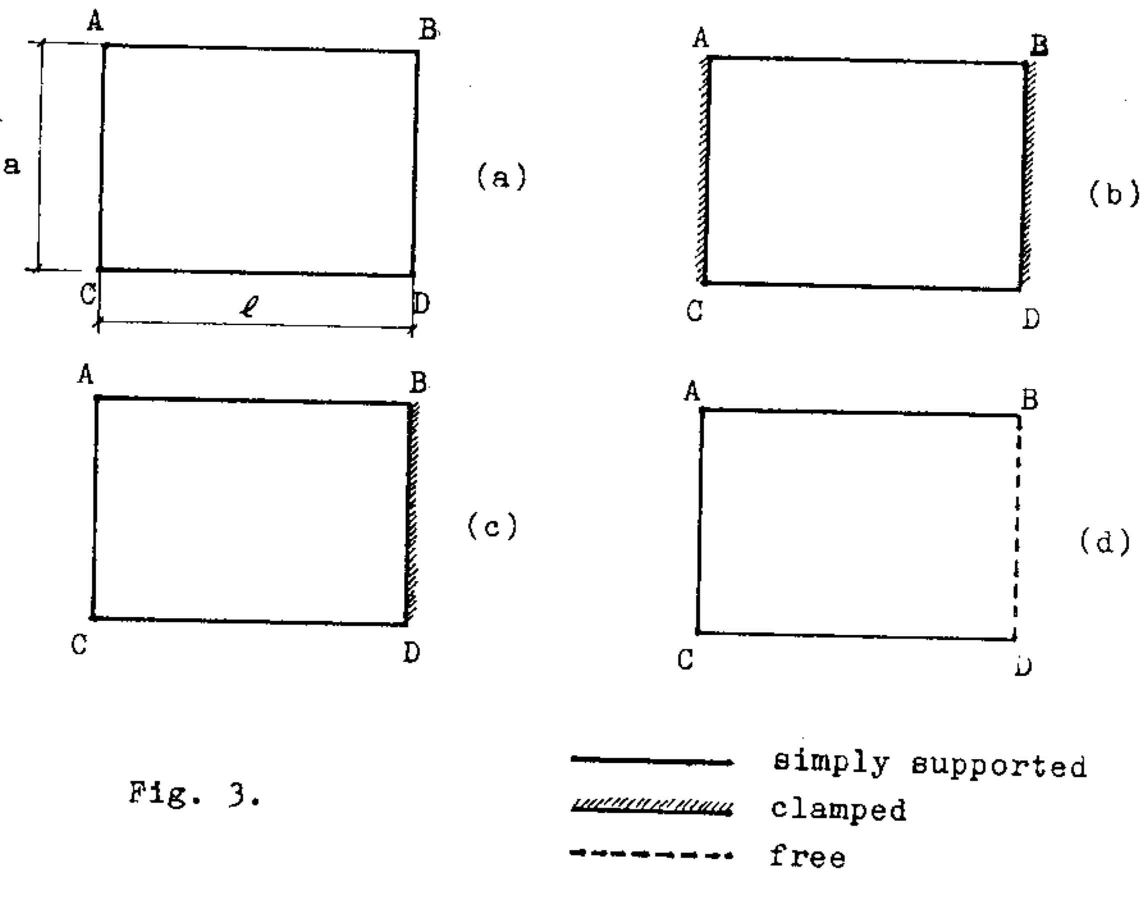


Fig. 3.

Table 1. Study of convergence. Square plate simply supported on four sides subjected to uniform distributed load of intensity q. Fig.3-a, $\nu = 0.3$

Δx	No of terms	Central Deflection	Central Mx	Central My
ℓ/20	1	41.0868	4.9130	5.1640
	2	40.5821	4.7580	4.7096
	3	40.6238	4.7892	4.8124
	4	40.6160	4.7779	4.7748
	5	40.6182	4.7832	4.7925
	6	40.6174	4.7803	4.7828
	7	40.6178	4.7820	4.7887
	8	40.6176	4.7809	4.7849
	9	40.6177	4.7817	4.7875
	10	40.6176	4.7811	4.7856
ℓ/40	1	41.0919	4.9184	4.1661
	2	40.5867	4.7634	4.7112
	3	40.6283	4.7946	4.8141
	4	40.6205	4.7833	4.7765
	5	40.6228	4.7886	4.7942
	6	40.6219	4.7857	4.7845
	7	40.6223	4.7875	4.7903
	8	40.6221	4.7863	4.7865
	9	40.6222	4.7871	4.7891
	10	40.6222	4.7866	4.7873
Exact [5]		40.60	4.79	4.79
Multiplier		$10^{-4}.q.a^4/B$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$

Table 2. Study of convergence. Square plate clamped on sides AC and BD, simply supported on the two other sides subjected to uniform distributed load of intensity q.Fig.3-b, $\nu = 0.3$

Δx	No of terms	Central Deflection	Central Mx	Central My	Mx at Middle of AC
ℓ/20	1	19.9368	3.4666	2.8305	-7.2849
	2	19.4528	3.3067	2.3914	-6.8562
	3	19.4944	3.3380	2.4940	-6.9351
	4	19.4866	3.3267	2.4564	-6.9117
	5	19.4888	3.3320	2.4741	-6.9205
	6	19.4880	3.3291	2.4644	-6.9166
	7	19.4884	3.3309	2.4703	-6.9185
	8	19.4882	3.3297	2.4665	-6.9175
	9	19.4883	3.3305	2.4691	-6.9181
	10	19.4882	3.3300	2.4672	-6.9177
ℓ/40	1	19.6990	3.4627	2.8080	-7.3582
	2	19.2159	3.3024	2.3694	-6.8947
	3	19.2574	3.3337	2.4721	-6.9905
	4	19.2496	3.3224	2.4345	-6.9579
	5	19.2519	3.3277	2.4522	-6.9720
	6	19.2510	3.3248	2.4425	-6.9650
	7	19.2514	3.3266	2.4484	-6.9688
	8	19.2512	3.3254	2.4445	-6.9666
	9	19.2513	3.3262	2.4472	-6.9679
	10	19.2513	3.3257	2.4453	-6.9671
Exact [5]		19.20	3.32	2.44	-6.97
Multiplier		$10^{-4}.q.a^4/B$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$

Table 3. Analysis of rectangular plates simply supported on four sides subjected to uniform distributed load of intensity q Fig.3-a , $\Delta x = \ell/40$, No of terms=7, $\nu = 0.3$

ℓ/a	Central Deflection	Central Mx	Central My	
1.0	40.6223 40.60	4.7875 4.79	4.7903 4.79	N.L.F.D Exact [5]
1.1	48.6852 48.50	4.9304 4.93	5.5498 5.54	N.L.F.D Exact [5]
1.2	56.4974 56.40	5.0067 5.01	6.2692 6.27	N.L.F.D Exact [5]
1.3	63.9104 63.80	5.0323 5.03	6.9392 6.94	N.L.F.D Exact [5]
1.4	70.8333 70.50	5.0208 5.02	7.5552 7.55	N.L.F.D Exact [5]
1.5	77.2201 77.20	4.9830 4.98	8.1159 8.12	N.L.F.D Exact [5]
2.0	101.2487 101.30	4.6347 4.64	10.1669 10.17	N.L.F.D Exact [5]
3.0	122.2809 122.30	4.0640 4.06	11.8843 11.89	N.L.F.D Exact [5]
4.0	128.1533 128.20	3.8432 3.84	12.3454 12.35	N.L.F.D Exact [5]
Multiplier	$10^{-4}.q.a^4/B$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$	

Table 4. Analysis of rectangular plates clamped on sides AC and BD, simply supported on the two other sides subjected to uniform distributed load of intensity q. Fig.3-b, $\Delta x = \ell/40$, No of terms=7, $\nu = 0.3$

ℓ/a	Central Deflection	Central Mx	Central My	My at Middle of AC	
1.0	19.2514 19.20	3.3266 3.32	2.4484 2.44	-6.9688 -6.97	N.L.F.D Exact [5]
1.1	25.3779 25.10	3.6967 3.71	3.0972 3.07	-7.8548 -7.87	N.L.F.D Exact [5]
1.2	32.0624 31.90	4.0086 4.00	3.7825 3.76	-8.6487 -8.68	N.L.F.D Exact [5]
1.3	39.0969 38.80	4.2600 4.26	4.4839 4.46	-9.3418 -9.38	N.L.F.D Exact [5]
1.4	46.2784 46.00	4.4531 4.48	5.1829 5.14	-9.9333 -9.98	N.L.F.D Exact [5]
1.5	53.4253 53.10	4.5926 4.60	5.8639 5.85	-10.4280 -10.49	N.L.F.D Exact [5]
2.0	84.6182 84.40	4.7314 4.74	8.7025 8.69	-11.7892 -11.91	N.L.F.D Exact [5]
3.0	116.8937 116.80	4.2090 4.19	11.4439 11.44	-12.1804 -12.46	N.L.F.D Exact [5]
Multiplier	$10^{-4}.q.a^4/B$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$	

Table 5. Analysis of rectangular plates clamped on side BD and simply supported on the other three sides subjected to uniform distributed load of intensity q. Fig.3-c, $\Delta = \ell/40$, No of terms=7 ν = 0.3

ℓ/a	Central Deflection	Central Mx	Central My	Mx at Middle of BD	
2.0	92.7700 93.00	4.6840 4.70	9.4203 9.40	-12.0280 -12.20	N.L.F.D Exact [5]
1.5	64.5309 64.00	4.7748 4.80	6.9149 6.90	-11.1473 -11.20	N.L.F.D Exact [5]
1.4	57.5301 58.00	4.7133 4.70	6.2698 6.30	-10.7951 -10.90	N.L.F.D exact [5]
1.3	50.2124 50.00	4.6061 4.50	5.5836 5.60	-10.3538 -10.40	N.L.F.D Exact [5]
1.2	42.7098 43.00	4.4437 4.40	4.8657 4.90	-9.8093 -9.80	N.L.F.D Exact [5]
1.1	35.2005 35.00	4.2170 4.20	4.1301 4.10	-9.1489 -9.20	N.L.F.D Exact [5]
1.0	27.9074 28.00	3.9189 3.90	3.3957 3.40	-8.3643 -8.40	N.L.F.D Exact [5]
1/1.1	31.7401 32.00	4.3355 4.30	3.3256 3.30	-9.1263 -9.20	N.L.F.D Exact [5]
1/1.3	38.0115 38.00	4.9889 5.00	3.1050 3.10	-10.3038 -10.30	N.L.F.D Exact [5]
1/1.5	42.5938 42.00	5.4423 5.40	2.8577 2.80	-11.1076 -11.10	N.L.F.D Exact [5]
1/2.0	48.9088 49.00	6.0212 6.00	2.3608 2.30	-12.1128 -12.20	N.L.F.D Exact [5]
Multiplier	$10^{-4}.q.L^4/B$	$10^{-2}.q.L^2$	$10^{-2}.q.L^2$	$10^{-2}.q.L^2$	

L is the smallest value of ℓ and a

Table 6. Analysis of rectangular plates free on side BD and simply supported on the other three sides subjected to uniform distributed load of intensity q. Fig.3-d, $\Delta x = \ell/40$, No of terms=7 ν = 0.3

ℓ/a	Max. Deflection	Max. My	Central My	Central Mx	
1/2	70.9325 71.00	6.0175 6.00	3.8506 3.90	2.2327 2.20	N.L.F.D Exact [5]
2/3	96.7861 96.80	8.3262 8.30	5.5127 5.50	3.0233 3.00	N.L.F.D Exact [5]
1/1.3	109.1829 109.20	9.4370 9.40	6.3942 6.40	3.3830 3.40	N.L.F.D Exact [5]
1/1.1	122.1195 123.20	10.5976 10.70	7.4183 7.40	3.7365 3.70	N.L.F.D Exact [5]
1.0	128.5015 128.60	11.1705 11.20	7.9869 8.00	3.8976 3.90	N.L.F.D Exact [5]
1.1	134.0327 134.10	11.6672 11.70	8.5353 8.50	4.0254 4.00	N.L.F.D Exact [5]
1.3	141.6098 141.70	12.3477 12.40	9.4333 9.40	4.1683 4.20	N.L.F.D Exact [5]
1.5	146.0688 146.20	12.7483 12.80	10.1242 10.10	4.2133 4.20	N.L.F.D Exact [5]
2.0	150.6418 150.70	13.1593 13.20	11.2482 11.30	4.1411 4.10	N.L.F.D Exact [5]
Multiplier	$10^{-4}.q.a^4/B$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$	$10^{-2}.q.a^2$	

Table 7. Central deflection of square plate simply supported on four sides subjected to uniform distributed central patch load of breadth Δx and intensity Q_o.Fig.4, $\Delta x = \ell/40$, No of terms=7, ν=0.3

ρ	Central Deflection
1.00	67.4904
0.90	74.1652
0.80	80.6699
0.70	86.9535
0.60	92.9461
0.50	98.5860
0.40	103.7666
0.30	108.3473
0.20	112.2017
0.10	115.3573
0.05	115.7702
0.04	115.8671
0.02	115.9975
0.01	116.0304
Multiplier	$10^{-4}.P.a^2/B$

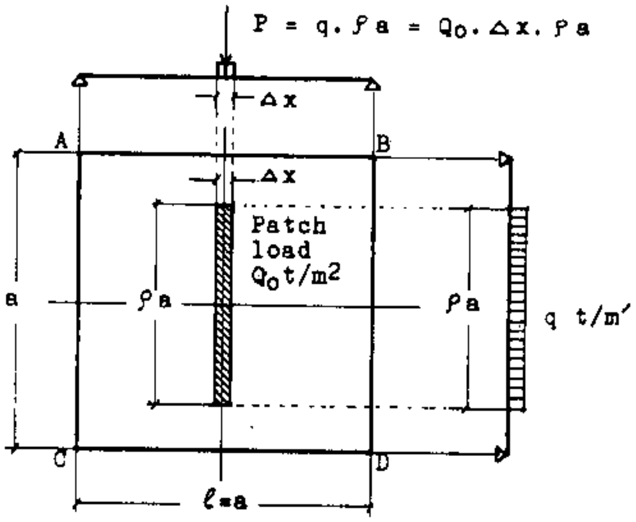


Fig. 4

Table 8. Central deflection of rectangular plates simply supported on four sides subjected to central concentrated load P of area ($\Delta x.\rho a$), ρ=0.01, Fig.4, $\Delta x = \ell/40$, No of terms=7, ν=0.3

ℓ/a	Central Deflection	
	N.L.F.D	Exact [5]
1.0	116.0304	116.00
1.1	126.7417	126.50
1.2	135.6654	135.30
1.4	148.8684	148.40
1.6	157.3188	157.00
1.8	162.5735	162.00
2.0	165.7946	165.10
Multiplier	$10^{-4}.P.a^2/B$	

CONCLUSION

A new semi analytical method for the analysis of plates in bending has been presented. The method permits the direct formulation of the problem, since it transforms the partial differential equation into an ordinary differential one, in which the simple approach of the finite difference method is applied. This method is simple in concept, easy to program, requires minimal input data, fairly small storage and short time for execution.

An analysis of elastic plates with two opposite simply supported ends is presented in this paper. The results obtained demonstrate the high accuracy of the method. The basic idea of the method can also be extended to problems for isotropic and orthotropic plates with different boundary conditions.

NOTATION

w = transverse deflection.
a = length of the nodal lines.
ℓ = length or width of the plate.
Δx = distance between the nodal lines.
E = modulus of elasticity.
t = thickness of the plate.
ν = poisson's ratio.
B = flexural rigidity.
$f_{m,k}$ = nodal line parameters.
Y_m = basic function.
q = load intensity.
$[S]_m$ = square band matrix.
$\{f\}_m$ = nodal line parameters vector.
$\{P\}_m$ = load vector.

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