Application of Fuzzy AHP on Determining the Dewatering Main Criteria Weights in Egypt

I. INTRODUCTION

It is difficult to establish a set of rules that exactly select the right dewatering system for every site or job. Guidelines can be constructed to help the selection process in many sites. The contractor/consultant should look at each construction site critically to decide whether the proposed dewatering method accommodates ground conditions, environment, and available budget. Furthermore, there are many systems available in the Egyptian market such as: Sumps, Shallow Well Systems, Wellpoint Systems, and Deep Well Systems, etc. Choosing an appropriate dewatering method from all the feasible alternatives at the planning stage is essential for the success of a project. In such a decision-making problem, the project manager or owner needs to discover decision criteria and evaluate the relative importance of each pair of the whole selected criteria. In essence, the planning for the construction of the dewatering system consists of management element including safety, and cost, site characteristics (e.g., soil conditions, groundwater conditions, excavation depth), and adjacent facility characteristics, etc. Usually, the selection process relies on the experience of contractor/consultant or their geotechnical engineers. This experience should be documented so that when an engineer leaves the company, his/her experience is
still owned by this company. There is a lack of models and tools that capture the experience of practitioners on how to select the appropriate dewatering system.

Over the last few years, artificial intelligence (AI) has been used successfully for modellng and evaluates almost all aspects of foundation constructions and related alternatives. AI approaches are useful tools to simulate a human’s decision-making process. However, AI methods usually require considerable computation time for solving the problem owing to complicated mathematical operations. Moreover, people are better at making relative comparisons as opposed to absolute judgments using AI approaches. On the other hand, the Analytical Hierarchy Process (AHP) proposed by Saaty (1980) [1] has been widely used for evaluating alternatives in the area of construction management. In spite of its popularity, the main drawback of AHP is its inability to sufficiently tackle the uncertainty and vagueness associated with the mapping of the decision-makers perception and judgment to exact ratios or numbers. Hence, Pan (2009) [2] proposed a fuzzy AHP employing triangular fuzzy numbers to describe fuzzy ratios so as to overcome the difficulty for decision-makers to express the strength of their judgments by exact values. Following their work, numerous fuzzy AHP methods have been developed and implemented (for instance; Buckely, 1985 [3]; Pan, 2009 [2]; Vahidnia, et al., 2009 [4]; Fazlollahtabar, et al., 2010 [5].

A. Objectives

This research focuses on determining the dewatering main criteria weights to facilitate the selection process of dewatering systems using the Fuzzy analytical hierarchy process.

II. FACTORS AFFECTING DEWATERING SYSTEMS SELECTION

Selecting an adequate dewatering system requires searching through many dewatering systems and their attributes. By reviewing the literature, it was found that many factors affect the selection process. Some factors may have high effect in the selection of a dewatering system, and at the same time, some factors may be considered not effective at all. For example, soil type is a very significant factor in the selection process, and on the other hand, the weather condition may not be an effective factor in Egypt due to the prevailing moderate climate throughout the entire four seasons of the year. So, to choose and identify the most important factors that have high effect in dewatering selection process, the factors had to be revised by dewatering expertise in Egypt. From the literature review, we can classify the factors affecting selection of groundwater control system into groups. These groups are basically depending on management element including safety, cost and project duration, site characteristics and adjacent facility characteristics. Each of the major group is divided into several main factors.

To determine the most important factors affecting the selection of dewatering systems in Egypt, semi-structured interviews were conducted with eighty experts representing major groundwater control companies working in Egypt. The purpose of these interviews was to gather the experts’ opinions towards the predetermined factors and their applicability in the Egyptian industry through a questionnaire survey. Visits were made to the dewatering experts in order to present the questionnaire to them and to feed them with instructions needed to fill out the questionnaire form. Subsequent visits were carried out to follow up and eventually collect the completed questionnaires.

After conducting the survey, it was found that there was variability in the experts’ results based on their experience. Therefore, these results are analyzed and combined to come out with one certain specific degree of importance for each factor to obtain the qualified factors. For each factor, the results were summed up to obtain a total weight representing the importance of each factor. Then the average weight (Aw) of each factor was determined by dividing each factor’s total weight by the number of results. Thereafter, it was supposed to decide which of the factors to be taken into consideration when selecting the dewatering system. So, the average weight obtained for each factor were summed up and divided by the number of factors to determine the factors average weight (Faw), which equaled 3.03. Then, the average weight (Aw) of each factor was compared with (Faw). Factors with (Aw) more than or equal to 3.03 were considered as qualified factors, while the others were disqualified. Table. 1 shows the qualified factors that will be taken into consideration, in the subsequent study.

<table>
<thead>
<tr>
<th>TABLE 1. QUALIFIED FACTORS AFFECTING DEWATERING PROCESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors</td>
</tr>
<tr>
<td>Ground conditions (soil type)</td>
</tr>
<tr>
<td>Excavation depth</td>
</tr>
<tr>
<td>Proximity to the nearest structure</td>
</tr>
<tr>
<td>Ground water head</td>
</tr>
<tr>
<td>Initial budget cost of the selected method</td>
</tr>
<tr>
<td>Excavation size</td>
</tr>
</tbody>
</table>

III. OUTLINE OF THE PROPOSED FAHP APPROACH

Inability of traditional AHP to deal with the imprecision and subjective in the pairwise comparison process has been improved in Fuzzy AHP. Instead of single crisp value, Fuzzy AHP used a range of values to incorporate decision maker’s uncertainty. From this range, decision maker can select the value that reflects his confidence and also he can specify his attitude like optimistic, pessimistic or moderate (Lee et. al, 2007) [6]. The proposed analysis is developed within the AHP framework. The analysis steps of the approach including the enhancements made to Pan’s model. The following subsections describe the method used in this paper.
A. Hierarchy developments

Hierarchy is the structural frame in traditional AHP, which is consisting of the overall goal, criteria and the alternatives. The goal, which is placed on the first level, is to express the conservation system and its overall satisfaction. The lowest level of the hierarchy is occupied by the alternatives. Between them are criteria and sub-criteria which achieve the overall goal.

B. Fuzzy pairwise comparisons

The typical fuzzy AHP decision problem consists of (1) a number of alternative Mi (i=1, 2, m), (2) a set of evaluation criteria Cj (j=1, 2, n), (3) a linguistic judgment aij, representing the relative importance of each pair criteria, and (4) a weighting vector, w= (w1, w2,...,wn). All the criteria on the same level of the hierarchy are compared to each of the criterion of the preceding upper level. A pairwise comparison is performed by using linguistic terms, by the decision maker. Due to the large number of alternatives and criteria, in addition to the differing nature and the uniqueness of the projects, therefore this paper divides the evaluation process into two phases. Phase one evaluates the common criteria affecting selecting an appropriate dewatering system in the Egyptian market by forming a decision group consisting of construction experts, after this the project decision team determine and evaluate the available alternatives with respect to the evaluated criteria. Because the assessment of importance by pairwise comparisons is generally subjective and ambiguous, this approach applied the triangular fuzzy number through symmetric triangular membership function. A linguistic variable is a variable whose values are linguistic terms. For the pairwise comparison, this paper defines five linguistic terms which are “Very unimportant”, “Less important”, “Equally important”, “More important” and “Very important” represented by numerical values 1–5, as shown in Table 2. A fuzzy number or linguistic variable can be represented by membership function, (x), as shown in Fig.1.

Fuzzy comparison matrix, \( \tilde{A} \), representing fuzzy relative importance of each pair elements is given by (1):

\[
\tilde{A} = \begin{bmatrix}
1 & \tilde{a}_{12} & \tilde{a}_{13} & \cdots & \tilde{a}_{1n} \\
\tilde{a}_{21} & 1 & \tilde{a}_{23} & \cdots & \tilde{a}_{2n} \\
\vdots & \vdots & \ddots & & \vdots \\
\tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1
\end{bmatrix}
\]  

(1)

Where, \( \tilde{a}_{ii} = 1 \), \( \tilde{a}_{ij} = \tilde{a}_{ji} \), \( \tilde{a}_{ij} \neq 0 \)

In the proposed approach, each reciprocal fuzzy number is characterized by its own representative membership values, rather than an inverse and reversed order of its corresponding positive fuzzy number. For example, if “is assessed as “more important”, a positive judgment that is represented by (3, 4, 5), its reciprocal element results in “less important”, a negative judgment that is characterized by (1, 2, 3). By this way, it can facilitate pairwise comparison operations and better reflect human’s judgments. To reflect particular degrees of uncertainty regarding the decision making process, the \( \alpha \)-cut concept is applied. The value of \( \alpha \) is between 0 and 1. \( \alpha=0 \) and \( \alpha=1 \), signify the degree of uncertainty is greatest and least, respectively. Selecting \( \alpha=0.50 \) indicates that environmental uncertainty is steady. Fig. 2 illustrates that a triangular fuzzy number regarding a given value can be denoted by (X \( \alpha \), L, X \( \alpha \), M, X \( \alpha \), U)

\[
X \alpha, L, X \alpha, M \text{ and } X \alpha, U \text{ represents the most-likely value, minimum value, and maximum value of the fuzzy number, respectively. The five membership functions in Fig.2 can be mathematically represented by by (2-6):}
\]

\[
X(\alpha) \text{ Very unimportant } = \begin{bmatrix}
1 \\
1 \\
2 - \alpha
\end{bmatrix}
\]  

(2)

\[
X(\alpha) \text{ Less important } = \begin{bmatrix}
1 + \alpha \\
2 \\
3 - \alpha
\end{bmatrix}
\]  

(3)

\[
X(\alpha) \text{ Equally important } = \begin{bmatrix}
2 + \alpha \\
3 \\
4 - \alpha
\end{bmatrix}
\]  

(4)

\[
X(\alpha) \text{ More important } = \begin{bmatrix}
3 + \alpha \\
4 \\
5 - \alpha
\end{bmatrix}
\]  

(5)

\[
X(\alpha) \text{ Very important } = \begin{bmatrix}
4 + \alpha \\
5 \\
5
\end{bmatrix}
\]  

(6)

Accordingly, a fuzzy comparison matrix can be defined as follows in (7):

\[
\tilde{A} = \begin{bmatrix}
1 & (\tilde{x}_{12, L}, \tilde{x}_{12, M}, \tilde{x}_{12, U}) & \cdots & (\tilde{x}_{1n, L}, \tilde{x}_{1n, M}, \tilde{x}_{1n, U}) \\
(\tilde{x}_{21, L}, \tilde{x}_{21, M}, \tilde{x}_{21, U}) & 1 & \cdots & (\tilde{x}_{2n, L}, \tilde{x}_{2n, M}, \tilde{x}_{2n, U}) \\
\vdots & \vdots & \ddots & \vdots \\
(\tilde{x}_{n1, L}, \tilde{x}_{n1, M}, \tilde{x}_{n1, U}) & \cdots & \cdots & 1
\end{bmatrix}
\]  

(7)

For instance, (X \( 12, L \), X \( 12, M \), X \( 12, U \)) in (7) shows the lower, middle and upper value of the first element compared with the second element at the higher level, respectively. To facilitate fuzzy weight computations, matrix \( \tilde{A} \) is further decomposed into three crisp matrices: the lower bound matrix \( \tilde{A}_L \), most-likely matrix, \( \tilde{A}_M \), and upper-bound matrix \( \tilde{A}_U \). These non-fuzzy comparison matrices are given by (8-10):
\[ A_L = \begin{bmatrix} 1 & x_{12,L} & \cdots & x_{1n,L} \\ x_{21,L} & 1 & \cdots & x_{2n,L} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1,L} & \cdots & \cdots & 1 \end{bmatrix} \quad (8) \]

\[ A_M = \begin{bmatrix} 1 & x_{12,M} & \cdots & x_{1n,M} \\ x_{21,M} & 1 & \cdots & x_{2n,M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1,M} & \cdots & \cdots & 1 \end{bmatrix} \quad (9) \]

\[ A_U = \begin{bmatrix} 1 & x_{12,U} & \cdots & x_{1n,U} \\ x_{21,U} & 1 & \cdots & x_{2n,U} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1,U} & \cdots & \cdots & 1 \end{bmatrix} \quad (10) \]

**Table 2: Fuzzy Importance Scale**

<table>
<thead>
<tr>
<th>Verbal judgment</th>
<th>Explanation</th>
<th>Fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very unimportant (1)</td>
<td>A criterion is strongly inferior to another</td>
<td>(1,1.2)</td>
</tr>
<tr>
<td>Less important (2)</td>
<td>A criterion is slightly inferior to another</td>
<td>(1,2,3)</td>
</tr>
<tr>
<td>Equally important (3)</td>
<td>Two criteria contribute equally to the object</td>
<td>(2,3,4)</td>
</tr>
<tr>
<td>More important (4)</td>
<td>Evaluation slightly favor one criterion over another</td>
<td>(3,4,5)</td>
</tr>
<tr>
<td>Very important (5)</td>
<td>Evaluation strongly favor one criterion over another</td>
<td>(4,5,5)</td>
</tr>
</tbody>
</table>

![Figure 1 Membership functions for linguistic values](image1.png)

![Figure 2 Triangular fuzzy intervals under α-cuts](image2.png)
C. Relative weight calculations

The Normalization of the Geometric Mean (NGM) method used in Buckley's model is applied to compute local weights and given by (Buckley’s 1985) [3] (11-12):

\[ w_i = \frac{g_i}{\sum_{i=1}^{n} g_i} \quad (11) \]

\[ g_i = \left( \prod_{j=1}^{n} a_{ij} \right)^{1/n} \quad (12) \]

In the above equations, \( g_i \) is geometric mean of criterion \( i \), \( a_{ij} \) is the comparison value of criterion \( i \) to criterion \( j \), \( w_i \) is the \( i \)-th criterion's weight, where \( w_i > 0 \) and, \( 1 \leq i \leq n \). The maximum eigenvalue \( \lambda_{max} \) is calculated as follows in (13):

\[ \lambda_{max} = Q w^T \quad (13) \]

Where \( Q \) is the sum of each column of matrix, \( Q \) is a vector size equal (n×1) and \( w^T \) is the normalized vector (1×n). Similarly, the weight of the k-th sub-criteria (\( k = 1, 2, \ldots K \)), with regard to the j-th main criterion, \( s_{kj} \), can be obtained by using the above procedure. Accordingly, the synthetic weight of the k-th sub-criterion (\( s_k \)) can be determined as follows by (14):

\[ s_k = w_j \times s_{kj} \quad (14) \]

By the same manner, the weight of the ith alternative (=1, 2, m) with respect to the k-th sub-criterion (\( e_{ik} \)) can be obtained. Consequently, the overall weight of the ith alternative (\( r_i \)) is given by (15):

\[ r_i = s_k \times e_{ik} \quad (15) \]

Finally, the overall weight of the ith alternative regarding all sub-criteria, \( R_i \), can be found by the following (16):

\[ R_i = \sum_{k=1}^{K} s_k \times e_{ik} \quad (16) \]

D. CONSISTENCY CHECKS

A comparison matrix is consistent if the maximum eigenvalue \( \lambda_{max} = n \), where \( n \) is the matrix size. The consistency index (CI) is used as a measurement of the deviation of the judgments expressed and defined as follows (Satty 1980) [1]:

\[ CI = \frac{\lambda_{max} - n}{n - 1} \quad (17) \]

The consistency ratio (CR) with an index calculated from the same values from randomly generated matrices, and is given by (18):

\[ CR = \frac{CI}{RI} \quad (18) \]

E. SYNTHESIS OF GROUP DECISIONS

Once the relative weight is calculated, it is required to aggregate manifold evaluators’ opinions into one. This paper employs the average of weights, which is much faster than most related methods and easy to implement. Defuzzification plays an important role when a conversion of a fuzzy number to a single representative value is required, the three fuzzy numbers lower, medium, and upper values are defuzzified into one crisp value as follows in (19-21) (Kwonga, et. al., 2003) [7]:

\[ w_L = \frac{\sum_{i=1}^{n} w_{L,i}}{n} \quad (19) \]

\[ w_M = \frac{\sum_{i=1}^{n} w_{M,i}}{n} \quad (20) \]

\[ w_U = \frac{\sum_{i=1}^{n} w_{U,i}}{n} \quad (21) \]

The three fuzzy numbers lower, medium and upper values can be defuzzified into one crisp value using the following (22). (Kwonga, et. al., 2003) [7]:

\[ M_{crisp} = \frac{w_L + 4 \times w_M + w_U}{6} \quad (22) \]

IV. CRITERIA HIERARCHY DEVELOPMENT

A criteria hierarchy was constructed by breaking down the decision problem. Nodes in the hierarchy represent main criteria that have sub-criteria as shown in Fig.3.

A. CONSTRUCTING FUZZY COMPARISON MATRIX

Once the hierarchy is established, the pairwise comparison evaluation took place by a decision group consisting of ten experts. Based on Fig.3, a series of questionnaires were designed and used to direct these experts to provide their comparison judgments using the linguistic scale defined in Fig.1. Comparisons were performed separately for each criterion in the hierarchy. Specific questionnaires for the three levels of the hierarchy were developed. As an example, the questionnaire used to evaluate sub criteria is shown in Table.

The comparison results of all main criteria with regard to the overall goal and sub criteria regarding the main criteria can be found in Table 4, Table 5, and Table 6 respectively.

B. ASSIGNING CRITERIA WEIGHTS

To better illustrate the procedure of this proposed model, only the pairwise comparison judgments regarding soil condition (B11), excavation depth (B12), proximity of the nearest structure (B13), and depth of water below ground level (B14) with respect to safety (B1) given by the first expert are
presented. First, applying the fuzzy numbers defined in Fig.1 and (2-6), and the fuzzy comparison matrices of under α = 0.50 are given below:

<table>
<thead>
<tr>
<th></th>
<th>B11</th>
<th>B12</th>
<th>B13</th>
<th>B14</th>
</tr>
</thead>
<tbody>
<tr>
<td>B11</td>
<td>1</td>
<td>4.5</td>
<td>3.5</td>
<td>4.5</td>
</tr>
<tr>
<td>B12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B13</td>
<td>1.5</td>
<td>4.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B14</td>
<td>1</td>
<td>4.5</td>
<td>4.5</td>
<td>1</td>
</tr>
</tbody>
</table>

The first row in $\tilde{A}_1$ represents the relative preference given by first expert. Applying Eqs.(8,9,10) the lower bound matrix $\tilde{A}_L$ and eigenvector estimation are derived as shown in Table.7.

Similarly, the matrices $\tilde{A}_1^*$ and $\tilde{A}_2^*$ with regard the first expert and the second expert can also be determined. It can be found that the eigenvector of $\tilde{A}_1$ results in (0.38, 0.13, 0.21, 0.28). These four values ordering from left to right represent the weight of B11 corresponding to B12, B13, and B14, respectively. By the same manner, the eigenvectors for $\tilde{A}_M$ and $\tilde{A}_1^*$ are given by (0.39, 0.12, 0.22, 0.27) and (0.36, 0.15, 0.23, 0.27), respectively. Therefore, the eigenvector of B11 (0.38, 0.39, 0.36) indicating the lower, medium, and upper relative weights of B11, respectively. Likewise, the relative weights of B12, B13, and B14 result in (0.13, 0.12, 0.15), (0.21, 0.22, 0.23), and (0.28, 0.27, 0.27).

C. CONSISTENCY CHECKS

After estimating relative weights, the consistency checks take place. By applying (11) to calculate the eigenvalue $\lambda_{\text{max}}$ for the medium matrix regarding the first experts can be determined as follows:

$$\lambda_{\text{max}} = (4.5x0.38+14.5x0.13+10x0.21+7.5x0.28) = 7.80$$

Using (15) and (16) to get CI and CR, respectively. Where RI is 0.9 corresponding to n= 4, by (Saaty, 1980) [1] as follows:

$$\text{CI} = (7.80- 4)/ (4-1) = 1.27.$$  
$$\text{CR} = 1.27/0.90 = 1.41.$$ 

Note that all the CI and CR values indicating that the comparison assessments based on the ten experts are consistent (Khader, 2009) [8].

D. SYNTHESIS OF GROUP DECISIONS

It is now needed to group the ten different experts’ measurements. Concerning soil condition by using (19-21) as follows:

$$\text{WL} = (3.65/10) = 0.365$$ 
$$\text{WM} = (3.71/10) = 0.371$$ 
$$\text{WM} = (3.71/10) = 0.371$$ 

Accordingly, the weight of soil condition can be estimated by using (21) as follows:

$$M_{\text{soil condition}} = (0.365 + 0.371 + 0.345) = 1.081$$

By using the foregoing procedures, the final main criteria weights regarding the overall goal and under $\alpha = 0.50$ are safety (0.68) and cost (0.32). Likewise, the final sub criteria weights regarding the main criteria under $\alpha = 0.0$, $\alpha = 0.50$, and $\alpha = 1.0$ result in Table 8.

Using the main criteria weights (Table.8) and (12) to get the synthetic weights of sub criteria. The results are shown in Table. 9. For example, the synthetic weight of soil condition at $\alpha = 0.50$ is computed as:

$$\text{Soil condition} = 0.36*0.67 = 0.24$$

It can be noted that the weights at the different degrees of $\alpha$-cuts are similar values due to the symmetric nature of the triangular fuzzy numbers. So, the charge of $\alpha$-cuts doesn’t change the results.

V. CONCLUSION

The conclusions obtained from this study can be summarized as follows:

1. Through interviews with construction industry experts, the most effective factors that affect the selection of the appropriate dewatering system were identified by experts, these factors are Ground conditions (soil type), Excavation depth. The proximity to the nearest structure, The depth of groundwater head, Excavation size, and The initial budget for the dewatering method.

2. Based on the data collected from 80 construction participants, a Fuzzy AHP analysis is developed to help contractors/consultants to determining the dewatering main criteria weights to facilitate the selection process of dewatering systems in construction field in the Egyptian market.

3. A fuzzy AHP analysis for selecting the appropriate dewatering system was developed and employed fuzzy linguistic terms for facilitating the comparisons between the subjective criteria since the decision makers feel much comfortable with using linguistic terms rather than providing exact crisp judgments.

4. The analysis employed the $\alpha$-cut concept to reflect various degrees of uncertainty in the decision-making process. It can be noted that the weights at the different degrees of $\alpha$-cuts are similar and very close due to the symmetric nature of the triangular fuzzy numbers. So, the charge of $\alpha$-cuts doesn’t change the results.

5. Finally, the proposed analysis is found to be capable of dealing with uncertainty factors, imprecise information, judgments and analyzing relative weights among criteria.
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Figure 3 The hierarchy for determining the dewatering main criteria weights

Table 3 Questionnaire used to evaluate sub criteria (Pan, 2009)

<table>
<thead>
<tr>
<th>Question</th>
<th>Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1. How important is Ground conditions (soil type) when it is compared to Excavation depth?</td>
<td>5</td>
</tr>
<tr>
<td>Q2. How important is Ground conditions (soil type) when it is compared to Proximity to the nearest structure (m)?</td>
<td></td>
</tr>
<tr>
<td>Q3. How important is Ground conditions (soil type) when it is compared to Ground water head?</td>
<td></td>
</tr>
<tr>
<td>Q4. How important is Proximity to the nearest structure (m) when it is compared to Excavation depth?</td>
<td></td>
</tr>
<tr>
<td>Q5. How important is Proximity to the nearest structure (m) when it is compared to Ground water head?</td>
<td></td>
</tr>
<tr>
<td>Q6. How important is Excavation depth when it is compared to Ground water head?</td>
<td></td>
</tr>
<tr>
<td>Q7. How important is Initial budget cost of the selected method when it is compared to Excavation size?</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Assessment results of the main criteria regarding the overall goal

<table>
<thead>
<tr>
<th>Pairwise criteria</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety vs. cost</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5 Assessment results of the sub-criteria with respect to safety

<table>
<thead>
<tr>
<th>Pairwise criteria</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground conditions vs Excavation depth</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<tr>
<td>Ground conditions vs Proximity to the nearest structure</td>
<td>4</td>
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<td>4</td>
</tr>
<tr>
<td>Ground conditions vs Ground water head</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
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</tr>
<tr>
<td>Proximity to the nearest structure vs Excavation depth</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<td>5</td>
</tr>
<tr>
<td>Proximity to the nearest structure vs Ground water head</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>Excavation depth vs Ground water head</td>
<td>1</td>
<td>1</td>
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<td>$\lambda_{max}$</td>
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<td>8.47</td>
<td>8.87</td>
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<td>1.49</td>
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<td>1.49</td>
<td>1.62</td>
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<td>1.53</td>
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<td>1.81</td>
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<td>1.86</td>
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<td>1.66</td>
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Table 6 Assessment results of the sub-criteria with respect to cost

<table>
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<tr>
<th>Pairwise criteria</th>
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<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
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</thead>
<tbody>
<tr>
<td>Budget cost vs Excavation size</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
<td>$\lambda_{max}$</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
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<td>2.00</td>
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<td>2.00</td>
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<td>0.00</td>
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### Table 7

**Lower Bound Matrix and Eigenvector Calculations**

<table>
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<tr>
<th></th>
<th>B11</th>
<th>B12</th>
<th>B13</th>
<th>B14</th>
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</thead>
<tbody>
<tr>
<td>B11</td>
<td>1</td>
<td>4.5</td>
<td>3.5</td>
<td>4.5</td>
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<tr>
<td>B12</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>B13</td>
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<td>4.5</td>
<td>1</td>
<td>1</td>
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<tr>
<td>B14</td>
<td>1</td>
<td>4.5</td>
<td>4.5</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
(1 \times 4.5 \times 3.5 \times 4.5)^{1/4} = 2.90 \\
(1 \times 1 \times 1 \times 1)^{1/4} = 1.00 \\
(1.5 \times 4.5 \times 3.1 \times 1)^{1/4} = 1.61 \\
(1 \times 4.5 \times 4.5 \times 1)^{1/4} = 2.12
\end{array}
\]

\[
\begin{array}{c}
(2.9/7.63) = 0.38 \\
(1/7.63) = 0.13 \\
(1.61/7.63) = 0.21 \\
(2.12/7.63) = 0.28
\end{array}
\]

\[
\sum = 4.5 \quad 14.5 \quad 10 \quad 7.5 \quad 7.63 \quad 1
\]

### Table 8

**Sub-criteria weights regarding the main criteria**

<table>
<thead>
<tr>
<th></th>
<th>Safety</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Ground conditions</td>
<td>Excavation depth</td>
</tr>
<tr>
<td>0</td>
<td>0.36</td>
<td>0.13</td>
</tr>
<tr>
<td>0.5</td>
<td>0.37</td>
<td>0.13</td>
</tr>
<tr>
<td>1</td>
<td>0.37</td>
<td>0.12</td>
</tr>
</tbody>
</table>

### Table 9

**Synthetic sub-criteria weights regarding \( \alpha = 0, 0.5 \) and 1**

<table>
<thead>
<tr>
<th></th>
<th>Safety</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Ground conditions</td>
<td>Excavation depth</td>
</tr>
<tr>
<td>0</td>
<td>0.2412</td>
<td>0.0871</td>
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<td>0.5</td>
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<td>1</td>
<td>0.2553</td>
<td>0.0828</td>
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</table>

### References